Online appendix for "Advising the management"

The online appendix presents the analysis of the extensions of the model.

A.1 Endogenous ownership structure

In this extension, we endogenize the decisions of the N potentially informed agents to become shareholders. Specifically, we consider the setup described in Section 5.1, where agent i's utility from holding stake α_i is given by (13). First, all N agents trade in a competitive market; then, the firm forms an optimal advisory committee B; and after that, the game proceeds as in the basic model. We assume that constant u_0 in the payoff specification U_i is sufficiently high, so that the equilibrium share price is positive.[27]

Note that this setup assumes that agents' trading is based on their preferences and prior beliefs regarding the firm's decision, but that agents do not trade again ex-post, after learning their private signals. There are two arguments for this simplifying assumption. One is tractability: the model in which agents both trade on private information and then decide whether to reveal it to the manager is very difficult to analyze. The second argument is more fundamental. As discussed in the literature review, prior research has extensively studied how market trading incorporates agents' private information into real decisions through its impact on *prices*. In contrast, our contribution is to examine how trading incorporates agents' information into real decisions through a different channel, *communication*: trading determines the firm's shareholder base and thus, determines which agents communicate their information to the manager via voting or being on the board. Assuming that agents to not trade based on private information allows us to abstract from the price channel and focus on the more novel communication channel.

Finally, note that another interpretation of this setup is that b_i and ρ_i capture agents' general preferences and beliefs regarding the firm (e.g., how congruent they are with the overall strategic direction the management is pursuing), and are not decision-specific. In this interpretation, the firm's shareholder base S captures the firm's long-term shareholders, and hence it is reasonable to assume that such long-term shareholders' ownership stakes are not affected by more transitory, decision-specific private information.

Analysis. We solve the model by backward induction. The analysis at the communication stage remains unchanged, and hence we only need to consider the trading stage. Using (13), the optimal ownership stake of agent i is

$$\alpha_i(p) = \max\left\{\frac{\mathbb{E}_i[U_i] - p}{\lambda}, 0\right\}. \tag{30}$$

As expected, a larger holding cost λ decreases the agent's demand for shares, while higher expected utility $\mathbb{E}_i[U_i]$ from holding each share increases his demand. In particular,

[27] The only role of this assumption is to make the interpretation more intuitive, but none of the results depends on u_0.

as Lemma 2 and expressions (11)-(12) for $\mathbb{E}_i[U_i]$ demonstrate, the shareholder's expected utility is higher when the manager learns more from other shareholders. This implies that an agent's optimal stake in the firm is affected by the firm' overall ownership structure, which will be important for the results that follow.

Market clearing implies $1 = \sum_{i=1}^{N} \alpha_i(p) = \sum_{i \in S} \frac{\mathbb{E}_i[U_i]-p}{\lambda}$, and hence the price is

$$p^* = \frac{1}{|S|}\left(\sum_{i \in S} \mathbb{E}_i[U_i] - \lambda\right). \tag{31}$$

Similarly to the basic model, the implications under endogenous ownership structure are, as we show next, very different depending on whether agents differ in their preferences or beliefs. We consider each of these cases separately.

A.1.1 Heterogeneity in beliefs

Suppose agents have heterogeneous beliefs: $\rho_i \neq \rho_m$ for some i, but $b_i = b_m = b$ for all i. Our first observation is that there always exists an equilibrium in which all informed agents become shareholders and the manager's action reflects all available information. Indeed, according to the proof of Proposition 3, if the shareholder base is $S = \{1, ..., N\}$, and hence the advisory vote includes all informed agents, there exists an equilibrium in which all shareholders report their information truthfully. Intuitively, if the manager learns all the N signals, there is no ex-post disagreement between him and any of the shareholders, which makes truthful communication by all shareholders indeed optimal. Moreover, because of no ex-post disagreement between agents, they all value the firm in the same way and thus acquire equal stakes, $\alpha_i = \frac{1}{N}$. In addition, there can also exist an inefficient equilibrium, in which only a subset of the agents become shareholders, the manager's decision is not based on all the information, and total holding costs are larger. The share price in this inefficient equilibrium is lower than in the efficient equilibrium. Specifically:

Proposition 7. *Suppose agents have heterogeneous beliefs. There always exists an efficient equilibrium in which all N agents become shareholders, acquire equal stakes $\alpha_i = \frac{1}{N}$ in the firm, the optimal committee is the set of all shareholders, and the efficient action $a = b + Z$ is undertaken. However, there can also exist an inefficient equilibrium, in which a strict subset of agents become shareholders, an inefficient action $a \neq b + Z$ is undertaken, and the share price is lower than in the efficient equilibrium.*

The source of equilibrium multiplicity is the following. If only a subset of agents are expected to become shareholders and communicate their information to the manager, there is still ex-post disagreement between the manager and the shareholders. Realizing this at the trading stage, shareholders who have substantially different beliefs from those of the manager (i.e., have large enough $|\rho_i - \rho_m|$), do not buy shares, making this equilibrium self-fulfilling.

One interesting implication of the above results is for the effect of index funds in the firm's ownership structure. There is an ongoing active debate about the role of passive (index) asset managers in corporate governance. Some market participants have expressed concerns that the presence of passive investors weakens governance and even proposed that they should be restricted from voting, noting that they may lack the adequate incentives to become informed. Others argue that index funds actively engage with their portfolio companies, even more so than investors who can easily exit.[28] An implication of our analysis is that the presence of index funds can make advisory shareholder voting more effective. Indeed, suppose investors expect that a large fraction of the firm's shareholders will consist of index funds, whose stake in the firm will not depend on whether their fund managers agree or disagree with the firm's CEO. In this case, the inefficient equilibrium may again cease to exist due to positive externalities in communication, as long as these passive investors are sufficiently informed and have diverse prior beliefs. This logic highlights a positive effect of passive investors on the voting of *other* shareholders, with a caveat that it relies on passive investors being informed.

A.1.2 Heterogeneity in preferences

Recall that when agents have heterogeneous preferences, communication externalities are negative. Since positive externalities were the key reason for equilibrium multiplicity under heterogeneous beliefs, it is intuitive to expect that equilibrium under heterogeneous preferences is unique. This is indeed the case, as shown in Proposition 8. In this equilibrium, unlike in the efficient equilibrium under heterogeneous beliefs, the shareholder base is generally restricted and consists of agents whose preferences are sufficiently aligned with those of the manager (small $|b_m - b_i|$). This is because under heterogeneous preferences, there are always ex-post disagreements between the manager and shareholders about the optimal course of action, regardless of the amount of information conveyed.

Proposition 8. *Suppose agents have heterogeneous preferences and for at least one agent the preference misalignment with the manager is sufficiently large, $|b_m - b_i| > \frac{1}{2}c_i$. In the unique equilibrium, agent i becomes a shareholder if and only if $|b_m - b_i|$ is sufficiently low. The equilibrium number of shareholders increases with λ. There exists cutoff $\lambda^* \in (0, \infty)$, such that:*

- *if $\lambda \leq \lambda^*$, then $B^* = S \setminus \{m\}$, i.e., the optimal advisory body includes all non-manager shareholders;*
- *if $\lambda > \lambda^*$, then $B^* \subset S \setminus \{m\}$, i.e., the optimal advisory body is a strict subset of non-manager shareholders.*

[28] See "Vanguard, Trian and the problem with 'passive' index funds", *Forbes*, HBS Working Knowledge (Feb 15, 2017) and "The case against passive shareholder voting," *CLS Blue Sky Blog* (August 2, 2017) for negative views on the governance role of passive investors; and "Passive investment, active ownership," *Financial Times* (April 6, 2014) for a positive view.

As Proposition 8 shows, the key factor that determines the optimal advisory body is the holding cost λ. Intuitively, λ affects how concentrated vs. dispersed the firm's ownership structure is, and hence how close shareholders' preferences are to those of the management. If holding costs are small, the equilibrium features concentrated ownership: investors with similar preferences to those of the manager end up holding large stakes in the firm. Under such concentrated ownership, shareholder voting is effective in its advisory role, in the sense that all shareholders share their information with the manager. Hence, the optimal advisory committee includes all of the firm's shareholders. In contrast, if holding costs are large, concentrated ownership becomes too expensive, so the firm's shareholder base is relatively diverse and includes many shareholders whose preferences are misaligned with those of the manager. In this case, advisory voting is no longer effective: there is no equilibrium in which all of the shareholders truthfully convey their views to the manager. Hence, advice is optimally provided by a board consisting of a subset of the firm's shareholders, rather than through an advisory vote.

A.2 Expertise of the manager

To analyze the effect of the manager's expertise on the board's advising effectiveness, we consider a small extension of the basic model by assuming that the manager knows a subset $\mathcal{M}$ of signals $\{\theta_i, i \neq m\}$ in addition to signal θ_m. If $\mathcal{M} = \emptyset$, then the manager only knows his private signal θ_m, as in the basic model. If $\mathcal{M} \neq \emptyset$, then the manager also knows some signals of the other agents, in addition to his private signal θ_m. We interpret an expansion/contraction of $\mathcal{M}$ (i.e., addition/removal of signals to/from $\mathcal{M}$) as an increase/decrease in the manager's expertise.[29] For simplicity, we assume that the costs of information acquisition are sufficiently small that all directors acquire information and focus on the effects of communication frictions.

Let us fix all parameters of the model and consider any board B, i.e., a subset of agents $\{1, ..., N\}$. We say that board B is efficient at providing advice to the manager if truthful communication by all members of B to the manager is an equilibrium. Without loss of generality, consider boards in which each member has some information that the manager does not already know: $\{\theta_i, i \in B\} \cap \mathcal{M} = \emptyset$.[30] The next proposition shows how the advising effectiveness of the board varies with the manager's information:

Proposition 9 (Manager's expertise). *Consider any board B with $\{\theta_i, i \in B\} \cap \mathcal{M} = \emptyset$.*

1. *If $|b_m - b_i| \leq \frac{1}{2}c_i$ $\forall i \in B$ and board B is efficient at providing advice under $\mathcal{M}$, then it is also efficient at providing advice if set $\mathcal{M}$ expands, i.e., as the manager becomes more informed.*

[29] An alternative way to model higher managerial expertise is to increase c_m, while normalizing $\sum_{i=1}^{N} c_i = 1$. This model leads to the same result. However, it cannot be used to analyze how firm value changes with the manager's expertise because a change in c_m changes the distribution of state Z in this formulation.

[30] Clearly, there is no benefit of adding an agent to the advisory board if he has no information that the manager does not already possess.

2. *If* $|\rho_m - \rho_i| \leq \frac{1}{2}$ $\forall i \in B$ *and board* B *is efficient at providing advice under* $\mathcal{M}$*, then it is also efficient at prodiving advice if set* $\mathcal{M}$ *contracts, i.e., as the manager becomes less informed.*

Intuitively, as the manager becomes more informed, there is less information relevant for the decision that neither the manager nor the board knows. This has different effects depending on the nature of communication frictions. When the friction of heterogeneous beliefs is relatively more important, the key consequence is the increase in the manager's congruence because there is less information that the manager and board members can disagree about. As a result, board members have stronger incentives to truthfully reveal their information to the manager, explaining the first statement of the proposition. In contrast, when communication is primarily hampered by conflicts of interest, greater managerial expertise decreases directors' costs of misreporting their information because the manager is expected to react less to each director's message. This explains the second statement of the proposition.

Since Proposition 8 applies to any committee, both optimal and suboptimal ones, we can conclude that the manager's expertise enhances the advisory role of the board if communication is mainly hampered by disagreement between the manager and board members, but impedes its advisory role if communication is mainly hampered by conflicts of interest.

Moreover, the latter effect implies that it can be Pareto improving to appoint a less informed manager. Intuitively, a less informed manager has worse private information, but obtains more information from the board members. The numerical example below illustrates that this second effect can dominate.

Example (Manager's expertise can be harmful).

There is a manager and 100 other agents, divided into two groups. The parameters are: $c_m = 0.3$, $b_m = 0.0475$, $b_i = 0$ $\forall i \neq m$, $\rho_i = \rho = 2$, $\tau = 4$. The first group are the relatively more informed agents: it contains $\bar{N} = 10$ agents with $c_i = \bar{c} = 0.05$. The second group are the relatively less informed agents: it contains $\underline{N} = 90$ agents with $c_i = \underline{c} = 0.2/90$ (for simplicity, $\sum_{i=1}^{N} c_i$ is normalized to one). Thus, the manager's signal θ_m has weight 30% in the state, the sum of all signals of the more informed agents has weight 50% in the state, and the sum of all signals of the less informed agents has weight 20% in the state. If the manager only knows signal θ_m (i.e., $\mathcal{M} = \emptyset$), then the optimal board is comprised of 5 agents from the first group. These agents report their signals to the manager truthfully, and the implied expected payoff of each agent $i \neq m$ is $V = -0.0093$ (the payoff of the manager is higher by b_m^2). In contrast, if the manager also knows one of the signals with $c_i = 0.2/90$ and the board is comprised of 5 agents from the first group, the IC constraint is violated and truthful revelation by all members of this board is not an equilibrium. Instead, the optimal board is comprised of 4 agents from the first group. The implied expected payoff of each agent $i \neq m$ is $V = -0.0109$, which is lower than if the manager is less informed. Thus, a reduction in the manager's information improves the values of all agents by promoting more efficient communication.

In contrast, the fact that a more informed manager improves the advisory role of the board when conflicts of interest play a small role relative to differences in beliefs, implies that in this case, both effects act in the same direction. As the manager becomes more informed, he both makes better decisions due to his own information and can also get better advice from other agents.

A.3 Heterogeneous interpretation of information

In this extension, we analyze the model with heterogeneous interpretations of signals, described in Section 5.3. Specifically, we assume that given a set of signals $\{\theta_j, j = 1, ..., N\}$, shareholder i believes that the state of the world is equal to (14).

Because the analysis of this model repeats the analysis of the basic model, we leave the details for the proofs and only highlight the novel parts. After the communication stage, if the manager knows subset R of signals, his optimal action is

$$a_m(\theta_R) = \gamma c\theta_m + \sum_{i\in R, i\neq m} c\theta_i + \frac{\rho_m + \sum_{i\in R}\theta_i}{\tau + |R|}(N - |R|)\, c. \tag{32}$$

As (32) shows, the manager places a higher weight on his signal θ_m than on any other signal. In contrast, the optimal action from shareholder i's perspective places a higher weight on signal θ_i than on any other signal.

The next proposition characterizes the constraint for truthful communication by agent i and the optimal advisory body:

Proposition 10. *(i) Suppose that the manager learns subset R of signals (which includes his own signal θ_m but not θ_i) and does not know all the other signals, $-R$. Then, agent i reports his signal truthfully if and only if*

$$\begin{array}{ll} \text{for } \rho_i \leq \rho_m: & \rho_m - \rho_i \leq \frac{1}{2}\frac{N+\tau}{N-|R|-1} + (\gamma - 1)\frac{\tau-\rho_i}{\tau+1}\frac{\tau+1+|R|}{N-|R|-1}, \\ \text{for } \rho_i > \rho_m: & \rho_i - \rho_m \leq \frac{1}{2}\frac{N+\tau}{N-|R|-1} + (\gamma - 1)\frac{\rho_i}{\tau+1}\frac{\tau+1+|R|}{N-|R|-1}. \end{array} \tag{33}$$

(ii) If $|\rho_m - \rho_i|$ is sufficiently high for each i, there exists a cutoff on committee size $N^{\gamma}_{\min} \geq 1$, which is weakly decreasing in γ, such that an equilibrium where all members of committee B truthfully communicate to the manager does not exist if $|B| < N^{\gamma}_{\min}$.

(iii) For any $\gamma \geq 1$, the optimal committee is the entire set of agents.

Proposition 10 shows that the overconfidence of each agent in the importance of his signal increases his incentives to report truthfully: the right-hand side of (33) increases in γ. Intuitively, if the agent perceives his signal to be more important than it actually is, he perceives lying to be costlier. Proposition 10 also shows that when agents have heterogeneous beliefs but common objectives, communication has positive externalities, as in the baseline model: the right-hand side of (33) increases as R expands. As a consequence of positive externalities, the optimal committee is again the entire set of informed agents (part (iii) of the proposition).

Overall, the key requirement for positive communication externalities is that communication of other agents to the manager moves the manager's and the agent's beliefs about the state closer to each other, which is consistent with heterogeneous interpretations of signals and does not require complete convergence of beliefs under full information.

A.4 A more general specification of differences in beliefs

Consider the baseline model but suppose that each agent i has a prior belief that φ is distributed according to Beta distribution with parameters $(\rho_i, \tau_i - \rho_i)$. We derive the incentive compatibility constraint for this specification and show that the main results of Section 3 (positive externalities of communication and the optimal committee being the full set of agents) extend to this specification.

Proposition 11. *Suppose that the manager learns subset R of signals (which includes his own signal θ_m but not θ_i) and does not know all the other signals, $-R$. Then agent i reports his signal truthfully if and only if*

$$\left|(b_m - b_i)\frac{(\tau_i+1)(\tau_m+|R|+1)}{\sum_{j\in -R\setminus\{i\}} c_j} + \frac{\tau_i-\tau_m}{2} + \rho_m(\tau_i+1) - \rho_i(\tau_m+1)\right|$$
$$\leq \frac{(\tau_i+1)c_i(\tau_m+|R|+1)}{2\sum_{j\in -R\setminus\{i\}} c_j} + \frac{\tau_i+1}{2} - \frac{\tau_i-\tau_m}{2}. \quad (34)$$

In particular, when $b_i = b_m \ \forall i$, it reduces to

$$\left|\frac{\tau_i-\tau_m}{2} + \rho_m(\tau_i+1) - \rho_i(\tau_m+1)\right| \leq \frac{(\tau_i+1)c_i(\tau_m+|R|+1)}{2\sum_{j\in -R\setminus\{i\}} c_j} + \frac{\tau_i+1}{2} - \frac{\tau_i-\tau_m}{2}. \quad (35)$$

This inequality is relaxed as R expands and is always satisfied if R includes all signals other than θ_i.

Since the right-hand side of (35) increases as R expands, information transmission has positive externalities as in the baseline model: more information revealed to the manager by some agents encourages other agents to report their information truthfully. For this reason, the argument of Proposition 2 also applies to this extended model. Since the right-hand side of (35) is infinite when R includes all signals other than agent i's signal, the argument of Proposition 3 applies to this extended model.

A.5 Model with common priors and two private signals

In this section, we consider a variation of our model in which agents have common priors, and each agent gets two private signals but can send only one binary message and hence effectively can communicate at most one signal. We show that this model with common priors and two private signals, one of which cannot be communicated, is different from the model with heterogeneous priors. In particular, we characterize the incentive compatibility conditions for truthful communication (of one of the two signals) and show that this model does not feature positive externalities of communication, which is a general property of the model with heterogeneous prior beliefs in the paper.

The reason why the model with two private signals, one of which cannot be communicated, is not equivalent to the model with different prior beliefs is because of the following difference between a private signal and a prior belief. Intuitively, if the decision-maker has a different prior belief from an agent, then the decision-maker believes that her prior is correct, while the agent's prior is incorrect, which creates an effective conflict of interest and harms communication. In contrast, if the agent possesses a signal that she cannot communicate to the decision-maker, the decision-maker believes that the agent's signal is relevant for the decision. Thus, different priors prevent informative communication unless the decision-maker is expected to obtain enough signals from other agents, so that the effect of priors becomes less relevant. In contrast, if the agent has the same prior as the decision-maker, even if he can communicate at most a binary message, this binary message will be informative since some information communicated to the decision-maker is better than none.

In particular, assume that all agents share the same prior $\rho_i = \rho_m = \rho$, but each of them receives two informative private signals. The state of the world is equal to the weighted sum of $2N$ signals:

$$Z = \sum_{i=1}^{N} c_i \theta_i + \sum_{j=N+1}^{2N} c_j \theta_j, \tag{36}$$

where agent i privately learns two signals, θ_i and θ_{i+N}. However, suppose that the agent can only send one binary message $m_i \in \{0, 1\}$. We will focus on equilibria in which message m_i is either fully informative about one of the two signals (about θ_i if $c_i > c_{i+N}$, and about θ_{i+N} if $c_i < c_{i+N}$) or fully uninformative. Thus, each agent i is able to communicate at most one of his two signals. Without loss of generality, suppose that $c_i > c_{i+N}$, so that signals from θ_1 to θ_N can be potentially communicated, while signals from $N + 1$ to $2N$ cannot be. For simplicity, we consider the special case in which φ is distributed uniformly over $[0, 1]$ (i.e., $\rho = 1$ and $\tau = 2$) and ignore costs of information acquisition (i.e., we assume that these costs are infinitesimal).

Consider any informed agent i and suppose that the manager knows the subset $R \subset \{1, ..., N\}$ of signals, where R includes the manager's own signal θ_m but not agent i's signal θ_i. The optimal action of the manager is the same as in the baseline model: If agent i reveals his signal truthfully,

$$a_m(\theta_R, \theta_i) \equiv b_m + c_i \theta_i + \sum_{k \in R} c_k \theta_k + \frac{1 + \theta_i + \sum_{k \in R} \theta_k}{3 + |R|} \sum_{j \in -R \setminus \{i\}} c_j, \tag{37}$$

and if agent i misreports, the manager's action is

$$a_m(\theta_R, 1-\theta_i) \equiv b_m + c_i(1-\theta_i) + \sum_{k\in R} c_k\theta_k \; + \frac{1+(1-\theta_i)+\sum_{k\in R}\theta_k}{3+|R|} \sum_{j\in -R\backslash\{i\}} c_j. \tag{38}$$

Agent i communicates signal θ_i truthfully if and only if

$$\sum_{\theta_{-i}} \left[(a_m(\theta_R,\theta_i) - Z - b_i)^2 - (a_m(\theta_R, 1-\theta_i) - Z - b_i)^2\right] P_i(\theta_{-i}|\theta_i, \theta_{i+N}) \leq 0 \tag{39}$$

for all $(\theta_i, \theta_{i+N}) \in \{0,1\}^2$.

Note that

$$a_m(\theta_R,\theta_i) - a_m(\theta_R, 1-\theta_i) = (2\theta_i - 1)\left(c_i + \frac{\sum_{j\in -R\backslash\{i\}} c_j}{|R|+3}\right).$$

Hence, plugging (37) and (38) into (39), we get that (39) is equivalent to

$$(2\theta_i - 1)\sum_{\theta_{-i}} (a_m(\theta_R,\theta_i) + a_m(\theta_R, 1-\theta_i) - 2Z - 2b_i)\, P_i(\theta_{-i}|\theta_i, \theta_{i+N}) \leq 0$$

or

$$(2\theta_i - 1)\sum_{\theta_{-i}} P_i(\theta_{-i}|\theta_i, \theta_{i+N}) \begin{pmatrix} 2(b_m - b_i) + c_i(1-2\theta_i) - 2c_{i+N}\theta_{i+N} \\ -2\sum_{j\in -R\backslash\{i,i+N\}} c_j\theta_j + \frac{2\cdot\sum_{k\in R}\theta_k + 3}{|R|+3}\sum_{j\in -R\backslash\{i\}} c_j \end{pmatrix} \leq 0. \tag{40}$$

Next, we use

$$\sum_{\theta_{-i}} \left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\theta_j\right) P_i(\theta_{-i}|\theta_i, \theta_{i+N}, \theta_R) = \frac{1+\sum_{k\in R}\theta_k + \theta_i + \theta_{i+N}}{|R|+4}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right)$$

and hence,

$$\begin{aligned} \sum_{\theta_{-i}} \left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\theta_j\right) P_i(\theta_{-i}|\theta_i, \theta_{i+N}) &= \frac{1+\frac{1+\theta_i+\theta_{i+N}}{4}|R| + \theta_i + \theta_{i+N}}{|R|+4}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right) \\ &= \frac{1+\theta_i+\theta_{i+N}}{4}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right). \end{aligned}$$

Hence, (40) simplifies to

$$(2\theta_i - 1)\left(\begin{array}{c} 2(b_m - b_i) + c_i(1-2\theta_i) - 2c_{i+N}\theta_{i+N} \\ -\frac{1+\theta_i+\theta_{i+N}}{2}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right) + \frac{\frac{1+\theta_i+\theta_{i+N}}{2}|R|+3}{|R|+3}\sum_{j\in -R\backslash\{i\}} c_j \end{array}\right) \leq 0.$$

There are four possible realizations of (θ_i, θ_{i+N}). For each of these four realizations, we get the following inequalities:

- for $\theta_i = 1$, $\theta_{i+N} = 1$:

$$2(b_m - b_i) \leq c_i + \frac{\frac{1}{2}|R|+3}{|R|+3}c_{i+N} + \frac{3}{2(|R|+3)}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right)$$

- for $\theta_i = 0$, $\theta_{i+N} = 0$:

$$2(b_i - b_m) \leq c_i + \frac{\frac{1}{2}|R|+3}{|R|+3}c_{i+N} + \frac{3}{2(|R|+3)}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right)$$

  These two cases can be summarized as:

$$2|b_m - b_i| \leq c_i + \frac{\frac{1}{2}|R|+3}{|R|+3}c_{i+N} + \frac{3}{2(|R|+3)}\left(\sum_{j\in -R\backslash\{i,i+N\}} c_j\right),$$

  which is more difficult to satisfy as R expands.

- for $\theta_i = 1$, $\theta_{i+N} = 0$:

$$2(b_m - b_i) \leq c_i - c_{i+N}$$

- for $\theta_i = 0$, $\theta_{i+N} = 1$:

$$2(b_i - b_m) \leq c_i - c_{i+N}$$

  These two cases can be summarized as:

$$2|b_i - b_m| \leq c_i - c_{i+N},$$

  where the right-hand side is positive because $c_i > c_{i+N}$. This inequality is unaffected by the communication strategy of others.

In particular, if all agents have the same preferences ($b_i = b_m$), then regardless of the composition of committee, all committee members communicate truthfully the signal that the communication protocol allows them to communicate. This contrasts with the positive externalities of communication in the model where agents have common preferences but different beliefs: In that model, the agent will not communicate truthfully unless the committee size is sufficiently large.

A.6 Proofs for the Online Appendix

Proof of Proposition 7

We prove the existence of the efficient equilibrium by showing that no agent wants to deviate from the strategies described in the proposition. Consider a subgame that happens after all N agents become shareholders. According to Proposition 3 and its proof, there exists an equilibrium in which all N agents communicate their information truthfully, and hence the optimal committee consists of the set of all agents (excluding the manager). Since all agents communicate their signals truthfully, the manager takes action $a_m = b + Z$ (a special case of Lemma 1). Now, consider the trading game, provided that each agent expects that all agents will become shareholders and the manager will undertake action $a_m = b + Z$. Given this expectation, Lemma 2 implies that the expected per-share payoff of each agent i (excluding the holding cost) is u_0. Hence, from (31), the market-clearing price reduces to $p^* = u_0 - \frac{\lambda}{N}$. From (30), at this price, the fraction of the firm purchased by agent i is $\alpha_i = \frac{u_0 - p^*}{\lambda} = \frac{1}{N}$. This equilibrium is efficient in the sense of maximizing the sum of the seller's proceeds and the utilities of all the agents, because it (1) achieves the most efficient action from all agents' point of view ($a_m = b + Z$), and (2) minimizes the agents' total holding costs by distributing the asset evenly among all agents ($\alpha_i = \frac{1}{N}$).

We prove the possibility of equilibrium multiplicity by construction of an example. Suppose there are two groups of agents, Γ_1 and Γ_2. Agents in the first group have the same prior beliefs as the manager: $\rho_i = \rho_m \; \forall i \in \Gamma_1$ (this group includes the manager). In contrast, agents in the second group have different beliefs than the manager: $\rho_i \neq \rho_m$ with $|\rho_m - \rho_i| = \delta > 0$. We show that if δ is sufficiently high, there exists an equilibrium in which only agents from group Γ_1 become shareholders and an inefficient action $a \neq b + Z$ is implemented. Consider a subgame that happens after trading, in which only agents from group Γ_1 became shareholders. Since $\rho_i = \rho_m$ for these agents, the IC condition for truthful communication is satisfied for each $i \in \Gamma_1 \backslash \{m\}$. Hence, the optimal committee is the set of all agents excluding the manager, $\Gamma_1 \backslash \{m\}$, and the manager learns all signals in Γ_1. Using Lemma 1, the action that the manager undertakes is

$$a_m(\theta_R) = b + c \sum_{i \in \Gamma_1} \theta_i + \frac{\rho_m + \sum_{i \in \Gamma_1} \theta_i}{\tau + |\Gamma_1|} c \left(N - |\Gamma_1| \right). \tag{41}$$

The expected per-share payoff of each agent $i \in \Gamma_1$ (excluding the holding cost) is

$$u_0 - c^2 \frac{\rho_m \left(\tau - \rho_m \right) \left(N + \tau \right)}{\tau(\tau + 1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|}.$$

Hence, from (31), the market-clearing price is

$$p^* = u_0 - c^2 \frac{\rho_m (\tau - \rho_m)(N+\tau)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} - \frac{\lambda}{|\Gamma_1|}. \tag{42}$$

From (30), at this price, the fraction of the firm purchased by agent $i \in \Gamma_1$ is $\alpha_i = \frac{1}{|\Gamma_1|}$. To show that this is an equilibrium, it remains to show that no agent $j \in \Gamma_2$ is better off deviating to buying shares at price p^* if δ is sufficiently high. Suppose that one agent $j \in \Gamma_2$ deviates to $\alpha_j > 0$. Using Proposition 1, the IC condition on truthful reporting for this agent, (9) with $R = \Gamma_1$, is violated if $\delta > \delta_1$, where

$$\delta_1 \equiv \frac{1}{2}\left[1 + \frac{\tau + |\Gamma_1| + 1}{N - |\Gamma_1| - 1}\right].$$

Hence, for $\delta > \delta_1$, if agent j deviates to $\alpha_j > 0$, the optimal committee and firm's action will remain the same because the agent will not communicate his information to the manager. Therefore, the expected per-share payoff of this agent (excluding the holding cost) is

$$\mathbb{E}_j[U_j] = u_0 - c^2 \frac{\rho_j (\tau - \rho_j)(N+\tau)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} - c^2 \delta^2 \left(\frac{N - |\Gamma_1|}{\tau + |\Gamma_1|}\right)^2.$$

Using (30), deviation to $\alpha_j > 0$ is unprofitable for agent j if $\mathbb{E}_j[U_j] < p^*$, where p^* is given by (42). Simplifying, a sufficient condition for this deviation to be unprofitable is that $\delta > \delta_2$, where

$$\delta_2 \equiv \frac{\tau + |\Gamma_1|}{N - |\Gamma_1|} \sqrt{\frac{\lambda}{c^2 |\Gamma_1|} + \frac{(N+\tau)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} \rho_m (\tau - \rho_m)}.$$

Hence, this strategy profile is indeed an equilibrium if $\delta > \max\{\delta_1, \delta_2\}$. Comparing the expression (42) and the price $p^* = u_0 - \frac{\lambda}{N}$ in the efficient equilibrium, it automatically follows that the price in the inefficient equilibrium is lower.

Proof of Proposition 8

Suppose agents have heterogeneous preferences: $b_i \neq b_m$ for some i, but $\rho_i = \rho_m = \rho$ for all i. Let S denote the equilibrium set of agents that become shareholders, $B \subseteq S$ be the equilibrium committee chosen out of these shareholders, and $|B|$ denote the equilibrium committee size. Then, the expected per-share payoff of each agent $i \in S$ (excluding the holding cost) is

$$u_0 - (b_m - b_i)^2 - c^2 \frac{\rho(\tau - \rho)(N+\tau)}{\tau(\tau+1)} \frac{N - |B| - 1}{\tau + |B| + 1}.$$

Plugging this expression into (31), we obtain the equilibrium price:

$$p^* = u_0 - c^2 \frac{\rho(\tau - \rho)(N+\tau)}{\tau(\tau+1)} \frac{N - |B| - 1}{\tau + |B| + 1} - \frac{1}{|S|}\left(\sum_{j \in S} (b_m - b_j)^2 + \lambda\right).$$

Plugging this price into the demand equation (30), we obtain:

$$\alpha_i^* = \max\left\{\frac{1}{|S|} + \frac{1}{\lambda}\left(\frac{1}{|S|}\sum_{j\in S}(b_m - b_j)^2 - (b_m - b_i)^2\right), 0\right\}. \tag{43}$$

Note that this expression does not depend on the composition of the committee. Hence, we can solve for the set of all equilibria in two steps: (1) solve for the equilibrium set of shareholders; (2) find the optimal committee given the solution to the first step.

Consider the first step. Note that

$$\alpha_i^* > 0 \Leftrightarrow \lambda + \sum_{j\in S}(b_m - b_j)^2 - |S|\,(b_m - b_i)^2 > 0.$$

Suppose, without loss of generality, that agents are ordered in the order of (weakly) increasing $|b_m - b_i|$, with the manager being number 1, and the agent with the highest $|b_m - b_i|$ being number N. Consider the function

$$d\,(k) \equiv \lambda + \sum_{j\leq k}(b_m - b_j)^2 - k(b_m - b_k)^2 > 0. \tag{44}$$

Note that $d\,(k)$ is monotone decreasing in k. To see this, take the difference:

$$d\,(k+1) - d\,(k) = k\left[(b_m - b_k)^2 - (b_m - b_{k+1})^2\right] \leq 0.$$

Therefore, there exists a unique K such that $d\,(k) > 0$ for all $k \leq K$, but $d\,(K+1) \leq 0$. Then, (43) implies that there is a unique equilibrium set of shareholders, which is given by $S = \{1, ..., K\}$, so that $|S| = K$.

Note also that from (44), the equilibrium number of shareholders K is increasing in λ; we denote this function $K\,(\lambda)$. In particular, if $\lambda \to 0$ (holding costs are negligible), the ownership structure is very concentrated and only agents with exactly the same preferences as the manager become shareholders. Conversely, if λ exceeds $N(b_m - b_N)^2 - \sum_{j=1}^{N}(b_m - b_j)^2$, then all agents become shareholders.

Next, consider the optimal advisory committee B given that $K\,(\lambda)$ agents become shareholders. This is the committee of the maximum size for which the IC constraint on truthful reporting is satisfied for all its members. The IC constraint for committee member i (using (10) with $R = B \cup \{m\}\backslash\{i\}$) is

$$(\tau + |B| + 1)\,|b_m - b_i| \leq (\tau + N)\,\frac{c}{2}. \tag{45}$$

Since agents are ranked in order of increasing $|b_m - b_i|$ and have the same quality of information, it is without loss of generality to consider committees that include all agents of sufficiently low rank (except the manager). Suppose we take an advisory committee consisting of agents $\{2, ..., k\}$ (recall that agent 1 is the manager). The size of this committee is

$k-1$, so (45) for agent k, who has the largest misalignment of preferences with the manager among all committee members, becomes

$$(\tau + k)\,|b_m - b_k| \le (\tau + N)\,\frac{c}{2}. \tag{46}$$

Since $|b_m - b_k|$ increases in k, the left-hand side expression $(\tau + k)\,|b_m - b_k|$ is increasing in integer k, taking value of zero for $k = 1$ and value exceeding $(\tau + N)\,\frac{c}{2}$ for $k = N$ (by assumption of the proposition). Therefore, there exists a unique integer $\hat{K} \le N - 1$, such that among committees of the form $\{2, ..., k\}$, the constraint (45) holds for all agents of this committee if and only if $k \le \hat{K}$. Since the optimal committee is that of the maximum size subject to truth-telling of all its members, the optimal committee is $\{2, ..., \min(\hat{K}, K(\lambda))\}$ and has size $\min\left(\hat{K}, K(\lambda)\right) - 1$. Thus, the optimal committee includes all non-manager shareholders if and only if $\hat{K} \ge K(\lambda)$. Since $\hat{K}$ does not depend on λ and $K(\lambda)$ is increasing in λ, there exists a cutoff λ^* such that the optimal committee includes all non-manager shareholders if and only if $\lambda \le \lambda^*$.

Proof of Proposition 9

Rewriting the IC constraint from Proposition 1 and using $(b_m - b_i)(\rho_m - \rho_i) \ge 0$, board B is efficient if and only if $\mathcal{I}_i \ge 0$ for all $i \in B$, where

$$\mathcal{I}_i \equiv \frac{\tau + |B| + |\mathcal{M}| + 2}{\sum_{j \in -B_m} c_j} \left(\frac{1}{2}c_i - |b_m - b_i|\right) + \frac{1}{2} - |\rho_m - \rho_i|\,,$$

where $-B_m$ is a set of all signal indices that are not known to the board or the manager. Consider an expansion of $\mathcal{M}$ by one element. If this element belongs to $\{\theta_i, i \in B\}$, then all statements of the proposition are vacuously true, as the IC constraints are unaffected. Thus, consider the case when this element does not belong to $\{\theta_i, i \in B\}$. In this case, an expansion in $\mathcal{M}$ increases $|\mathcal{M}|$ and decreases $\sum_{j \in -B_m} c_j$. Suppose that $|b_m - b_i| \le \frac{1}{2}c_i$ $\forall i \in B$. Then, an expansion in $\mathcal{M}$ increases $\mathcal{I}_i$ for any i. Hence, if $\mathcal{I}_i \ge 0$ for all i for some $\mathcal{M}$, then $\mathcal{I}_i \ge 0$ $\forall i$ for any expansion in set $\mathcal{M}$. This proves the first statement of the proposition.

To prove the second statement, rewrite the IC constraint from Proposition 1 as $\mathcal{J}_i \ge 0$, where

$$\mathcal{J}_i \equiv \frac{\sum_{j \in -B_m} c_j}{\tau + |B| + |\mathcal{M}| + 2} \left(\frac{1}{2} - |\rho_m - \rho_i|\right) + \frac{1}{2}c_i - |b_m - b_i|\,.$$

Again consider an expansion of $\mathcal{M}$ by one element that does not belong to $\{\theta_i, i \in B\}$. Suppose that $|\rho_m - \rho_i| \le \frac{1}{2}$ $\forall i \in B$. Then, an expansion in $\mathcal{M}$ reduces $\mathcal{J}_i$ for any i, because it increases $|\mathcal{M}|$ and decreases $\sum_{j \in -B_m} c_j$. Hence, if $\mathcal{J}_i \ge 0$ for all i for some $\mathcal{M}$, then $\mathcal{J}_i \ge 0$ $\forall i$ for any contraction in set $\mathcal{M}$.

Proof of Proposition 10

Proof of part (i)**.** Suppose that the manager expects agent i to report his signal truthfully, and consider agent i's decision whether to do so. If agent i reveals his signal truthfully, the manager's action is

$$a_m(\theta_R, \theta_i) \equiv c\theta_i + \gamma c\theta_m + \sum_{k\in R\setminus\{m\}} c\theta_k \; + \frac{\rho_m + \theta_i + \sum_{k\in R}\theta_k}{\tau + 1 + |R|}(N - |R| - 1)\,c. \tag{47}$$

In contrast, if agent i misreports, the manager's action is

$$a_m(\theta_R, 1-\theta_i) \equiv c(1-\theta_i) + \gamma c\theta_m + \sum_{k\in R\setminus\{m\}} c\theta_k \; + \frac{\rho_m + (1-\theta_i) + \sum_{k\in R}\theta_k}{\tau + 1 + |R|}(N - |R| - 1)\,c. \tag{48}$$

Truthful reporting is optimal if and only if

$$\sum_{\theta_{-i}\in\{0,1\}^{N-1}} \left[(a_m(\theta_R,\theta_i) - Z_i)^2 - (a_m(\theta_R, 1-\theta_i) - Z_i)^2\right] P_i(\theta_{-i}|\theta_i) \le 0 \tag{49}$$

for each $\theta_i \in \{0,1\}$, where $Z_i = \gamma c\theta_i + c\sum_{j\neq i}\theta_j$. Simplifying, (49) reduces to

$$(2\theta_i - 1)\left[c(1 - 2\gamma\theta_i) + 2(\gamma - 1)c\frac{\rho_i + \theta_i}{\tau + 1} + \frac{2(\rho_m - \rho_i) + 1 - 2\theta_i}{\tau + 1 + |R|}(N - |R| - 1)\,c\right] \le 0.$$

If $\theta_i = 1$, we have:

$$\rho_m - \rho_i \le \frac{1}{2}\frac{N+\tau}{N - |R| - 1} + (\gamma - 1)\frac{\tau - \rho_i}{\tau + 1}\frac{\tau + 1 + |R|}{N - |R| - 1},$$

which is trivially satisfied for $\rho_m \le \rho_i$, but may be violated if $\rho_m > \rho_i$. If $\theta_i = 0$, we have:

$$\rho_i - \rho_m \le \frac{1}{2}\frac{N+\tau}{N - |R| - 1} + (\gamma - 1)\frac{\rho_i}{\tau + 1}\frac{\tau + 1 + |R|}{N - |R| - 1},$$

which is trivially satisfied for $\rho_m \ge \rho_i$, but may be violated if $\rho_m < \rho_i$. Combining the two cases proves part (i) of the proposition.

Proof of part (ii)**.** For any agent i with sufficiently high $|\rho_i - \rho_m|$, (33) is violated for a sufficiently small committee, unless γ is too high. Specifically, the condition for this is:

$$\begin{aligned} \rho_m - \rho_i \;&>\; \frac{1}{2}\frac{N+\tau}{N-2} + (\gamma - 1)\frac{\tau - \rho_i}{\tau + 1}\frac{\tau + 2}{N - 2} \\ \text{or } \rho_i - \rho_m \;&>\; \frac{1}{2}\frac{N+\tau}{N-2} + (\gamma - 1)\frac{\rho_i}{\tau + 1}\frac{\tau + 2}{N - 2} \end{aligned}$$

for any i. In this case, there exists no committee consisting of a single agent that can communicate information truthfully (R in the case of a single-member committee is simply the manager's own signal). Since (33) relaxes as R expands, there exists $N^{\gamma}_{\min} \geq 1$ such that there is no committee consisting of $N^{\gamma}_{\min}$ or fewer agents for which truthful communication of information to the manager is an equilibrium. For any R, (33) relaxes as γ increases. Therefore, $N^{\gamma}_{\min}$ is (weakly) decreasing in γ.

Proof of part (iii)**.** Consider the committee consisting of all agents, $B = \{1, ..., N\}\backslash\{m\}$. Since the right-hand sides of inequalities (33) equals infinity in this case ($|R| = N-1$), there is an equilibrium in which all agents in this advisory body communicate information truthfully to the manager. The argument identical to the proof of Lemma 2 implies that every agent is better off if the manager is more informed ($\mathbb{E}_i[U_i|R]$ increases as R expands). Therefore, $B = \{1, ..., N\}\backslash\{m\}$ is the optimal committee.

Proof of Proposition 11

The proof largely repeats the proof of Proposition 1 in the paper. Using $a_m(\theta_R, \theta_i)$ and $a_m(\theta_R, 1-\theta_i)$, the IC becomes

$$\textstyle\sum_{\theta_R,\theta_{-R}} P_i(\theta_R, \theta_{-R}|\theta_i) \left[c_i(2\theta_i - 1) + \left(\sum_{j\in -R\backslash\{i\}} c_j\right) \frac{2\theta_i - 1}{\tau_m + |R| + 1}\right] \times$$
$$\left[2(b_m - b_i) + c_i(1-2\theta_i) - 2\textstyle\sum_{k\in -R\backslash\{i\}} c_k\theta_k + (\sum_{k\in -R\backslash\{i\}} c_k)\frac{2(\rho_m + \sum_{k\in R}\theta_k)+1}{\tau_m + |R| + 1}\right] \geq 0.$$

Note that $P_i(\theta_R, \theta_{-R}|\theta_i) = P_i(\theta_{-R}|\theta_R, \theta_i)P_i(\theta_R|\theta_i)$. Since $\left[c_i + \frac{\sum_{j\in -R\backslash\{i\}} c_j}{\tau_m + |R| + 1}\right] > 0$, this is equivalent to

$$-(2\theta_i - 1)\times\left[2(b_m - b_i) + c_i(1-2\theta_i) - 2\frac{\rho_i + \theta_i}{\tau_i + 1}\sum_{k\in -R\backslash\{i\}} c_k + (\sum_{k\in -R\backslash\{i\}} c_k)\frac{2\rho_m + 2|R|\frac{\rho_i + \theta_i}{\tau_i + 1} + 1}{\tau_m + |R| + 1}\right] \geq 0$$

or equivalently,

$$-(2\theta_i - 1)\times\left[2(b_m - b_i) + c_i(1-2\theta_i) + (\sum_{k\in -R\backslash\{i\}} c_k)\left[\frac{2\rho_m(\tau_i + 1) + \tau_i + 1 - 2(\rho_i + \theta_i)(\tau_m + 1)}{(\tau_i + 1)(\tau_m + |R| + 1)}\right]\right] \geq 0$$

Considering two cases ($\theta_i = 1$ and $\theta_i = 0$) and simplifying the expressions (similar to the proof of Proposition 1), we obtain (34). It is easy to see that (34) is equivalent to the condition in Proposition 1 of the paper when $\tau_i = \tau$ for all i.