Advising the Management*

Ali Kakhbod† Uliana Loginova‡ Andrey Malenko§
Nadya Malenko¶

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Abstract

We study the optimal size and composition of an advisory committee when shareholders differ in preferences and beliefs and strategically acquire and communicate information. If shareholders and management have similar objectives but disagree due to different beliefs, and information is cheap, the optimal advisory body includes all shareholders. Conversely, if agents have conflicting preferences or information is sufficiently costly, the optimal advisory body is a strict subset of shareholders. Thus, advisory voting (board) is optimal in the former (latter) case. Similar implications hold if the committee also has authority, but unlike purely advisory committees, committees with authority are more diverse.

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†MIT, Department of Economics. Email: akakhbod@mit.edu.
‡McKinsey & Company. Email: uloginova@gmail.com.
§Boston College, Carroll School of Management. Email: malenkoa@bc.edu.
¶Boston College, Carroll School of Management. Email: nadya.malenko@bc.edu.

1 Introduction

Information relevant to corporate decisions is dispersed among many partially informed parties, such as the firm's managers, employees, shareholders, customers, and industry participants. No manager, even the most experienced and talented one, is fully informed about the optimal course of action, so managers regularly seek advice from other informed agents. In modern firms, there exists a large heterogeneity in advisory structures. On many decisions, advice is provided by a relatively small group of people, such as the board of directors. In fact, advising the management is considered one of the most important functions of the board (e.g., Business Roundtable, 1990). In many recently listed technology firms, such as Facebook, Snap, and Spotify, the advisory role is de facto the only role of the board, as their management has virtually full decision-making authority by holding superior-voting shares. For small companies, including startups, that do not have a formal board of directors, having an advisory board is considered a critical element of the company's success.[1] On other decisions, advice to decision-makers is provided by a large group of people. For example, shareholders provide advice through a non-binding vote on governance proposals, corporate social responsibility policies, and executive compensation (say on pay). Although these votes are often purely advisory, firms often learn from and respond to their results (Ferri, 2012). Other examples include employee and customer surveys, which are both regularly conducted by companies.[2]

Why is there heterogeneity in the means of providing advice? When is it optimal to seek advice from a large group of people (e.g., shareholders through an advisory vote) vs. a small group of people (e.g., the board)? If advisory voting is indeed informative, why are not shareholders consulted on a greater variety of corporate decisions? And how do optimal board size and composition depend on whether the board has authority in addition to its advisory role?

The goal of this paper is to address these questions by studying the optimal design of the advisory body. We propose a simple and tractable model that captures the three key features of the advisory process. First, relevant information is dispersed among multiple

[1] See, e.g. "Who advises the entrepreneur?" (Harvard Business Review, October 22, 2013) and the 2014 BDC study "Advisory boards: An untapped resource for businesses".

[2] According to Watson Wyatt's 2001 survey of 500 publicly traded companies, 79% regularly surveyed their employees.

agents, which creates value from advising. Second, acquiring information is costly for the agents. Finally, information may not be perfectly communicated to the manager because agents may have different preferences or beliefs from the manager.

Specifically, the firm needs to make a decision, whose value depends on the unknown state of the world. Multiple agents – the firm's shareholders and other stakeholders – are potentially informed about the state, and the firm's advisory committee is some subset of the agents. For example, a committee that includes all shareholders corresponds to advisory voting, while a committee composed of selected shareholder representatives corresponds to the board of directors. Committee members (advisors) first decide whether to acquire private signals and then communicate with the partially informed manager by sending non-verifiable messages ("cheap talk"). The manager then chooses which action to take.

Frictions in communication can arise if, given the same information, the advisor wants to take a different action than the manager – the advisor may then have incentives to misreport his signal. This may happen for two reasons – heterogeneous preferences and heterogeneous beliefs. For example, consider a firm deciding on the scale of production in a new market. Given the same information, the manager may prefer a larger scale than a shareholder for two reasons: he may get private benefits from running bigger operations (different preferences) or he may have "more bullish" priors about the growth of the market (different beliefs).[3] Our model features heterogeneity in both preferences and beliefs and shows that these communication frictions have drastically different implications for the quality of advice and optimal advisory structures.

We start by showing that an advisor's incentives to truthfully reveal his information to the manager are affected by whether other committee members reveal their information. If agents have the same preferences but differ in prior beliefs, these communication externalities are positive: as more committee members share their information with the manager, others have stronger incentives to do so as well. In contrast, if agents have different preferences but common beliefs, communication externalities are negative: as more agents share their information, others have stronger incentives to misreport. As a

[3]The empirical literature finds the presence of both heterogeneous preferences (e.g., Matvos and Ostrovsky (2010), Cvijanovic et al. (2016), and Bolton et al. (2020), among others) and heterogeneous beliefs among shareholders (Li, Maug, and Schwartz-Ziv, 2019).

result, under heterogeneous beliefs, a committee can only be effective in its advisory role if its size is sufficiently large. In contrast, only small advisory committees are effective under heterogeneous preferences.

The intuition for positive externalities is that heterogeneous priors become less relevant as the manager becomes more informed. Simply speaking, differences in prior beliefs generate disagreement only over the information that is unknown, so there is less disagreement when more information is learned. Thus, as more other committee members share their information, the manager becomes more congruent with the advisor, which improves communication between them. The reason for negative externalities is that as the manager becomes more informed, he reacts less to the advisor's messages. Intuitively, when misrepresenting his signal, the advisor faces a trade-off: he wants to move the manager closer to his own preferred decision, but is afraid to make too big of an impact and move the manager's decision too much, away even from his own preferred decision. This concern encourages truthful communication when the manager strongly reacts to the agent's messages, but is not sufficient to constrain misreporting when the manager's reaction to the agent's advice is small, which is when he receives advice from many other agents.

This logic leads to the following results about the optimal composition of the advisory body.[4] First, if the cost of information acquisition is sufficiently low, the optimal advisory body is the set of all informed agents under heterogeneous beliefs, but is a strict subset of informed agents under heterogeneous preferences. Second, if the cost of acquiring information is substantial, committee size cannot be too large; otherwise, its members would lack incentives to become informed in the first place. In this case, the optimal advisory body is always a strict subset of informed agents. Practically, including all informed agents in the advisory body can be interpreted as holding an advisory shareholder vote. Our analysis implies that such a vote will be efficient in its advisory role if shareholders and the manager have common interests but different beliefs, and acquiring information is not too costly for the shareholders.[5] On the other hand, a committee that is a strict sub-

[4]We assume that there is an infinitesimal cost of including each committee member, and so the optimal committee does not include agents who do not provide any advice.

[5]If there are informed non-shareholder stakeholders such as employees, an advisory shareholder vote alone is not optimal. However, it is still a necessary part of the optimal advisory process, together with seeking advice from other informed stakeholders.

set of all informed agents can be interpreted as an advisory board. Our results suggest that in contrast to shareholder voting, such a board will be efficient in its advisory role for issues where conflicts of interests are particularly important, or for issues that require extensive and costly analysis.

While our basic setting considers committees that have a purely advisory role, in many cases the managerial team does not have full authority over the decision: the decision proposed by the manager must be approved by either a shareholder vote or the board. We show that our main insights are robust to giving approval authority to the committee: the optimal committee includes all shareholders (in the form of a shareholder vote) if the source of conflict is heterogeneous beliefs, and a strict subset of all shareholders (in the form of a board) if the source of conflict is heterogeneous preferences.

The new implication is that the composition of the optimal board with authority is different from that of a purely advisory board. While the optimal advisory board is homogenous in that all its directors should have preferences sufficiently aligned with the manager, the optimal board with authority is diverse: it should include directors with preferences similar to the average shareholder but also a director who is biased in the *opposite* direction from the manager. The role of this director is not to give advice, but rather to reduce the bias in the manager's proposed action by threatening to not approve it. In practice, such a director could be a representative of a labor union or another individual with the opposite bias from the manager. Interestingly, the presence of such a director does not harm communication between the board and the manager, despite the fact that she and the manager have opposing interests. This result arises due to our focus on the board as a collective body and distinguishes our paper from existing studies in which the board consists of a single agent and a higher misalignment with the manager necessarily harms its advisory role (e.g., Adams and Ferreira, 2007).

We consider several extensions of the basic model. First, we note that in practice, some informed agents may choose not to become shareholders in the first place. Such agents will then have little incentive or means to communicate their information to the management. We extend the model to endogenize the firm's ownership structure and show that if agents differ in prior beliefs, multiple equilibria can arise. On the one hand, there always exists an efficient equilibrium, in which all informed agents become shareholders and communicate their information via an advisory vote, so the manager makes the optimal decision and firm value is high. In addition, there can exist an inefficient equilibrium, in which only a few informed agents become shareholders, so the manager makes suboptimal decisions and firm value is low. Intuitively, if an investor expects the firm to have few shareholders and the manager's decision to be therefore based on little information, he expects to disagree with the manager's decisions ex-post, and hence does not invest in the firm in the first place. One implication of this result is that the presence of passive (index) investors, whose stake in the firm does not depend on whether they agree or disagree with management, can make advisory voting more effective.

Second, we ask how the board's advisory role depends on the manager's expertise. We show that a more informed manager enhances the board's advising effectiveness if heterogeneity in beliefs is relatively more important than heterogeneity in preferences, but impedes its advisory role otherwise. Interestingly, this second effect implies that if conflicts of interest are substantial, a more informed manager does not always make more informed decisions, and it can be Pareto improving to appoint a less informed manager.

Together, our analysis offer novel implications regarding the advising effectiveness of shareholder votes and boards of directors, the effects of board size and board committee structure, and the role of the manager's expertise, which we discuss in Section 6.

Our paper is related to the literature on the advisory role of the board (Adams and Ferreira, 2007; Harris and Raviv, 2008; Baldenius et al., 2014; Chakraborty and Yilmaz, 2017; Levit, 2020). Differently from this literature, which mostly analyzes the board as a single agent, we examine the board as a collective body and study the interactions among directors. Hence, the focus of our paper is on the effects of size and composition of the advisory body and in particular, the optimality of advisory board vis-is advisory voting – a question that has not been studied in the literature before. The analysis of multiple directors also allows us to provide novel implications about the optimal board size: while there is substantial empirical literature on board size, which we discuss in Section 6.2, theoretical analysis of this question is limited. The exception is Harris and Raviv (2008), who point out that large board size impedes information acquisition by directors. While this effect is also present in our paper, our main focus is on the impact of board size on the board's communication with the manager.[6] Our other contribution to this literature is to

[6]Warther (1998), Baranchuk and Dybvig (2009), Malenko (2014), Chemmanur and Fedaseyeu (2017), and Donaldson et al. (2019) study interactions between multiple directors within the board, but do not

highlight the role of board diversity when the board has authority in addition to playing an advisory role. This relates our paper to the empirical literature on board diversity (see Ferreira (2010) for a survey; see also Section 6.2) and complements Donaldson et al. (2019), who study the effect of diversity on board deadlock.

Since our paper provides a unified framework that studies both boards and shareholder voting, it is also related to the literature on strategic voting, which examines how voting aggregates dispersed information (e.g., Feddersen and Pesendorfer, 1997; Maug and Yilmaz, 2002; and Persico, 2004; among others). Unlike the majority of this literature, where voting is binding, shareholder voting in our model primarily plays an advisory role, which leads to very different mechanisms. In this sense, the closest paper on strategic voting is Levit and Malenko (2011), who also analyze non-binding voting. However, our paper focuses on heterogeneous beliefs in addition to conflicting preferences, and also features different economic forces due to the different way we view the role of advisory votes. As a result, unlike in their paper, information aggregation in our setting crucially depends on the number of shareholders (see Section 6.1 for details).

Our paper is also related to the literature on heterogeneous priors. Morris (1995) provides an overview of the heterogeneous prior assumption and discusses why it is consistent with rationality. Our model also features rational agents: although they have different priors, they are not dogmatic and rationally update their beliefs in a Bayesian way after receiving new information. Overall, there is growing empirical evidence suggesting that heterogeneous beliefs are important to explain corporate finance decisions and the dynamics of asset prices and volume,[7] and Li et al. (2019) find evidence that shareholders have different beliefs when they vote in shareholder meetings. Accordingly, there is a large theoretical literature studying the implications of heterogeneous priors.[8] The contribution of our paper is to apply the insights of this literature to decision analysis. The closest papers are Garlappi et al. (2017, 2020), who study group decision-making under heterogeneous beliefs but without private information and communication, and

study the board's advisory role and do not feature the mechanisms that arise in our paper. Khanna and Schroder (2015) examine multiple committee members acquiring and communicating information, but focus on optimal contracting with committee members.

[7]E.g., Kandel and Pearson (1995), Diether et al. (2002), Malmendier and Tate (2005), Dittmar and Thakor (2007), and Thakor and Whited (2011), among others. See Hong and Stein (2007) for a survey.

[8]Examples in the finance literature include Harris and Raviv (1993), Kandel and Pearson (1995), Boot, Gopalan, and Thakor (2006), Banerjee et al. (2009), and Banerjee and Kremer (2010), among many others.

Che and Kartik (2009), Van den Steen (2010), and Alonso and Camara (2016), who study communication under heterogeneous beliefs but with only one sender and not via cheap talk, and thus do not feature the forces highlighted in our paper.

Within the literature on cheap talk (Crawford and Sobel, 1982), our paper is closest to the papers analyzing communication by multiple imperfectly informed senders (Austen-Smith, 1993; Battaglini, 2004; Morgan and Stocken, 2008; Galeotti et al., 2013). Our analysis of the case of conflicting preferences is related to Morgan and Stocken (2008) and their result that full information revelation is an equilibrium in a poll with a small sample, but not with a large one. We contribute to this literature in several ways. Most importantly, we emphasize that the results under heterogeneous preferences (which these papers focus on) are the opposite of those under heterogeneous beliefs. We also show how both communication frictions can be simultaneously captured in a simple tractable model. Finally, we highlight how in the corporate setting, ownership structure is endogenously determined by investors' preferences and beliefs, and how this, in turn, affects communication between shareholders and management.

Finally, our extension to the endogenous ownership structure relates our paper to the literature on feedback effects from financial markets to the real economy, comprehensively surveyed in Bond et al. (2012). In both this literature and our paper, financial markets affect how much information is incorporated into corporate decisions and thereby affect fundamental firm value. But differently from this literature, where information aggregation occurs through prices, the real effect of financial markets in our paper is through their impact on the ownership structure: trading determines the shareholder base and thus affects which agents share their information with the manager via voting or being on the board.

The paper proceeds as follows. Section 2 describes the setup. Section 3 studies the externalities in communication and the optimal composition of the advisory committee. Section 4 discusses the results in light of the choice between voting and the board and analyzes the case when the advisory body has authority to overrule the manager's proposed action. Section 5 considers several extensions of the model. Section 6 discusses the empirical predictions, and Section 7 concludes.

2 Setup

In this section, we present a simple model, which captures heterogeneous preferences, heterogeneous beliefs, and dispersed private information, and has tractable and intuitive solutions.

The firm needs to make a decision, denoted by a continuous action $a \in \mathbb{R}$. The value of this decision depends on the unknown state of the world Z. There is a set of N shareholders indexed by i, $i \in \{1, ..., N\}$, characterized by their preferences b_i, such that the payoff of shareholder i from action a given state Z is

$$U_i(a, Z, b_i) = u_0 - (a - Z - b_i)^2, \tag{1}$$

where $u_0 > 0$ is a constant.

One of these agents, indexed by $m \in \{1, ..., N\}$, is the manager, who decides on action a after getting information from its advisory committee. We consider two variations of the model. In the first, the manager has full decision-making authority over the choice of a, and the committee plays a purely advisory role. This setting is applicable to situations in which the managerial team has full authority due to super-voting shares or significant stock ownership in the firm. Examples include many publicly-traded technology companies, such as Facebook, Snap, and Alphabet, as well as many private firms. In the second variation of the model, the committee has authority to not approve the action proposed by the manager. This setting is applicable to situations in which the managerial team needs approval from the board or the shareholders to implement a certain action. We start by describing and analyzing the first variation of the model and examine the second variation in Section 4.1.

The information structure is as follows. The state of the world is equal to the weighted sum of N signals:

$$Z = \sum_{i=1}^{N} c_i \theta_i, \tag{2}$$

where $c_i > 0$ and θ_i are independent and identically distributed. Signals θ_i can be thought of as different factors relevant to the decision. Information about these relevant factors is dispersed among the shareholders of the company: agent $i \neq m$ can incur cost $\kappa \geq 0$

to privately observe signal θ_i, and is uncertain about other signals. For simplicity, we assume that the manager is endowed with his signal θ_m.[9] An agent with a higher c_i can be interpreted as being more informed. Such information structure is common in the literature (e.g., Harris and Raviv, 2005, 2008; Chakraborty and Yilmaz, 2017) and captures the idea that shareholders may have different areas of expertise and thus have information relevant to different aspects of the decision. It also assumes that if agents combine their information together, they know the state completely. In Section 3.2, we discuss the robustness of the results to a generalization in which there is some residual information that no agent knows.

For example, in the context of M&A decisions, a could be the choice of how much to bid for a potential target, and signals θ_i could capture the synergies from the merger, the intrinsic value of the target, the number of potential competing bidders and their bids, the costs of integrating the two companies, and other relevant factors. Different shareholders have information about these different aspects of the decision, with some being more important than others. We develop this M&A example further below, as we explain the intuition behind the results (see Section 3.1).

Note also that another interpretation of the N informed players is that they include not only the firm's shareholders, but also its employees, customers, industry participants, and other stakeholders who care about the firm's decision and have relevant expertise. For this reason, we will often refer to these players as simply "agents."

The setup so far captures heterogeneous preferences and information dispersed among agents. In addition, to capture the possibility that agents may have heterogeneous beliefs, we assume that they have different priors about the distribution of signals θ_i. Intuitively, some agents can be ex-ante more bullish about the prospects of the acquisition, while others can be more bearish. To capture this feature in a tractable way and derive closed form solutions, we make the following distributional assumption. We assume that θ_i is a binary signal equal to 1 with probability φ and 0 with probability $1-\varphi$, and agents may potentially disagree about φ: agent i's prior of φ is characterized by the Beta distribution with parameters $(\rho_i, \tau-\rho_i)$.[10] Parameter φ captures the intrinsic

[9] This allows us to abstract from the effect of the committee structure on the manager's private information and instead focus on the committee members' information acquisition and communication decisions.

[10] That is, given the agent's belief ρ_i, the density of φ is $f_i(\varphi) = \frac{\varphi^{\rho_i-1}(1-\varphi)^{\tau-\rho_i-1}}{Beta(\rho_i,\tau-\rho_i)}$, where $Beta(\rho, \tau-\rho) =$

value of the decision: when φ is higher, the state is likely to be higher, so the optimal action is higher as well. Since the expected value of the Beta distribution is $\frac{\rho_i}{\tau}$, parameter ρ_i captures how optimistic agent i is about the state: those with a higher ρ_i are ex-ante more optimistic.[11] The case $\rho_i = \rho$ captures common priors: for example, if $\rho_i = 1$ and $\tau = 2$, all agents believe that φ is uniformly distributed on $[0,1]$. While agents may have different prior beliefs, they update their beliefs rationally (according to Bayes' rule) when they receive new information.

To summarize, each agent is characterized by his preference parameter b_i, belief ρ_i (which captures whether he is ex-ante bullish or bearish about the state), and private signal about the state θ_i with relative importance c_i. Parameters b_i, ρ_i, and c_i are publicly known.

We assume that parameters b_i and ρ_i satisfy $(b_i - b_m)(\rho_i - \rho_m) \geq 0$ for any i. This assumption is made so that agent i can be interpreted as biased towards a higher or lower action relative to the manager, where this bias can come from different preferences and/or different prior beliefs.[12] This assumption is automatically satisfied if only one communication friction is present, i.e., if the agents either have common preferences or common priors.

The timeline is as follows. There is an advisory body (committee) B, which is a subset of all agents excluding the manager, $B \subseteq \{1, ..., N\} \backslash \{m\}$. First, all agents in the advisory body simultaneously decide whether to incur a private cost to acquire their private signals. For simplicity, we assume that agents' information acquisition decisions are observed, and this happens after the communication stage.[13] Next, all informed members

$\frac{\Gamma(\rho)\Gamma(\tau-\rho)}{\Gamma(\tau)}$ and $\Gamma(\cdot)$ is the gamma function.

[11] Notice, however, that ρ_i also affects other moments, not only the mean. In Section A.4 of the Online Appendix, we consider a more flexible specification in which agent i's prior of φ is characterized by the Beta distribution with parameters $(\rho_i, \tau_i - \rho_i)$. The main results of the baseline model extend to this setting.

[12] Such an interpretation would not be possible without this assumption because the two sources of the bias could offset each other, making the agent effectively unbiased relative to the manager.

[13] Without this assumption, an agent who deviates from his equilibrium strategy and does not invest in information, may want to mislead the manager and try to send a signal that he did not in fact acquire. Making the above assumption makes such deviations impossible and hence simplifies the analysis. In addition, assuming that information acquisition is observed after the communication stage rather than before simplifies the incentive compatibility constraint on information acquisition, because it implies that other agents do not change their behavior when one agent deviates to not acquiring information. However, most of the analysis remains unchanged if information acquisition decisions are unobserved: the only difference is an additional incentive compatibility constraint on information acquisition, which does not

of the advisory body simultaneously communicate their information to the manager (via cheap talk). Finally, the manager takes the action that maximizes his payoff.

We look for equilibria in pure strategies at the information acquisition and communication stages. Because signals are binary, it is without loss of generality to consider a binary message space at the communication stage: the communication strategy of agent i is a mapping from his signal $\theta_i \in \{0,1\}$ into a binary non-verifiable message $m_i \in \{0,1\}$. Thus, in equilibrium, an agent who has information either communicates it truthfully (i.e., $m_i(\theta_i) = \theta_i$ up to relabeling) or sends an uninformative (babbling) message (i.e., $m_i(0) = m_i(1)$).

If, for a given advisory body B, there exists an equilibrium in which all members of B acquire information and truthfully communicate it to the manager, we assume that this equilibrium is played. Clearly, no agent would acquire information if he does not plan to communicate it, so the second part of this equilibrium selection is immaterial. The first part of the equilibrium selection is natural, since any alternative equilibrium would imply that advisory body B is equivalent to a smaller advisory body, which excludes members of B that do not acquire information.

3 Analysis of the model

3.1 Externalities in communication

We start by considering the communication stage. Since we focus on pure strategy equilibria at the information acquisition stage, the set of agents at the communication stage consists of two subsets: those known to be informed and those known to be uninformed. The manager ignores any messages sent by the latter group of agents, and hence we can focus on incentives of the former group of agents to communicate truthfully.

We first characterize the action taken by the manager for a given outcome of the communication stage. Suppose that after communicating with the advisory body, the manager knows subset $R \subseteq \{1,...,N\}$ of signals ("revealed" signals) and does not know all the other signals, $-R \equiv \{1,...,N\} \backslash R$. We use R and $-R$ to denote signal indices and $\theta_R \equiv \{\theta_i, i \in R\}$ and $\theta_{-R} \equiv \{\theta_i, i \in -R\}$ to denote the corresponding subsets of signal realizations.

change the results qualitatively.

The subset θ_R includes the manager's own signal θ_m and the signals of those members of the advisory body who become informed and communicate truthfully.

Given the quadratic payoff function, the optimal action of the manager is the sum of b_m and his expectation of the state given his prior ρ_m and the signals he learned $(\theta_i, i \in R)$:

$$a_m(\theta_R) = b_m + \mathbb{E}_m(Z|\theta_i, i \in R) = b_m + \sum_{i \in R} c_i \theta_i + \sum_{j \in -R} c_j \, \mathbb{E}_m\Big[\theta_j | \theta_i, i \in R\Big]. \tag{3}$$

The subscript m in the expectation operator $\mathbb{E}_m$ highlights that the manager uses his own prior ρ_m to update his beliefs about the unknown signals $\theta_j, j \in -R$, from his knowledge of signals $\theta_i, i \in R$. In the appendix, using the properties of the Beta distribution, we derive a simple expression for the manager's posterior beliefs, $\mathbb{E}_m(\varphi|\theta_R) = \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|}$, which gives the following result:

Lemma 1 (Optimal action of the manager). *Suppose that after the communication stage, the manager knows subset R of signals. Then his optimal action is*

$$a_m(\theta_R) = b_m + \sum_{i \in R} c_i \theta_i + \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|} \sum_{j \in -R} c_j, \tag{4}$$

where $|R|$ is the number of signals in R.

The higher are the revealed signals $\theta_i, i \in R$, the higher is the manager's posterior belief about the state, and hence the higher is his optimal action (e.g., offer price for a target) given this information. For any given information set, a higher bias b_m induces the manager to take a higher action. Likewise, a higher ρ_m, capturing more optimistic ex-ante beliefs, also induces the manager to take a higher action. However, while the effect of heterogeneous preferences b_m does not depend on the manager's information, heterogeneity in beliefs becomes less important as the manager becomes more informed and updates his beliefs. In particular, as the set R expands, the term $\frac{1}{\tau + |R|} \sum_{j \in -R} c_j$, and hence the effect of ρ_m on the manager's action, decreases. In the extreme case, if $b_m = b_j$ and $R = \{1, ..., N\}$, the manager's optimal action coincides with the optimal action from the perspective of any other agent.

Using Lemma 1, we next examine when a given committee member will truthfully reveal his information to the manager. Consider any informed agent i and suppose that the manager knows subset $R \subset \{1,...,N\}$ of signals, where R includes the manager's own signal θ_m but not agent i's signal θ_i. The manager does not know all the other signals, i.e., agent i's signal θ_i and all signals in the subset $-R\backslash\{i\}$, where as before, $-R \equiv \{1,...,N\}\backslash R$. Suppose the manager believes the agent's message and uses it to update his belief about the state according to (4). If agent i reveals his signal truthfully, the manager's action is

$$a_m(\theta_R, \theta_i) \equiv b_m + c_i\theta_i + \sum_{k\in R} c_k\theta_k + \frac{\rho_m + \theta_i + \sum_{k\in R}\theta_k}{\tau + 1 + |R|} \sum_{j\in -R\backslash\{i\}} c_j. \tag{5}$$

In contrast, if agent i misreports and claims that his signal is $1-\theta_i$, the manager's action is

$$a_m(\theta_R, 1-\theta_i) \equiv b_m + c_i(1-\theta_i) + \sum_{k\in R} c_k\theta_k + \frac{\rho_m + (1-\theta_i) + \sum_{k\in R}\theta_k}{\tau + 1 + |R|} \sum_{j\in -R\backslash\{i\}} c_j. \tag{6}$$

Because agent i does not know the realization of all other agents' signals, he compares his expected payoff from actions $a_m(\theta_R, \theta_i)$ and $a_m(\theta_R, 1-\theta_i)$ given his signal θ_i and his own prior belief about the distribution of those $N-1$ signals, and reports his signal truthfully if and only if:

$$\sum_{\theta_{-i}\in\{0,1\}^{N-1}} \left[(a_m(\theta_R,\theta_i) - Z - b_i)^2 - (a_m(\theta_R, 1-\theta_i) - Z - b_i)^2\right] P_i(\theta_{-i}|\theta_i) \le 0, \tag{7}$$

where θ_{-i} is the set of all signals except θ_i and $P_i(\theta_{-i}|\theta_i)$ is agent i's belief given his signal θ_i and his own prior.

The next result characterizes the necessary and sufficient conditions for (7) to hold.

Proposition 1 (IC constraint for truthful reporting). *Suppose that the manager learns subset R of signals (which includes his own signal θ_m but not θ_i) and does not know all the* *other signals, $-R$. Then agent i reports his signal truthfully if and only if*

$$\left|(b_m - b_i) + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau + |R| + 1}(\rho_m - \rho_i)\right| \le \frac{1}{2}\left[c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau + |R| + 1}\right]. \tag{8}$$

The left-hand side of (8) illustrates the agent's benefit from misreporting the signal, while the right-hand side illustrates his cost of misreporting. As (8) shows, communication from the agent to the manager is inhibited both by the difference in their preferences, captured by $b_m - b_i$, and by the difference in their beliefs, captured by $\rho_m - \rho_i$. In particular, conflicting preferences create incentives for misreporting as is standard in cheap talk games: the agent wants to tilt the manager's action in the direction of his preference, b_i. Similarly, if the agent and manager have different priors — for example, if the manager is ex-ante more optimistic about the state, the agent will want to correct this "bias in beliefs" by reporting a more negative signal. Overall, these results imply that, other things equal, a committee will be less effective in its advisory role if the manager is sufficiently different from the committee members in either preferences or prior beliefs about the optimal decision.

Note also that regardless of the source of communication frictions, agent i is more likely to report his signal truthfully if his information is more important: the IC constraint (8) is relaxed when c_i increases. Intuitively, the agent faces a trade-off: while he wants to tilt the manager in the direction of his optimal action (the benefit of misreporting), he is also afraid to tilt it too much, away even from his own optimal action, i.e., to "overshoot" (the cost of misreporting). As the agent's information becomes more important and hence the manager is expected to react more strongly to the agent's message, this fear makes the agent more reluctant to misreport.

Finally, the key implication of (8) is that the agent's incentive to communicate truthfully depends on how much information the manager obtains from *other* committee members, i.e., the set R: this is captured by the term $\frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau+|R|+1}$ on both sides of (8). However, as we show next, while expanding R relaxes the agent's IC constraint under heterogeneous preferences ($\rho_i = \rho_m$), it tightens the IC constraint under heterogeneous beliefs ($b_i = b_m$). To explain the intuition more clearly, we rewrite (8) for each of these two cases separately. For the remainder of the paper, we will use the following terminology.

Definition. We say that agents have ***heterogeneous preferences*** if $b_i \neq b_m$ for at least some i, but $\rho_i = \rho_m = \rho$ for all i. We say that agents have ***heterogeneous beliefs*** if $\rho_i \neq \rho_m$ for at least some i, but $b_i = b_m = b$ for all i.

Case 1. Heterogeneity in beliefs

In this case, $b_i = b_m = b$, so agent i reports his signal truthfully if and only if[14]

$$\left|\rho_m - \rho_i\right| \leq \frac{1}{2}\left[1 + c_i \frac{\tau + |R| + 1}{\sum_{j \in -R\setminus\{i\}} c_j}\right]. \tag{9}$$

Hence, the more information the manager gets from other committee members (i.e., the higher is $|R|$ and the lower is $\sum_{j\in -R\setminus\{i\}} c_j$), the more likely it is that agent i will also truthfully communicate his information. We refer to this effect as the *positive externality effect* of information transmission, because more information revealed to the manager by some agents encourages other agents to also report truthfully. The reason is that as the manager learns more information from others, he becomes more congruent with the agent. This happens for two related and complementary reasons. First, heterogeneous prior beliefs generate disagreement only over the information that is still unknown — once a certain piece of information gets revealed, there is nothing to disagree about. (This is captured by the manager learning signals θ_R out of $\theta_{\{1,\ldots,N\}}$.) The second reason, which is related and also aligns the manager's and agent's views, is that once a signal about the state is revealed, agents update their posteriors about the distribution of the state. Hence, even if an agent's and manager's prior beliefs were initially very different, they become closer to each other following the revelation of information by other agents. (This is captured by the manager updating his beliefs about φ and hence signals θ_{-R} after learning signals θ_R.)

Together, these two effects imply that the manager's and agent's optimal actions become more congruent as the manager learns more information from his other advisors, increasing the agent's incentives for truthful communication. To see this most starkly, consider the extreme case where the manager knows all the signals except agent i's signal:

[14]Here if $-R\setminus\{i\}$ is an empty set, the right-hand side of (9) is equal to infinity, i.e., (9) is always satisfied.

$R = \{1,...,N\}\backslash\{i\}$ and so $-R\backslash\{i\}$ is an empty set. In this case, truthfully reporting the last remaining signal θ_i results in the manager taking the action that is optimal from the agent's perspective, and hence is always incentive compatible.[15]

To illustrate this intuition in the context of our M&A example, suppose that the acquisition target is a pharmaceutical company with a pipeline of drugs it is developing. The optimal offer price then crucially depends on the success probability of each drug, but given that such drugs have not been developed before, different members of the advisory body may have very different priors about the likelihood of success of each drug. If they think the manager is too optimistic about the likelihood of success, they may try to counteract this optimism by more negative (non-truthful) reports about the aspects of the merger they are knowledgeable about. If, however, the advisory body includes an expert who knows the state of research on a particular disease, he can inform the manager about whether a given drug will be a failure or a success. Once this information is revealed to the manager, the uncertainty in target value coming from the success of this drug disappears, increasing the manager's congruence with other advisors (the first effect described above). Moreover, other advisors also realize that following the expert's report, the manager will adjust his prior belief about the likelihood of success of *other* drugs towards its correct value. This makes the manager even more congruent with other advisors and further decreases their incentives to misreport (the second effect described above).

Case 2. Heterogeneity in preferences

In this case, $\rho_i = \rho_m = \rho$, so agent i reports his signal truthfully if and only if

$$|b_m - b_i| \leq \frac{1}{2}\left[c_i + \frac{\sum_{j\in -R\backslash\{i\}} c_j}{\tau + |R| + 1}\right]. \tag{10}$$

[15]Note that the effect of heterogeneous priors is different from the effect of heterogeneous information under common beliefs. In Section A.5 of the Online Appendix, we analyze a variation of our model where priors are common and each agent gets two private signals, but can only send one binary message and hence effectively can communicate at most one signal. We show that this setting leads to different results. Intuitively, under heterogeneous priors, the decision-maker believes that the agent's prior is irrelevant for the decision, which creates an effective conflict of interest. In contrast, under common priors, if the agent possesses a signal she cannot communicate, the decision-maker believes that this signal is relevant for the decision.

In contrast to the case of belief heterogeneity, the more information the manager gets from other committee members (i.e., the higher is $|R|$ and the lower is $\sum_{j\in -R\setminus\{i\}} c_j$), the less likely it is that agent i will truthfully communicate his information. We refer to this effect as the *negative externality effect* of information transmission, because more information obtained by the manager from some agents harms the credibility of other agents. Intuitively, as the manager learns more information from others, he reacts less to the agent's message. Hence, the agent is less worried that misreporting the signal may tilt the manager's action too far away from his own optimal action and has stronger incentives to misreport (this intuition is similar to the earlier intuition for why a higher c_i encourages truthful communication).[16]

These externalities in communication imply the following result about the the size of the advisory body.

Proposition 2 (Committee size and communication). *Consider a committee B of informed agents.*

(i) If agents have heterogeneous beliefs, there is a cutoff on committee size $N_{\min}$, such that an equilibrium where all committee members truthfully communicate to the manager can only exist if $|B| \geq N_{\min}$.

(ii) If agents have heterogeneous preferences, there is a cutoff on committee size $N_{\max}$, such that an equilibrium where all committee members truthfully communicate to the manager can only exist if $|B| \leq N_{\max}$.

Proposition 2 implies that whether the manager should seek advice from a large vs. a small group of people crucially depends on the source of communication frictions. Under heterogeneous beliefs, the positive externalities in communication are not strong enough to overcome advisers' incentives to misreport unless the advisory body is large enough. This leads to the first statement of the proposition. The opposite result obtains under heterogeneous preferences: due to negative externalities in communication, truthful information revelation by all committee members is only possible if the committee

[16] This effect is also present in the case of heterogeneous beliefs, but as we show, it is dominated by the positive externality effect.

is small enough. This second statement is similar to the result of Morgan and Stocken (2008) about information revelation in polls when constituents have heterogeneous preferences.

3.2 Optimal advisory body

We now use the results in the previous section to analyze the optimal composition of the advisory committee. In order to define the optimal committee, it is useful to derive each agent's ex-ante expected utility as a function of a given committee. The next result shows that each agent is ex-ante better off if the manager is ex-post more informed:

Lemma 2 (Ex-ante payoffs). *Suppose that in equilibrium, the manager learns subset R of the signals and does not learn all the other signals, $-R$. Then the ex-ante equilibrium payoff of agent i (not accounting for his information acquisition costs) is given by*

$$\mathbb{E}_i[U_i|R] = u_0 - (b_m - b_i)^2 - \mathsf{A}_{im}(R) - \mathsf{B}_i(R) - \mathsf{C}_{im}(R), \tag{11}$$

where

$$\begin{aligned} \mathsf{A}_{im}(R) &= \tfrac{2(b_m-b_i)(\rho_m-\rho_i)}{\tau+|R|} \textstyle\sum_{j\in -R} c_j, \\ \mathsf{B}_i(R) &= \tfrac{\rho_i(\tau-\rho_i)}{\tau(\tau+1)} \left(\textstyle\sum_{j\in -R} c_j^2 + \tfrac{\left[\sum_{j\in -R} c_j\right]^2}{\tau+|R|} \right), \\ \mathsf{C}_{im}(R) &= \left[\tfrac{\rho_m-\rho_i}{\tau+|R|} \textstyle\sum_{j\in -R} c_j \right]^2. \end{aligned} \tag{12}$$

In particular, $\mathbb{E}_i[U_i|R]$ is increasing in $|R|$ and decreasing in any c_k, $k \in -R$.

Intuitively, agent i's utility from the manager's action is determined by how much information the manager's action reflects and by how different the manager's action is from the agent's optimal action given this information. Term $\mathsf{B}_i(\cdot)$ reflects the former — the loss in the agent's expected utility due to residual variance in the state, i.e., the fact that the manager's action does not reflect signals in $-R$. All the other terms capture the latter: term $(b_m - b_i)^2$ is the loss in agent i's utility due to conflicting preferences, while terms $\mathsf{A}_{im}(\cdot)$ and $\mathsf{C}_{im}(\cdot)$ reflect the additional effects due to the ex-post belief divergence between the agent and the manager.

Lemma 2 shows that for any agent in the economy, his ex-ante utility before acquiring and learning his private signal is higher when more information is known to the manager ex-post, i.e., when the set R is larger and $c_j, j \in -R$, is lower. For example, when all agents' information has the same importance, i.e., $c_i = c$ for all i, then each agent simply wants to maximize $|R|$, the number of signals that the manager learns from the advisory body.

We assume that the objective function in choosing the optimal committee is to maximize the sum of all N shareholders' expected utilities (11), net of their information acquisition costs. If $\kappa = 0$, our results generalize to any objective function that weakly increases in the expected utility of each agent and strictly increases in the expected utility of at least one agent.[17] We also make an assumption (which is similar to that in Persico, 2004) that there is an infinitesimally small positive cost of including each agent in the advisory body. This assumption ensures that it is optimal to search for the optimal advisory committee among the set of committees in which all members acquire information and truthfully communicate it to the manager. Thus, committee B is *optimal* if it maximizes the sum of expected utilities of all N agents, net of information acquisition costs of committee members, subject to information acquisition and truthful communication being optimal for all committee members.

We start with the case in which the cost κ of acquiring information is sufficiently small, so that committee members find it optimal to acquire information if they expect to communicate it (Proposition 3). This allows us to focus on the results that rely solely on communication frictions. In Proposition 4, we consider general information acquisition costs and study how communication frictions interact with the information acquisition friction. In both cases, the composition of the optimal committee depends on the nature of communication frictions – heterogeneous beliefs vs. heterogeneous preferences.

Proposition 3. *Suppose $\kappa \leq \kappa_l$, where $\kappa_l > 0$ is defined in the proof.*

(i) If agents have heterogeneous beliefs, the optimal advisory body is the entire set of agents: $B^* = \{1, ..., N\} \backslash \{m\}$.

(ii) If agents have heterogeneous preferences and for at least one agent the preference misalignment with the manager is sufficiently large, $|b_m - b_i| > \frac{1}{2} c_i$, the optimal advisory body

[17] This is because if $\kappa = 0$, the optimal committee is the same from any agent i's point of view, as Lemma 2 and the proof of Proposition 3 imply.

is a strict subset of all agents: $B^* \subset \{1,...,N\} \backslash \{m\}$.

Intuitively, under heterogeneous beliefs, the positive externality effect implies that the more agents are included in the advisory body and share their information, the more likely it is that other agents will also truthfully report their information. In fact, if the advisory body is the set of *all* informed agents, there is an equilibrium in which each agent reports his information truthfully. Indeed, if the manager learns all the N signals, there is no ex-post disagreement between him and other agents, and this, in turn, implies that truthful communication by all agents is indeed optimal. Because this maximizes the information available to the manager, this advisory body is optimal, leading to the first statement of the proposition. Formally, this can be seen from (11)–(12). Under heterogeneous beliefs, $\mathsf{A}_{im}(R) = 0$ and $\mathsf{B}_i(R)$ and $\mathsf{C}_{im}(R)$ decrease as set R expands, reaching zero when R includes all agents.

In contrast, under heterogeneous preferences, the negative externality effect implies that if the advisory body is too large (e.g., if it consists of all informed agents) and the misalignment in preferences is substantial, there is no equilibrium in which all members report their information truthfully. As long as there is an infinitesimal cost of including agents who do not contribute any information, the optimal advisory body will be a strict subset of all agents. Formally, this can be seen from a combination of Proposition 2 and Lemma 2. Under heterogeneous preferences, $\mathsf{A}_{im}(R) = \mathsf{C}_{im}(R) = 0$ and $\mathsf{B}_i(R)$ decreases as set R expands. Thus, the optimal committee should include the highest number of agents who report truthfully. But by Proposition 2, it is only a subset of all potentially informed agents.

Our setup assumes that together, agents in the economy know the state with certainty. The model can be easily extended to capture some residual information that none of the agents knows, and Proposition 3 will continue to hold as long as the amount of this residual unknown information is not too large. If it is very large and an agent's prior beliefs are very different from the manager's, then even if all other agents reveal their signals to the manager, it may not be sufficient to align their very different priors, giving the agent incentives to misreport. Importantly, the positive and negative externality effects continue to hold regardless of the amount of residual unknown information.

We next consider the general case, allowing for any possible information acquisition costs. To derive concrete implications about the optimal advisory committee, we make the simplifying assumption that all signals are equally informative for the rest of the paper.[18]

Assumption 1. *All signals are equally informative:* $c_i = c$ *for all* i.

If the cost of information κ is sufficiently large, the requirement that all committee members invest in information acquisition imposes an additional restriction on the size of the advisory body: regardless of whether agents differ in preferences or beliefs, the advisory body cannot be too large. Specifically:

Lemma 3 (Committee size and information acquisition). *Consider any committee* B *for which truth-telling conditions (8) are satisfied. Then, all members of committee* B *find it optimal to acquire information if and only if* $|B| \leq N_B(\kappa)$*, where* $N_B(\kappa)$ *decreases in* κ*. In particular,* $N_B(\kappa) < N-1$ *for any* $\kappa > \kappa_l$*. In the special case of* $\rho_i = \rho\ \forall i$*,* $N_B(\kappa) = N(\kappa)$*, i.e., it is independent of* B*.*

Intuitively, the larger is the size of the committee, and hence the larger is the aggregate information that the committee members possess, the lower is the marginal value of any additional signal. Therefore, if the committee is large enough, it is not optimal for all of its members to acquire information. This implies, in particular, that when agents have heterogeneous beliefs but information acquisition costs are non-trivial, then, differently from the result in Proposition 3, the optimal advisory body is no longer the set of all agents. Otherwise, some agents would end up not acquiring information in the first place, and hence it would be optimal to exclude them.

Lemma 3 imposes an upper bound on the size of the optimal committee, which complements the restrictions on the committee size from the requirement that all its members communicate truthfully (Proposition 2). Note also that because information acquisition has positive externalities on other agents, there is never overinvestment in it,

[18] The assumption $c_i = c$ implies that, as long as the IC constraints on information acquisition and communication are satisfied, committee size is a sufficient statistic for value U_i of each agent i. Indeed, according to Lemma 2, if $c_i = c$, each agent simply wants to maximize the number of signals the manager learns.

so the optimal committee is simply the committee of the maximum size that features information acquisition and truthful communication by all members. The next proposition characterizes the solution.

Proposition 4. *(i) If agents have heterogeneous beliefs, the optimal advisory body includes all agents with sufficiently low* $\left|\rho_i - \rho_m\right|$*, and its size* N^* *is:*

- *if* $\kappa \leq \kappa_l$, $N^* = N - 1$ *(the set of all agents);*
- *if* $\kappa \in (\kappa_l, \kappa_h)$, $N^* \in [N_{\min}, N-1)$ *and decreases in* κ*, where* $N_{\min}$ *is defined in Proposition 2;*
- *if* $\kappa \geq \kappa_h$, $N^* = 0$.

(ii) If agents have heterogeneous preferences, the optimal advisory body includes all agents with sufficiently low $|b_i - b_m|$*. Its size is* $N^* = \min\left(N(\kappa), N_{\max}\right)$ *and decreases in* κ*, where* $N(\kappa)$ *and* $N_{\max}$ *are defined in Lemma 3 and Proposition 2, respectively.*

In both cases, as information acquisition costs κ increase, the size of the optimal committee decreases, in order to maintain its members' incentives to acquire information. When agents have heterogeneous preferences, this smaller committee size improves communication due to negative communication externalities, and hence the optimal committee size gradually decreases in κ until it reaches zero. In contrast, when agents have heterogeneous beliefs, positive communication externalities imply that as the committee becomes smaller, each agent's incentives to misreport his information increase. At some point, when $\kappa = \kappa_h$, communication breaks down completely, and hence, interestingly, *any* advisory committee becomes completely ineffective. As a result, when $\kappa = \kappa_h$, the optimal committee size jumps down to zero.

4 Shareholder voting vs. board

Propositions 3 and 4 have direct implications for the use of advisory shareholder voting vs. advisory boards. Under advisory voting, all of the firm's shareholders are asked to convey their views to the management via the vote. The resulting decision-making protocol resembles our model with $B = \{1,...,N\}\backslash\{m\}$. One interpretation of Proposition 3 is that as long as shareholders' costs of information acquisition are not too large, such an advisory vote is the optimal decision-making protocol if agents have aligned preferences, but disagree due to differences in beliefs.

In contrast, when decisions are made with the help of a board, the management does not ask all shareholders for their opinions and gets feedback only from the board members. Propositions 3 and 4 suggest that this is always optimal if agents have heterogeneous preferences but common beliefs, regardless of their costs of information acquisition: negative externalities in communication limit the size of the optimal advisory committee. In addition, Lemma 3 and Proposition 4 imply that when information is sufficiently costly, then even under heterogeneous beliefs, restricting the size of the advisory body may be optimal to incentivize information acquisition in the first place, and this can make advisory voting no longer optimal. In Section 6, we discuss the implications of these results for different types of corporate decisions.

In our model, when the advisory board is optimal, it includes a subset of all shareholders of the firm. In practice, it is not uncommon for board members to come from outside of the firm's shareholders. The model can capture this possibility via a more general interpretation of the set of informed agents. Specifically, suppose that in addition to the firm's shareholders, there are also many other stakeholders, such as customers, employees, and industry participants, who care about the firm's decisions. Some of these stakeholders are uninformed, but some are informed. Under the more general interpretation, the set of N informed agents includes not only the firm's shareholders, but also all informed stakeholders. Then, in cases where the optimal committee is a subset of informed agents (i.e., board), this board may include a combination of some informed stakeholders and some shareholders, consistent with the observed practices. Similarly, in cases where the optimal committee consists of all informed agents, then an advisory shareholder vote is still a necessary part of the optimal advisory process, but it can be useful to combine such an advisory vote with asking the opinion of other informed stakeholders – e.g., by also conducting employee or customer surveys.[19] While the interpretation is different,

[19]No uninformed stakeholders will be included in the optimal committee if it plays a purely advisory role. As the next subsection shows, this is not true when the committee has formal authority over decision-

the main result is the same.

4.1 Voting vs. board with authority

The previous sections analyzed the optimal composition of purely advisory bodies. This analysis applies to companies in which management has de facto control rights over major corporate decisions due to a sufficiently large ownership or shares with superior voting power. In this subsection, we analyze the implications for other companies, where the advisory body also has authority to approve the decisions proposed by management. We show that the main result about when voting vs. board is optimal is the same as under purely advisory committees. The novel element is the implication for board composition: while purely advisory boards should be homogeneous, boards with authority should be diverse.

Specifically, building on the more general interpretation of the set of agents discussed above, consider the following setup. There is a set of N potentially informed agents, some of whom are shareholders and some of whom are other stakeholders of the firm. In addition, there are infinitely many uninformed agents with biases distributed according to some positive density $f(\cdot)$ over the real line: $f(b) > 0\ \forall b \in (-\infty, \infty)$. Each uninformed agent i's preference is described by (1).

For tractability, we impose additional restrictions, which were not present in the basic model. First, we focus on the case in which information acquisition costs are infinitesimal. Second, we now assume that the manager is uninformed since incorporating private information of the manager will result in a difficult signaling problem. This means that the value added from the manager's presence is not in possessing additional information but rather in having a unique skill that is necessary for the implementation of the project. Finally, to simplify the characterization of the optimal committee, we assume that informed agent's biases are symmetrically distributed around zero with increasing distance,[20] and that the biases of shareholders are also distributed symmetrically around zero with the median shareholder's bias being zero.

making in addition to giving advice.

[20] I.e., agent 1's bias is zero ($b_1 = 0$); agent 2's bias is $b_2 \geq 0$; agent 3's bias is $b_3 = -b_2 \leq 0$; agent 4's bias is b_4 with $b_4 - b_2 \geq b_2$; agent 5's bias is $b_5 = -b_4$; and so on with agent $2i$'s bias being $b_{2i} \geq 0$ with $b_{2i} - b_{2i-2} \geq b_{2i-2} - b_{2i-4}$, and agent $2i+1$'s bias being $b_{2i+1} = -b_{2i}$. Note that by construction N is even.

The timing is as follows. There is a committee B, which is a subset of all agents excluding the manager. First, all informed members of the committee simultaneously communicate their information to the manager. Next, the manager decides on the action a to propose to the committee. Finally, all committee members simultaneously vote whether to accept the action proposed by the manager or to get the status-quo utility $\bar{u}$, which we normalize to zero. the voting rule is $N_B \leq |B|$: if N_B or more committee members vote for the manager's action a, it gets undertaken. Otherwise, all agents get the status-quo payoff.

Using the M&A example discussed above, we can interpret this setting in the following way. The status-quo utility $\bar{u} = 0$ is the stand-alone value of the bidding firm, while $U_i(a, Z, b_i)$ is the expected value of the bidding firm to shareholder i from bidding a. The role of the committee is two-fold. First, its members communicate with the manager, who then proposes the acquisition price a. Second, its members vote whether to do the deal for a or to not pursue the acquisition. If the committee includes all stakeholders, the vote is similar to the shareholder vote on approving the deal. If the committee includes only a subset of stakeholders, the vote represents the requirement that all acquisitions must be approved by the board.

We next search for the optimal committee B and voting rule N_B. As before, the objective function is to maximize the sum of utilities of all shareholders, which under the assumption of biases symmetrically distributed around zero, is equivalent to maximizing the utility of the median (unbiased) shareholder.

First, consider the case in which agents differ in beliefs but have the same objectives. The next proposition shows that as in the case of pure advisory committees, the optimal committee includes all informed agents.

Proposition 5. *If agents have heterogeneous beliefs, the optimal committee with authority is the entire set of informed agents, $B^* = \{1, ..., N\} \backslash \{m\}$, and any voting rule is optimal.*

The intuition for Proposition 5 is similar to that for part (i) of Proposition 3. Since communication of different agents has positive externalities, the best advice is provided when the committee includes all informed agents. Under this optimal committee, all agents communicate signals to the manager truthfully, and the manager proposes the efficient action $a = Z$. Because there is no ex-post disagreement between the manager and other agents, the voting rule does not matter, as the agents rubber-stamp the action proposed by the manager. Overall, whether the committee is purely advisory or has formal authority makes no difference in this case.

Second, consider the case in which agents differ in preferences but have the same prior beliefs. The next result shows that as in the case of purely advisory committees, the board, rather than a shareholder vote, is optimal. However, the optimal board now also includes a agent who is biased in the opposite direction from the manager.

Proposition 6. *Suppose that agents have heterogeneous preferences, that $\bar{u} \le \bar{u}^*$, where $\bar{u}^*$ is given by (28) in the appendix, and $b_m \neq 0$. Consider a committee that includes all informed agents whose biases are sufficiently low ($|b_i| \le \beta$, where β is given by (27) in the appendix), and one uninformed agent with bias b_u,* such that $b_u b_m < 0$, *given by (29) in the appendix. This committee together with a unanimity rule for approving the manager's proposal is optimal.*

Thus, as long as $|b_i| > \beta$ for some i, the optimal committee includes a strict subset of informed agents, which we interpret as the board of directors. The intuition is similar to the case of purely advisory committees in Proposition 4: because of negative communication externalities, it is not possible to sustain truthful communication by all informed agents.

The new element, compared to the case of pure advisory boards in Proposition 4, is the composition of the board. The optimal board in Proposition 4 is homogeneous: all its directors are sufficiently aligned with the manager, i.e., have low $|b_i - b_m|$. In contrast, the optimal board in Proposition 6 is diverse: part of its members include directors aligned with the median shareholder, i.e., those with low $|b_i|$, but one of the directors is strongly biased, in the direction opposite of the manager. Because she has no relevant private information, this director plays no advisory role, but creates value by imposing a credible threat on the manager to veto his proposed action if it is too biased. The presence of this more extreme director makes the effective bias of the manager zero, to match the preferences of the median shareholder.

It is also interesting to compare the size of the optimal advisory board with the size of the optimal board with authority:

Corollary to Proposition 6. *If agents have heterogeneous preferences, the optimal purely advisory board is smaller than the optimal board with authority.*

This happens for two reasons. First, the board with authority includes an agent biased against the manager who does not provide advice; the board that is purely advisory has no reason to include such an agent. Second, the presence of this agent makes the de-facto bias of the manager zero, which allows for more members on the board while still satisfying the constraint for their truthful communication (due to the distribution of agents' biases being more concentrated around zero). This second effect implies that the presence of a director biased against the manager does not harm the board's advisory role. Thus, the board's advisory and monitoring role are not necessarily in conflict (as is the case when the board consists of a single agent, e.g., Adams and Ferreira, 2007): a diverse board in which some directors oppose the manager's bias and others provide advice plays both roles effectively.

5 Extensions

In this section, we consider several extensions of the model. For simplicity, we focus on the model with purely advisory committees. The complete analysis of these extensions is presented in the Online Appendix, and we present the summary of the main results here.

5.1 Endogenous ownership structure

So far, we have assumed that the firm's ownership structure consists of all N agents. In reality, potentially informed agents may choose not to become shareholders. In this case, they do not care that much about the firm's decisions and hence may have little incentive to communicate their information, as well as to acquire it in the first place. Moreover, even if such agents had relevant information and wanted to share it with the manager, they would generally have little opportunity to do so.

In this section, we extend the setup in Section 2 to capture this feature and endogenize the decisions of the N informed agents to become shareholders. If agent i does not become a shareholder, he does not acquire and/or communicate his potential information θ_i to the manager. Specifically, at the beginning of the game, all N agents participate in the market for the firm's shares, during which the total stock of the firm is sold in a competitive market. Suppose that the stock is in unit supply, so that holding α_i shares is equivalent to holding fraction α_i of the firm. We assume that agent i's utility from buying stake α_i is given by

$$\alpha_i \left(\mathbb{E}_i[U_i] - p \right) - \frac{\lambda}{2} \alpha_i^2, \tag{13}$$

where U_i is the agent's utility from each share and is given by (1), p is the share price, and $\lambda > 0$ captures either the holding cost due to limited diversification and risk aversion or the transaction cost due to limited liquidity. The firm's shareholder base, $S \subseteq \{1, ..., N\}$, is thus endogenously determined and consists of all agents who hold a positive number of shares after the trading stage: $S = \{i : \alpha_i > 0\}$. The firm forms an optimal advisory committee B out of its shareholders, $B \subseteq S$, to maximize shareholder value. For example, $B = S$ corresponds to an advisory vote of all shareholders, and $B \subset S$ corresponds to a board composed of selected shareholder representatives. After that, the game proceeds as in the basic model. For simplicity, we assume that κ is zero in this extension.

As the analysis in the Online Appendix demonstrates, if agents have heterogeneous beliefs, there always exists an equilibrium in which all informed agents become shareholders and the manager's action reflects all available information. Indeed, assuming that the manager learns all N signals, there is no ex-post disagreement between him and any of the shareholders, which makes truthful communication by all shareholders indeed optimal. Moreover, because of no ex-post disagreement between agents, they all value the firm in the same way and thus acquire equal stakes, $\alpha_i = \frac{1}{N}$, despite their ex-ante differences in beliefs. This equilibrium is efficient in that it maximizes total welfare, both because the firm's decision uses the maximum available information, and because agents' total holding costs are minimized since the asset is evenly divided among them.

However, we show that there can also exist an inefficient equilibrium, in which only a subset of the agents become shareholders. In this equilibrium, the manager's decision is not based on all the available information and total holding costs are larger since the firm is not equally divided among N agents. The share price in this inefficient equilibrium is lower than in the efficient equilibrium. The source of equilibrium multiplicity are positive externalities in communication. Intuitively, if only a subset of agents are expected to become shareholders and communicate their information to the manager, there is still ex-post disagreement between the manager and the shareholders who have different prior beliefs. Realizing this at the trading stage, shareholders who have substantially different beliefs from those of the manager do not buy shares in the first place. The set of shareholders thus only includes a relatively narrow set of investors whose prior beliefs are sufficiently close to those of the manager, making this equilibrium self-fulfilling.

This result demonstrates that the effectiveness of advisory voting depends on the firm's ownership structure. In the basic model, advisory voting always resulted in efficient decision-making (Proposition 3) because we assumed that the shareholder base included all N agents. In contrast, when the shareholder base is endogenously determined, it may no longer be optimal to rely on advisory voting alone: this may lead to an inefficient equilibrium, in which only investors with very similar priors to those of the management become shareholders. As we discuss in more detail in the Online Appendix, advisory voting can then be made more effective by the presence of passive (index) investors, whose stake in the firm does not depend on whether they agree or disagree with management. The advice received by management from informed passive investors can eliminate the inefficient equilibrium and thereby improve decision-making.

5.2 Expertise of the manager

Our next extension connects the advisory role of the board to the expertise of the manager. We introduce a measure of the manager's expertise by assuming that the manager knows a subset $\mathcal{M}$ of signals $\{\theta_i, i \neq m\}$ in addition to signal θ_m. We interpret an expansion/contraction of $\mathcal{M}$ as an increase/decrease in the manager's expertise.

We show in the Online Appendix that if agents have heterogeneous beliefs (preferences) and board B is efficient at providing advice under $\mathcal{M}$, then it is also efficient at providing advice if set $\mathcal{M}$ expands (contracts), i.e., as the manager becomes more (less) informed. In other words, whether the manager's expertise and the advisory role of the board are complements or substitutes depends on the nature of communication frictions. The intuition is close to the intuition behind positive and negative externalities in Section 3.1. As the manager becomes more informed, there is less information relevant for the decision that neither the manager nor the board knows. If agents have heterogeneous beliefs, the key consequence is the increase in the manager's congruence and enhanced communication because there is less information that the manager and directors can disagree about. In contrast, if agents have heterogeneous preferences, greater managerial expertise decreases directors' costs of misreporting because the manager is expected to react less to each director's message.

We conclude that the manager's expertise enhances the advisory role of the board if the source of conflict are heterogeneous beliefs, but impedes its advisory role if the source of conflict are heterogeneous preferences. Interestingly, the latter result implies that it can be Pareto improving to appoint a less informed manager, even if the less informed manager has the same preferences as the more informed manager and the advisory board can be chosen optimally. Intuitively, when heterogeneity in preferences is substantial, the manager's expertise has two opposing effects. First, a more informed manager has better private information, which improves decision-making. Second, a more informed manager obtains less information from the board members, which harms decision-making. The numerical example in the Online Appendix illustrates that this second effect can dominate.

5.3 Heterogeneous interpretation of information

In our basic model, agents have different prior beliefs about the distribution of signals, but they process information in similar ways. As a result, if all N agents shared their private signals with each other, they would perfectly learn state Z and disagreement would go away, i.e., there would be full convergence of beliefs. However, it is also natural to expect that different agents can interpret signals differently in the sense that they can disagree about the mapping between signals and the state. In this case, disagreement can remain even if all information is shared. A large literature on trading in financial markets models differences of opinion arising from different interpretation of information.[21]

In this section, we extend the model to capture different interpretations of signals by shareholders. For simplicity, we focus on the case of identical preferences ($b_i = b_m = 0$) and $\kappa = 0$. We also assume that $Z = \sum_{i=1}^{N} c\theta_i$, i.e., that the signals of all agents are

[21] For example, Harris and Raviv (1993), Kandel and Pearson (1995), Banerjee et al. (2009), and Banerjee and Kremer (2010).

equally important. To model different interpretations of signals in a tractable way, we follow models of differences of opinion in which agents disagree about the precision of signals, and each agent's belief about the precision of his signal is higher than other agents' beliefs about it (e.g., Banerjee et al., 2009; Kyle et al., 2018). Specifically, we assume that each shareholder overestimates the importance of his signal: given a set of signals $\left\{\theta_j, j = 1, ..., N\right\}$, shareholder i believes that the state of the world is equal to

$$Z_i = \gamma c \theta_i + \sum_{j \neq i} c \theta_j, \tag{14}$$

where $\gamma \geq 1$ captures the extent of heterogeneous interpretation of signals. In this setting, even if all agents fully shared their information, they would still disagree about the state and thus the optimal action.

We show in the Online Appendix that the overconfidence of each agent decreases his incentives to misreport. Intuitively, if the agent perceives his signal to be more important than it actually is, he perceives lying to be costlier, since lying leads to a more inaccurate decision as perceived by the agent. Importantly, we also show that the conclusions of the basic model continue to hold in this extension. Specifically, communication has positive externalities for the same reason as before: although each agent overestimates the importance of his private signal, for any given realization of his private signal, communication by other agents brings posterior beliefs of the manager and the agent closer to each other. As a consequence of positive externalities, the optimal committee is again the entire set of informed agents. However, differently from the basic model, this committee does not result in fully efficient decision-making, because the manager overweighs his signal when taking the action.

To summarize, the key property for our results is that communication by other agents moves the manager's and agent's beliefs about the state closer to each other. As the above extension shows, this property is consistent with heterogeneous interpretations of signals and does not require complete convergence of beliefs under full information. This property holds in a large class of models of different beliefs, but not in all of them.

6 Implications

There is a large empirical literature that studies the advisory role of the board and advisory shareholder voting. Our model offers new predictions about the advising effectiveness of the board and shareholder votes, as well as the role of board size, composition, and board committee structure.

6.1 Advisory role of the board and shareholder voting

The literature on the board's advisory role studies how the presence of directors with a certain type of expertise is related to corporate policies and performance. For example, Dass et al. (2014) analyze directors' expertise in related industries, Guner et al. (2008) study the role of financial expertise, while Harford and Schonlau (2013) and Field and Mkrtchyan (2016) focus on the role of directors' experience in mergers and acquisitions. The unique prediction of our model is that the advisory role of a director (i.e., whether his information will influence the manager's decisions) should not be viewed in isolation, but crucially depends on the expertise of other board members. Specifically, the analysis of communication externalities in Section 3.1 implies the following prediction:

Prediction 1. *When directors and manager have conflicting preferences (beliefs), the advisory role of a given director is weakened (enhanced) by the expertise of other directors.*

To test the above prediction, one could use similar measures of directors' advising effectiveness as those used in the empirical literature above. In addition, one needs to proxy for heterogeneity in preferences and beliefs, which can be done by exploiting heterogeneity both across firms and across different types of decisions. For example, across firms, conflicting preferences between shareholders and the manager are more likely in firms with poor governance practices and ineffective executive compensation since in such firms, the manager's incentives are not aligned (leading to higher $|b_m - b_i|$). Across decisions, heterogeneity in preferences is likely to arise if the decision involves a clear conflict of interest, such as an investment/acquisition or an increase in the scale of production that brings large private benefits to the manager. To capture heterogeneity of beliefs, one could rely on several measures of belief heterogeneity proposed by the literature (e.g., Thakor and Whited, 2011; Diether et al., 2002; Malmendier and Tate, 2005). Across decisions, there is likely to be strong heterogeneity in prior beliefs about the success of a brand new technology, or the development of an innovative drug. Likewise, there is often substantial disagreement about the effect of corporate governance policies, even among parties with similar interests, such as shareholders with similar portfolios.[22]

Our next prediction connects the advisory role of the board to the expertise of the manager. Are the two complements or substitutes? Both views have been expressed in the empirical literature, but have not been formally explored in a unified framework.[23] As Section 5.2 shows, the manager's expertise has a negative effect on the board's advising effectiveness under heterogeneous preferences, but a positive effect under heterogeneous beliefs:

Prediction 2. *When directors and manager have conflicting preferences (beliefs), the advisory role of the board is weakened (enhanced) by the expertise of the manager.*

Our analysis also has implications for the advisory role of shareholder votes. Voting on many types of proposals, such as say-on-pay and proposals sponsored by shareholders, is non-binding and primarily plays an advisory role. The literature has studied the effectiveness of such votes' advisory role by measuring management's responsiveness to the vote tally (e.g., Ertimur et al., 2010; Ferri, 2012; Cut al., 2012; and others). Our analysis predicts that the advising effectiveness of the vote depends on the number of shareholders. Specifically:

Prediction 3. *When shareholders and manager have conflicting preferences (beliefs), the advising effectiveness of shareholder voting is higher when the firm has a smaller (larger) number*

[22]See, e.g., "A Lack of Consensus on Corporate Governance", *The New York Times* (September 29, 2015), discussing shareholder disagreements on the issue of CEO-chairman separation. Another example is the debate about proxy access, when different shareholders and governance experts disagreed about the optimal terms of proxy access, such as the minimum size and holding period requirements. See "The Proxy Access Debate", *The New York Times* (October 9, 2009).

[23]For example, Armstrong et al. (2010) discuss that managers' informational advantage may impede the advisory role of outside directors. On the other hand, Sundaramurthy et al. (2014), which is the only paper we know that tests the relationship between CEO expertise and directors' advisory role, hypothesizes that directors' experience and expertise are more impactful when the manager has greater expertise as well.

of shareholders.

Formally, this prediction follows from a slight variation of the model in which N is the overall number of signals that determine the state and $S \subseteq \{1,...,N\}$ is the set of the firm's shareholders, which is also the set of agents communicating to the manager. The comparative statics of the constraint for truthful communication with respect to $|S|$ immediately implies Prediction 3.[24]

6.2 Board size and composition

A number of empirical studies examine the determinants and effects of board size (e.g., Yermack, 1996; Coles et al., 2008; Jenter et al., 2019; and others). One drawback of large boards that is sometimes discussed in this literature is the free-rider problem, whereby a larger board size discourages each individual director from exerting effort. A related effect is also present in our framework: as Lemma 3 and Proposition 4 show, the larger the board, the lower are each director's incentives to acquire private information, leading the optimal board size to decrease in the costs of information acquisition. In addition, our model predicts that a smaller board is optimal if there is stronger misalignment in preferences between the manager and potential directors. Indeed, when conflicts of interest are substantial, negative externalities in communication imply that directors will misreport their information unless the board is sufficiently small (we formally show this prediction at the end of the proof of Proposition 4). We summarize these predictions as follows:

Prediction 5. *The optimal advisory board is larger if:*

[24]Note that this prediction is different from another model of advisory voting, by Levit and Malenko (2011): in their paper, if the manager is conflicted, the vote does not aggregate information regardless of the number of shareholders. The reason our results differ is due to a different economic mechanism: unlike in our paper, the mechanism in Levit and Malenko (2011) works through shareholders conditioning their decisions on being pivotal. This is due to the different way we view the role of advisory votes. In their paper, the vote only matters in specific cases – when it changes the manager's decision from "not implement" the proposal to "implement", once the vote tally exceeds a certain cutoff. In contrast, we view the vote tally as affecting decisions even away from the cutoff – e.g., because it affects the extent to which the proposal is implemented. In practice, both roles are important: the literature shows both a monotonic increase in the probability and extent of proposal implementation as a function of the vote tally (e.g., Ertimur et al., 2013) and a discrete jump in the probability of implementation around certain cutoffs (e.g., Cut al., 2012).

(i) *the manager's and directors' preferences are more aligned;*

(ii) *directors' costs of becoming informed are lower.*

In addition, building further on the analysis of the manager's expertise in Section 5.2, we expect that under heterogeneous preferences, it is optimal to have a larger advisory board if the manager is less informed. This prediction is broadly consistent with the evidence in Coles et al. (2008), who study board size in the context of the board's advisory role. They find that boards of firms whose CEOs require more advice (which they proxy, e.g., by firm diversification and size) have larger boards, and this larger board size is driven by a larger number of outside directors. In the context of our model, the set of outside directors can be interpreted as the firm's advisory committee (which is also the interpretation of Coles et al., 2008), while inside directors can be interpreted as the manager.

Another new implication compares purely advisory boards and boards that have some decision-making authority, such as in firms with vs. without dual-class shares. As Proposition 6 and its corollary show, the advisory board is smaller and includes directors whose preferences are more homogeneous. In contrast, boards with authority are larger and more diverse. We summarize these implications as follows:

Prediction 6. *The optimal advisory board is smaller than the optimal board with authority. In addition, the optimal advisory board is more homogeneous in that all its directors' preferences are aligned with the management. In contrast, the optimal board with authority is diverse and includes directors aligned with the median shareholder and those biased against the management.*

The last part of this prediction is broadly consistent with Adams and Ferreira (2009), who show that women are more likely to be appointed to monitoring committees (audit, corporate governance, and nominating) than male directors. It will be interesting to test the other implication of Prediction 6, that unlike monitoring committees, diverse directors are less likely to be appointed to advisory committees.

Effect of board size on performance. Since board composition is chosen endogenously, our analysis does not generally predict any specific correlation between board size and performance. It does, however, predict how performance will be affected by exogenous changes in board size. One example is the 1976 law in Germany, which introduced new requirements for supervisory board size. Jenter et al. (2019) study this regulation via regression-discontinuity and difference-in-differences analyses and conclude that forcing firms to have larger boards lowers performance and value. This evidence is consistent with our model if heterogeneity in preferences is strong: differences in preferences are particularly likely in German supervisory boards, which contain shareholder and employee representatives, but no executive directors. Indeed, our model predicts that adding members to the optimal board characterized in Proposition 4 decreases value because it does not lead to more communication and better advice but imposes additional costs, for example, the costs of new board members' compensation.[25]

6.3 Board committees

Our paper also offers a new perspective on the use of board committees. Board committees are widespread: in addition to those required by the Sarbanes-Oxley Act and exchange listing standards (compensation, audit, and governance/nomination committees), firms also form finance, technology, strategy, mergers and acquisitions, ethics, and other types of committees.[26] Moreover, these committees play an advisory role: their task is to collect information and give recommendations to the board, which retains decision-making power.

Our model can be loosely applied to study the optimal board committee structure by interpreting the set of N informed agents as the entire board of directors, and the decision-maker m as the chairman of the board or the median director. Then, the key implication of Propositions 3 and 4 is that if directors have strong conflicts of interests regarding the decision or high information acquisition costs, it is optimal to form a subcommittee of the board, while if they have heterogeneous beliefs and low information acquisition costs, it is optimal for the entire board to contribute its views. Note that compensation, audit, and governance/nomination committees, which are required for

[25]These costs are assumed to be infinitesimally positive in our setting, but could be significant in practice.
[26]See, e.g., Adams et al. (2020), Chen and Wu (2016), and Reeb and Upadhyay (2010).

public firms, are responsible for highly controversial issues that are likely to raise significant conflicts of interest, justifying the use of committees for such issues. On the other hand, Chen and Wu (2016) document that the use of other types of committees – such as technology, strategy, and ethics – is rare, which could be consistent with such issues generating differences of opinion.

Moreover, Chen and Wu (2016) also find that the use of board committees is higher in firms where the positions of the CEO and chairman of the board are combined. This is again in line with our predictions because in such firms, the key decision-maker on the board is an insider rather than an independent director, and hence conflicts of interest are more likely.

Finally, Reeb and Upadhyay (2010) show that the number of committees is positively associated with measures of corporate complexity, consistent with the above prediction about the role of directors' information acquisition costs.

7 Conclusion

When information about the company is dispersed among multiple shareholders and stakeholders, the manager can benefit from seeking advice of these informed parties. Two key examples of such provision of advice to management are through the company's board of directors and advisory shareholder voting. The goal of this paper is to study the optimal design of the advisory body and derive implications for the use of advisory voting vis-is advisory board. We analyze a setting where members of the advisory body decide whether to acquire private information and communicate it to a partially informed manager, who then makes the decision. In this setting, communication can be inhibited by two key frictions – heterogeneous preferences and heterogeneous prior beliefs, which have very different implications for optimal advisory structures.

When agents have heterogeneous prior beliefs, communication exhibits positive externalities: as more advisors reveal their information to the manager, the incentives of other advisors to reveal their information become stronger. In contrast, when agents have heterogeneous preferences, communication externalities are negative. As a result, the optimal advisory committee is the set of all potentially informed agents if agents have heterogeneous beliefs and their costs of acquiring information are low, but a strict subset of informed agents if agents have heterogeneous preferences or collecting information is sufficiently costly. This implies that it is better to seek advice from a small advisory board on issues involving a significant misalignment in preferences. Conversely, advisory shareholder voting, combined with the manager seeking the advice of other informed stakeholders, is more effective for issues involving considerable heterogeneity in beliefs.

While the basic model deals with purely advisory committees, we show that these implications also extend to a setting in which the committee has authority over approving the action proposed by the management. Specifically, shareholder voting is optimal if agents have heterogeneous beliefs, while a board is optimal if agents have heterogeneous preferences. At the same time, the composition of the optimal board with authority is different from the optimal advisory board. While the latter contains directors with preferences sufficiently aligned with the management, the former is diverse and has directors aligned with the median shareholder as well as those biased in the opposite direction from the management, such as labor union representatives.

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Appendix

Appendix A contains the proofs of the main results. Appendix B analyzes an extension about the role of the manager's expertise, which is used for Prediction 2 in Section 6.

Appendix A: Proofs

Proof of Lemma 1

Since θ_i is a binary signal equal to 1 with probability φ and 0 with probability $1-\varphi$, the manager's optimal action (4) can be written as:

$$a_m(\theta_R) = b_m + \sum_{i\in R} c_i\theta_i + \mathbb{E}_m\left[\varphi|\theta_i, i\in R\right]\sum_{j\in -R} c_j \ .$$

Let $\mathbf{1}_R \equiv \sum_{i\in R}\theta_i$ be the number of signals in R equal to 1. The conditional probability that $\mathbf{1}_R$ signals out of $|R|$ are equal to one given φ is $P(\mathbf{1}_R|\varphi) = \binom{|R|}{\mathbf{1}_R}\varphi^{\mathbf{1}_R}(1-\varphi)^{|R|-\mathbf{1}_R}$. Since the prior distribution is Beta and the likelihood function is Binomial, the posterior distribution is also Beta but with different parameters (this is a known property of the Beta distribution). Formally, let $P_i(\mathbf{1}_R)$ be agent i's assessed probability that $\mathbf{1}_R$ signals out of $|R|$ are equal to 1 (over all possible values of φ). Using Bayes rule, agent i's posterior belief of φ, $P_i(\varphi|\mathbf{1}_R)$, is

$$\begin{aligned} P_i(\varphi|\mathbf{1}_R) &= \frac{f_i(\varphi)P(\mathbf{1}_R|\varphi)}{P_i(\mathbf{1}_R)} = \frac{\varphi^{\rho_i-1}(1-\varphi)^{\tau-\rho_i-1}}{Beta(\rho_i,\tau-\rho_i)}\frac{1}{P_i(\mathbf{1}_R)}\binom{|R|}{\mathbf{1}_R}\varphi^{\mathbf{1}_R}(1-\varphi)^{|R|-\mathbf{1}_R} \\ &= \frac{1}{Beta(\rho_i,\tau-\rho_i)P_i(\mathbf{1}_R)}\binom{|R|}{\mathbf{1}_R}\times\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}, \end{aligned}$$

which is some constant that does not depend on φ times $\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}$. Since the posterior beliefs must integrate to one over possible values of φ, this automatically implies that the posterior belief also follows a Beta distribution with parameters $(\rho_i+\mathbf{1}_R, \tau-\rho_i+|R|-\mathbf{1}_R)$ and density

$$P_i(\varphi|\mathbf{1}_R) = \frac{1}{Beta(\rho_i+\mathbf{1}_R,\tau-\rho_i+|R|-\mathbf{1}_R)}\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}.$$

It is known that the mean of a Beta distribution with parameters (α,β) is $\frac{\alpha}{\alpha+\beta}$. Therefore, using these expressions and the above posterior distribution, agent i's expected value of φ is $\mathbb{E}_i(\varphi|\mathbf{1}_R) = \frac{\rho_i+\mathbf{1}_R}{\tau+|R|}$, which proves the lemma.

Auxiliary Lemma A.1

Suppose $\varphi \sim Beta(\rho, \tau-\rho)$ and $X = \{x_1, x_2, \cdots, x_n\}$, where $x_i \in \{0,1\}$ are independent draws with $x_i = 1$ with probability φ. Let $\mathbf{1}_X \equiv \sum_{i=1}^n x_i$. Then

$$\begin{aligned}\mathbb{E}_X[\mathbf{1}_X] &= n\frac{\rho}{\tau} \\ \mathbb{E}_X[\mathbf{1}_X^2] &= n\rho\,\frac{\tau-\rho+n(\rho+1)}{\tau(\tau+1)}.\end{aligned}$$

Proof. It is known that the first two moments of a random variable X distributed according to a Beta distribution with parameters α and β are $\mathbb{E}[X] = \frac{\alpha}{\alpha+\beta}$ and $\mathbb{E}\left[X^2\right] = \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$. Hence, $\mathbb{E}[\varphi] = \frac{\rho}{\tau}$ and $\mathbb{E}\left[\varphi^2\right] = \frac{\rho(\rho+1)}{\tau(\tau+1)}$. Using this, we get

$$\mathbb{E}[1_X] = \mathbb{E}\left[\sum_{i=1}^n x_i\right] = n\mathbb{E}[x_i] = n\mathbb{E}[\varphi] = n\frac{\rho}{\tau}$$

and

$$\begin{aligned}\mathbb{E}\left[1_X^2\right] &= \mathbb{E}\left[\sum_{i=1}^n x_i^2 + \sum_{i\neq j} x_i x_j\right] = \mathbb{E}\left(n\mathbb{E}\left[x_i^2|\varphi\right] + n(n-1)\,\mathbb{E}[x_i|\varphi]^2\right) \\ &= n\mathbb{E}[\varphi] + n(n-1)\,\mathbb{E}\left[\varphi^2\right] = \frac{n\rho}{\tau(\tau+1)}(\tau-\rho+n(\rho+1)).\end{aligned}$$

Proof of Lemma 2

Let $\mathbf{1}_R = \sum_{i\in R}\theta_i$ denote the number of signals 1 in R. Using Lemma 1, we obtain agent i's ex-ante payoff, $\mathbb{E}_i(a_m(\theta_R) - Z - b_i)^2$, as follows:

$$\mathbb{E}_i[U_i|R] = u_0 - (b_m - b_i)^2 - U_1 - U_2, \tag{15}$$

where

$$\begin{aligned}U_1 &\equiv 2(b_m - b_i)\,\mathbb{E}_i\left[\left(\frac{\rho_m + \mathbf{1}_R}{\tau+|R|}\sum_{k\in-R} c_k - \sum_{k\in-R} c_k\theta_k\right)|R\right], \\ U_2 &\equiv \mathbb{E}_i\left[\left(\frac{\rho_m + \mathbf{1}_R}{\tau+|R|}\sum_{k\in-R} c_k - \sum_{k\in-R} c_k\theta_k\right)^2 |R\right].\end{aligned}$$

Using independence of θ_k conditional on φ, and Auxiliary Lemma A.1, U_1 simplifies to

$$U_1 = 2\left(b_m - b_i\right)\frac{\rho_m - \rho_i}{\tau + |R|}\left(\sum_{k\in -R} c_k\right) = \mathsf{A}_{im}(R). \tag{16}$$

To simplify U_2, we use the law of iterated expectations:

$$\begin{aligned} U_2 &= \mathbb{E}_i\left[\left(\frac{(\rho_m + \mathbf{1}_R)\sum_{k\in -R} c_k}{\tau + |R|}\right)^2 - 2\frac{(\rho_m + \mathbf{1}_R)(\rho_i + 1_R)\left(\sum_{k\in -R} c_k\right)^2}{(\tau + |R|)^2}|R\right] \\ &\quad + \mathbb{E}_i\left[\mathbb{E}_i\left[\left(\sum_{k\in -R} c_k\theta_k\right)^2 |\theta_R, R\right]|R\right], \end{aligned} \tag{17}$$

where we used $\mathbb{E}_i\left[\sum_{k\in -R} c_k\theta_k|\theta_R, R\right] = \left(\sum_{k\in -R} c_k\right)\mathbb{E}_i\left[\varphi|\theta_R, R\right] = \left(\sum_{k\in -R} c_k\right)\frac{\rho_i + 1_R}{\tau + |R|}$. Consider the last term under the expectation sign:

$$\begin{aligned} \mathbb{E}_i\left[\left(\sum_{k\in -R} c_k\theta_k\right)^2 |\theta_R, R\right] &= \mathbb{E}_i\left[\sum_{k\in -R} c_k^2 Var_i\left[\theta_k|\varphi, R\right] + \varphi^2\left(\sum_{k\in -R} c_k\right)^2 |\theta_R, R\right] \\ &= \mathbb{E}_i\left[\sum_{k\in -R} c_k^2\varphi\left(1-\varphi\right) + \varphi^2\left(\sum_{k\in -R} c_k\right)^2 |\theta_R, R\right] \\ &= \frac{\rho_i + 1_R}{\tau + |R|}\left(\sum_{k\in -R} c_k^2 + \left(\left(\sum_{k\in -R} c_k\right)^2 - \sum_{k\in -R} c_k^2\right)\frac{\rho_i + 1_R + 1}{\tau + |R| + 1}\right), \end{aligned}$$

where the second equality is due to $Var_i\left[\theta_k|\varphi, R\right] = \varphi\left(1-\varphi\right)$ and the last equality is due to the fact that the agent i's posterior distribution of φ conditional on θ_R is Beta with parameters $\rho_i + 1_R$ and $\tau + |R| - \rho_i - 1_R$, whose first and second moments are, respectively, $\frac{\rho_i+1_R}{\tau+|R|}$ and $\frac{(\rho_i+1_R)(\rho_i+1_R+1)}{(\tau+|R|)(\tau+|R|+1)}$. Plugging this expression into (17) and simplifying using Auxiliary Lemma A.1,

$$
\begin{aligned}
U_2 - \mathsf{C}_{im}(R) &= \mathbb{E}_i\left[\frac{\left(\sum_{k\in -R} c_k^2\right)(\rho_i + 1_R)}{\tau + |R|} - \left(\frac{\left(\sum_{k\in -R} c_k\right)(\rho_i + 1_R)}{\tau + |R|}\right)^2 |R\right] \\
&\quad + \left(\left(\sum_{k\in -R} c_k\right)^2 - \sum_{k\in -R} c_k^2\right)\mathbb{E}_i\left[\frac{(\rho_i + 1_R + 1)(\rho_i + 1_R)}{(\tau + |R| + 1)(\tau + |R|)}|R\right] \\
&= \left(\frac{\left(\sum_{k\in -R} c_k\right)^2}{\tau + |R|} + \sum_{k\in -R} c_k^2\right)\mathbb{E}_i\left[\frac{(\rho_i + 1_R)(\tau + |R| - \rho_i - 1_R)}{(\tau + |R|)(\tau + |R| + 1)}\right] \\
&= \left(\frac{\left(\sum_{k\in -R} c_k\right)^2}{\tau + |R|} + \sum_{k\in -R} c_k^2\right)\frac{\rho_i(\tau - \rho_i)}{\tau(\tau + 1)} = \mathsf{B}_i(R).
\end{aligned}
$$

Combining with (15) and (16) gives (11)-(12). This immediately shows that the ex-ante payoff of any agent i is increasing in $|R|$ and is decreasing in c_k for any $k \in -R$. In other words, when the manager learns an additional signal, $\mathsf{A}_{im}(\cdot)$, $\mathsf{B}_i(\cdot)$ and $\mathsf{C}_{im}(\cdot)$ are reduced. Indeed, the greater information and the smaller unknown part of the state Z imply that the residual variance $\mathsf{B}_i(\cdot)$ decreases. The first and third terms, i.e., $\mathsf{A}_{im}(\cdot)$ and $\mathsf{C}_{im}(\cdot)$, decrease as well, because agent i expects the additional signal to "persuade" the manager, such that they have a smaller expected divergence in their ex-post beliefs. This intuition holds in the opposite direction when $c_k, k \in -R$ increases.

Proof of Proposition 1

Plugging (5) and (6) into (7) gives

$$
\begin{aligned}
0 \geq \sum_{\theta_{-i}\in\{0,1\}^{N-1}} &\left[c_i(2\theta_i - 1) + \left(\sum_{j\in -R\setminus\{i\}} c_j\right)\cdot\frac{2\theta_i - 1}{\tau + |R| + 1}\right] \\
&\times\left[2(b_m - b_i) + c_i(1 - 2\theta_i) - 2\sum_{j\in -R\setminus\{i\}} c_j\theta_j + \frac{2(\rho_m + \mathbf{1}_R) + 1}{\tau + |R| + 1}\sum_{j\in -R\setminus\{i\}} c_j\right] P_i(\theta_{-i}|\theta_i).
\end{aligned}
$$

Note that the first multiple in each term equals $(2\theta_i - 1)[c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau + |R| + 1}]$, where $c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau + |R| + 1}$ is positive and is constant across all terms in the sum. Thus, the above inequality is equivalent to

$$0 \geq (2\theta_i - 1)\sum_{\theta_{-i}} P_i(\theta_{-i}|\theta_i)\left(2(b_m - b_i) + c_i(1-2\theta_i) - 2\sum_{j\in -R\setminus\{i\}} c_j\theta_j + \frac{2(\rho_m + \mathbf{1}_R)+1}{\tau+|R|+1}\sum_{j\in -R\setminus\{i\}} c_j\right).$$

Since $\sum_{\theta_{-R\setminus\{i\}}}\left(\sum_{j\in -R\setminus\{i\}} c_j\theta_j\right)P_i(\theta_{-R\setminus\{i\}}|\theta_i,\theta_R) = \frac{\rho_i+\mathbf{1}_R+\theta_i}{\tau+|R|+1}\sum_{j\in -R\setminus\{i\}} c_j$, we can further simplify it to

$$(2\theta_i - 1)\left[2(b_m - b_i) + c_i(1-2\theta_i) + \frac{2(\rho_m - \rho_i)+1-2\theta_i}{\tau+|R|+1}\sum_{j\in -R\setminus\{i\}} c_j\right] \leq 0.$$

We consider two separate cases. If $\theta_i = 0$, the above inequality becomes:

$$2(b_m - b_i) + c_i + \frac{2(\rho_m - \rho_i)+1}{\tau+|R|+1}\sum_{j\in -R\setminus\{i\}} c_j \geq 0,$$

and if $\theta_i = 1$, it becomes

$$2(b_m - b_i) - c_i + \frac{2(\rho_m - \rho_i)-1}{\tau+|R|+1}\sum_{j\in -R\setminus\{i\}} c_j \leq 0,$$

Together we get (8), which completes the proof.

Proof of Proposition 2

Proof of part (i). Consider advisory committee B, where all members are informed, and define $B_m \equiv B\cup\{m\}$. Full information revelation by all members of B to m is an equilibrium if and only if (8) holds for any agent i in B, where $R = B_m\setminus\{i\}$. Since $|R| = |B_m| - 1 = |B|$, (9) for all i in B becomes

$$\left|\rho_m - \rho_i\right| \leq \frac{1}{2}\left[1 + c_i\frac{\tau+|B|+1}{\sum_{j\in\{1,\ldots,N\}\setminus B_m} c_j}\right] \;\forall i \in B,$$

which is equivalent to

$$|B| \geq \left[\max_{i\in B}\frac{2\left|\rho_m - \rho_i\right| - 1}{c_i}\right]\sum_{j\in\{1,\ldots,N\}\setminus B_m} c_j - \tau - 1.$$

Hence, truth-telling by all members of the advisory body cannot be an equilibrium if $|B|$ is below

$$N_{\min} \equiv \min_{B \subseteq \{1,2,\cdots,N\} \backslash \{m\}} \left[\max_{i \in B} \frac{2\left|\rho_m - \rho_i\right| - 1}{c_i} \right] \sum_{j \in \{1,\ldots,N\} \backslash B_m} c_j - \tau - 1.$$

Proof of part (ii). Likewise, (10) for each i in B becomes

$$|b_m - b_i| \leq \frac{1}{2}\left[c_i + \frac{\sum_{j \in \{1,\ldots,N\} \backslash B_m} c_j}{\tau + |B| + 1}\right] \forall i \in B \Leftrightarrow \left[\max_{i \in B} 2\,|b_m - b_i| - c_i\right] \leq \frac{\sum_{j \in \{1,\ldots,N\} \backslash B_m} c_j}{\tau + |B| + 1}.$$

Hence, truth-telling by all members of the advisory body cannot be an equilibrium if $|B|$ is above

$$N_{\max} \equiv \max_{B \subseteq \{1,2,\cdots,N\} \backslash \{m\}} \frac{\sum_{j \in \{1,\ldots,N\} \backslash B_m} c_j}{\max_{i \in B} 2\,|b_m - b_i| - c_i} - \tau - 1,$$

which is strictly smaller than $N - 1$ if heterogeneity in preferences is sufficiently strong.

Proof of Proposition 3

We start with part (i). Suppose that the advisory body is $B = \{1,\ldots,N\} \backslash \{m\}$. We show that there exists $\kappa_l > 0$ such that for any $\kappa \leq \kappa_l$, there exists an equilibrium in which all agents in the advisory body acquire information and truthfully reveal it to the manager. Consider such an equilibrium and any agent $i \in B$. Then $R = \{1,\ldots,N\} \backslash \{i\}$ and $-R \backslash \{i\} = \emptyset$, so using (8), the IC constraint on communication of agent i is equivalent to $0 \leq c_i$, which is always satisfied. Consider the agent's decision to acquire information in this equilibrium. If agent i, $i \neq m$ acquires his signal, his expected utility is $\mathbb{E}_i[U_i|\{1,\ldots,N\}] - \kappa$, where $\mathbb{E}_i[U_i|R]$ is given by (11). If the agent deviates and does not acquire his signal, his expected utility is $\mathbb{E}_i[U_i|\{1,\ldots,N\} \backslash \{i\}]$: because information acquisition decisions are observed after the communication stage, the agent's deviation does not change other agents' incentives to communicate truthfully, but at the decision-making stage, the manager will make his decision knowing that the agent is uninformed. Define:

$$\kappa_l \equiv \min_{i \in \{1,\ldots,N\} \backslash \{m\}} \{\mathbb{E}_i[U_i|\{1,\ldots,N\}] - \mathbb{E}_i[U_i|\{1,\ldots,N\} \backslash \{i\}]\}. \tag{18}$$

Then, the IC constraint on information acquisition is satisfied for any $\kappa \leq \kappa_l$. Hence, for $\kappa \leq \kappa_l$, such an equilibrium indeed exists.

Moreover, according to Lemma 2, the sum of expected utilities of all agents in this equilibrium is $Nu_0 - (N-1)\kappa$, since $\mathbb{E}_i[U_i|\{1,\ldots,N\}] = u_0$ $\forall i$. Consider any other advisory body, and let R be the equilibrium set of signals that the manager knows after the communication stage. Since no agent acquires information if he does not expect to communicate it, the number of acquired signals is $|R|-1$. Hence, the sum of expected utilities of all agents in this equilibrium is $W(R) \equiv \sum_{i=1}^{N} \mathbb{E}_i[U_i|R] - \kappa(|R|-1)$. We next show that it is strictly smaller than $Nu_0 - (N-1)\kappa$. Indeed,

$$W(R) = \mathbb{E}_m[U_m|R] + \sum_{i\in R\backslash\{m\}} (\mathbb{E}_i[U_i|R] - \kappa) + \sum_{i\notin R} \mathbb{E}_i[U_i|R]. \tag{19}$$

Consider each term separately. First, by definition of κ_l, information acquisition is individually optimal for each agent in the committee that includes all agents for any $\kappa \leq \kappa_l$, and hence

$$\kappa \leq u_0 - \mathbb{E}_i[U_i|\{1,...,N\}\backslash\{i\}] \ \forall i.$$

Recall from Lemma 2 that $\mathbb{E}_i[U_i|R]$ increases as R expands. Hence, for any $i \notin R$, we have $\mathbb{E}_i[U_i|R] \leq \mathbb{E}_i[U_i|\{1,...,N\}\backslash\{i\}]$, and thus,

$$\kappa \leq u_0 - \mathbb{E}_i[U_i|\{1,...,N\}\backslash\{i\}] \leq u_0 - \mathbb{E}_i[U_i|R] \ \forall i \notin R. \tag{20}$$

In addition, $\mathbb{E}_m[U_m|R] < u_0$ and $\mathbb{E}_i[U_i|R] < u_0$ for any $i \in R\backslash\{m\}$. Combining these inequalities with (20) and using (19), we have

$$W(R) < u_0 + \sum_{i\in R\backslash\{m\}} (u_0 - \kappa) + \sum_{i\notin R} (u_0 - \kappa) = Nu_0 - (N-1)\kappa,$$

as required. Therefore, $B = \{1,...,N\}\backslash\{m\}$ is indeed optimal. The intuition is that information acquisition creates positive externalities on other shareholders. Since information acquisition is individually optimal in the committee that includes all shareholders, each shareholder that is not part of the smaller committee is worse off. Since information acquisition by these shareholders creates value for all other shareholders, all other shareholders are also worse off when the committee does not include them.

We next prove part (ii). Suppose, by contradiction, that the optimal committee is the set of all agents except the manager, $B = \{1,...,N\}\backslash\{m\}$. Given the infinitesimal positive cost of including each committee member, this committee can only be optimal if all its members truthfully communicate their information to the manager. Consider agent i for whom $|b_m - b_i| > \frac{c_i}{2}$. Applying (10) for $R = \{1,...,N\}\backslash\{i\}$, we have $-R\backslash\{i\} = \emptyset$, and hence (10) becomes $|b_m - b_i| \leq \frac{c_i}{2}$. Hence, the IC constraint is violated for agent i, a contradiction. Therefore, the optimal advisory body is a strict subset of $\{1,...,N\}\backslash\{m\}$.

Proof of Lemma 3

Suppose there is an equilibrium in which all members of the committee acquire information, which, in turn, requires that they communicate it truthfully. Consider the information acquisition decision of any member i of the committee, $i \in B$. Since truth-telling conditions (8) are satisfied, his expected payoff is $\mathbb{E}_i[U_i|\{m\},B]$. If he deviates from his equilibrium strategy and does not acquire information, his utility is $\mathbb{E}_i[U_i|\{m\},B\backslash\{i\}]$ (similar to the argument in the proof of Proposition 3). Hence, the incentive compatibility condition on information acquisition is equivalent to

$$\mathbb{E}_i[U_i|\{m\},B] - \mathbb{E}_i[U_i|\{m\},B/\{i\}] \geq \kappa. \tag{21}$$

Simplifying (11)-(12) for $c_i = c$ and denoting $\mathcal{G}(|B|) \equiv \frac{N-|B|-1}{\tau+|B|+1}$, we get

$$\begin{aligned} u_i(|B|) \equiv \mathbb{E}_i[U_i|\{m\},B] = u_0 - (b_m - b_i)^2 - 2c(b_m - b_i)(\rho_m - \rho_i)\mathcal{G}(|B|) \\ -c^2 \tfrac{\rho_i(\tau-\rho_i)(N+\tau)}{\tau(\tau+1)}\mathcal{G}(|B|) - c^2\left[(\rho_m - \rho_i)\mathcal{G}(|B|)\right]^2. \end{aligned} \tag{22}$$

Note that $\mathcal{G}(|B) > 0$ and that $\mathcal{G}(|B|)$ and hence $\mathcal{G}^2(|B|)$ decrease in $|B|$. In addition, $\mathcal{G}''(|B|) > 0$ and hence $\left(\mathcal{G}^2\right)''(|B|) > 0$ as well. It follows that the function $u_i(|B|)$ is increasing and concave in $|B|$, and hence $\mathbb{E}_i[U_i|\{m\},B] - \mathbb{E}_i[U_i|\{m\},B/\{i\}] = u_i(|B|) - u_i(|B|-1)$ is decreasing in $|B|$. Let $N_i(\kappa)$ be the highest value of $|B|$ for which $u_i(|B|) - u_i(|B|-1) \geq \kappa$, and note that $N_i(\kappa)$ is weakly decreasing in κ. Then, (21) is equivalent to $|B| \leq N_i(\kappa)$. Thus, the incentive compatibility condition on information acquisition is satisfied for all committee members $i \in B$ if and only if $|B| \leq N_B(\kappa) \equiv \min_{i\in B} N_i(\kappa)$. Since $N_i(\kappa)$ is weakly decreasing in κ for any i, $N_B(\kappa)$ is also weakly decreasing in κ. Note that according to the proof of Proposition 3, $\kappa_l = u(N-1) - u(N-2)$, and hence $N_B(\kappa) < N-1$ for any $\kappa > \kappa_l$. Finally, in the special case of $\rho_i = \rho_j$,

$$u_i(|B|) - u_i(|B|-1) = c^2 \frac{\rho(\tau-\rho)(N+\tau)}{\tau(\tau+1)}\left(\mathcal{G}(|B|-1) - \mathcal{G}(|B|)\right),$$

which is independent of i. Therefore, the highest value of $|B|$ for which $u_i(|B|) - u_i(|B|-1) \geq \kappa$ is the same for every agent i. Denoting it by $N(\kappa)$, we conclude that $N_B(\kappa) = N(\kappa)$ in this case.

Proof of Proposition 4

Since $c_i = c$, the expected utility of agent i depends on committee B only via its size $|B|$. First, we prove that the problem of choosing the optimal committee reduces to maximizing its size subject to the IC constraints on information acquisition and truth-telling of all its members. For contradiction, suppose that such committee B is not optimal. Then, there exists committee B' with $|B'| < |B|$ that yields a strictly higher sum of all agents' expected utilities net of information acquisition costs. Since information acquisition is optimal for all members of committee B, then $u_i(|B|) - u_i(|B|-1) \geq \kappa \; \forall i \in B$, where $u_i(\cdot)$ is defined by (22). Since $u_i(\cdot)$ is increasing in $|B|$ and $|B'| \leq |B|-1$, we have $u_i(|B|) - u_i(|B'|) \geq \kappa$ for all $i \in B$ and $u_i(|B|) - u_i(|B'|) \geq 0$ for all $i \notin B$. Combining these two sets of inequalities, it follows that

$$\sum_{i=1}^{N} \left(u_i(|B|) - u_i\left(\left|B'\right|\right)\right) \geq \kappa |B| \geq \left(|B| - \left|B'\right|\right) \kappa.$$

Hence, committee B' does not yield a strictly higher sum of all agents' expected utilities net of information acquisition costs, a contradiction. Thus, the problem indeed reduces to maximizing the committee size subject to the IC constraints on information acquisition and truth-telling of all its members.

Consider part (i), when $b_i = b_m = b$ for all i. First, consider the conditions for which the committee consisting of all $N-1$ agents except the manager satisfies IC constraints on information acquisition and truthful communication. According to the proof of Lemma 3, information acquisition is incentive compatible for all $N-1$ agents if and only if $u_i(N-1) - u_i(N-2) \geq \kappa \; \forall i$. Using $b_i = b_m = b$ for all i and $\mathcal{G}(N-1) = 0$, this constraint reduces to

$$c^2 \frac{\rho_i(\tau - \rho_i)(N+\tau)}{\tau(\tau+1)(\tau+N-1)} + c^2 \left[\frac{\rho_m - \rho_i}{\tau + N - 1}\right]^2 \geq \kappa \; \forall i. \tag{23}$$

Then, κ_l from (18) simplifies to:

$$\kappa_l = c^2 \min_{i \in \{1,\ldots,N\} \setminus m} \left\{ \frac{\rho_i(\tau - \rho_i)(N+\tau)}{\tau(\tau+1)(\tau+N-1)} + \left[\frac{\rho_m - \rho_i}{\tau + N - 1}\right]^2 \right\}.$$

Then, information acquisition is incentive compatible for all $N-1$ agents if and only if $\kappa \leq \kappa_l$. As already shown in the proof of Proposition 3, if $b_i = b_m = b$ for all i, (8) implies that the committee consisting of all $N-1$ members satisfies IC conditions for truthful communication of all members. Therefore, the committee consisting of all $N-1$ agents is optimal if $\kappa \leq \kappa_l$.

Consider $\kappa > \kappa_l$. Then, condition (23) is violated for at least one agent $i \in \{1, \ldots, N\} \setminus m$. Hence, the optimal committee is a subset of all agents, implying $N^* < N-1$. According to Proposition 2, there is a cutoff $N_{\min}$ such that truthful communication by all committee members is not an equilibrium if $|B| < N_{\min}$. Therefore, unless the optimal committee is $B = \emptyset$, it must be the case that $|B| \in [N_{\min}, N-1)$. Recall that we are searching for the committee of the highest size that satisfies the IC constraints on information acquisition and communication for all its members. The IC constraints on information acquisition become more stringent as κ increases, while the IC constraints on truthful communication do not depend on κ. Therefore, if a certain committee satisfies both sets of constraints for some κ, it also satisfies them for any lower κ, so N^* is (weakly) decreasing in κ. Finally, let κ_h denote the lowest κ for which there is no committee $B \neq \emptyset$ that satisfies the IC constraints on information acquisition and truthful communication for all its members. Then, $N^* \in [N_{\min}, N-1)$ for $\kappa \in (\kappa_l, \kappa_h)$ and $N^* = 0$ for $\kappa \geq \kappa_h$.

Consider part (ii). Using $c_i = c$ and (8) with $R = B \cup \{m\} \backslash \{i\}$, the IC condition for truth-telling of all members of committee B simplifies to

$$|b_m - b_i| \leq \frac{c}{2} \frac{\tau + N}{\tau + |B| + 1} \ \forall i \in B. \tag{24}$$

Reorder the agents in the order of (weakly) increasing $|b_m - b_i|$, with the manager being number 1, and the agent with the highest $|b_m - b_i|$ being number N. Then, $N_{\max}$ is given by the highest integer for which

$$\left| b_m - b_{N_{\max}+1} \right| \leq \frac{c}{2} \frac{\tau + N}{\tau + N_{\max} + 1}$$

is satisfied. It follows that there exists a committee with $|B|$ members satisfying the IC conditions for truth-telling of all its members if and only if $|B| \leq N_{\max}$. According to Lemma 3, any committee with $|B|$ members for which IC conditions for truth-telling are satisfied, satisfies the IC constraints on information acquisition if and only if $|B| \leq N(\kappa)$. Given this and the fact that the problem of choosing the optimal committee is equivalent to maximizing its size, we conclude that $N^* = \min\{N_{\max}, N(\kappa)\}$. This concludes the proof of the proposition.

In addition, the above analysis implies that the optimal size of the committee increases in the degree of preference alignment between the manager and potential directors, i.e., Prediction 5 (i) in Section 6. Indeed, the optimal committee is one of the largest possible size that allows information acquisition and truthful communication by all its members. As (24) implies, to find the committee of the largest size that allows truth-telling conditional on information acquisition, the solution is to rank all agents by the degree of their preference misalignment with the manager $|b_m - b_i|$, and then gradually expand committee B by including agents with progressively larger preference misalignments, up to the point when the next included agent's IC constraint for truth-telling is violated. In this setting, if we now increase the degree of misalignment by scaling up $|b_i - b_m|$ by some constant C for all i, then the maximum committee size that allows truthful communication will monotonically decrease in C and will become zero as C becomes large enough. According to Lemma 3, any committee with $|B|$ members, for which IC conditions for truth-telling are satisfied, satisfies the IC constraints on information acquisition if and only if $|B| \leq N(\kappa)$. Given this, the optimal committee size weakly decreases in C.

Proof of Proposition 5

The strategy profile outlined in the proposition implements action $a = Z$, yielding the highest possible payoff of u_0 to all parties for any realization Z. Hence, if this strategy profile constitutes an equilibrium, the committee from the statement of the proposition is optimal. To show that this strategy profile constitutes an equilibrium, consider whether deviations are profitable at each stage. At the committee approval stage, since $u_0 > \bar{u} = 0$, no committee member benefits from deviating to vote against the proposed action $a = Z$. Next, consider the manager's decision on which action to propose. Proposing $a = \sum_{i=1}^{N} cm_i$, where m_i is agent i's message, yields the manager the expected utility of u_0 (since she expects that all committee members communicate truthfully, $m_i = \theta_i$). In contrast, deviating to proposing any other action yields the expected payoff strictly below u_0. Finally, consider the decision of each committee member on whether or not to report his signal truthfully. Since (9) is satisfied, deviation to non-truthful communication is suboptimal for each committee member. Hence, the strategy profile from the proposition constitutes an equilibrium. Finally, since each committee member votes to approve the action proposed by the manager, the voting rule is irrelevant.

Proof of Proposition 6

We prove the proposition in three steps.

Step 1. Consider an auxiliary problem, in which the planner chooses committee B and commits to action $a\left(\sum_{j\in B} m_j\right) \equiv \kappa_0 + \kappa_1 \left(\sum_{j\in B} m_j\right)$ as a function of messages $\{m_j\}$ reported by committee members for some constants κ_0 and $\kappa_1 \geq 0$. Here B, κ_0, and κ_1 are chosen to maximize the value of the median shareholder subject to the incentive compatibility (IC) constraints that all committee members report their signals truthfully:

$$\max_{B,\kappa_0,\kappa_1} u_0 - \sum_{\theta_B\in\{0,1\}^{|B|}} \left(\sum_{\theta_{-B}\in\{0,1\}^{N-|B|}} \left(\kappa_0 + \kappa_1 \sum_{j\in B} \theta_j - c \sum_{i=1}^{N} \theta_i \right)^2 P\left(\theta_{-B}|\theta_B\right) \right) P\left(\theta_B\right) \tag{25}$$

subject to

$$\sum_{\theta_{-i}\in\{0,1\}^{N-1}} \left(\begin{array}{c} \left(\kappa_0 + \kappa_1 \sum_{j\in B} \theta_j - c \sum_{k=1}^{N} \theta_k - b_i\right)^2 \\ -\left(\kappa_0 + \kappa_1 \left(\sum_{j\in B\setminus i} \theta_j + 1 - \theta_i\right) - c \sum_{k=1}^{N} \theta_k - b_i\right)^2 \end{array} \right) P\left(\theta_{-i}|\theta_i\right) \leq 0 \tag{26}$$

for all $i \in B$. We will prove that this problem is solved by $\kappa_0 = \frac{\rho}{\tau+|B|}\left(N - |B|\right)c$, $\kappa_1 = c + \frac{N-|B|}{\tau+|B|}c$, and B that includes all informed agents with sufficiently low absolute biases so that $|b_i| \leq \left(1+\frac{N-|B|}{\tau+|B|}\right)\frac{c}{2}$.

To prove this result, first fix B and consider problem (25) without IC constraints. It is solved by

$$\kappa_0+\kappa_1\sum_{j\in B}\theta_j = c\mathbb{E}\left[\sum_{i=1}^{N}\theta_i|\theta_B\right] = c\left(\sum_{i\in B}\theta_i+(N-|B|)\frac{\rho+\sum_{i\in B}\theta_i}{\tau+|B|}\right),$$

which implies $\kappa_0 = \frac{\rho}{\tau+|B|}(N-|B|)\,c$ and $\kappa_1 = c+\frac{N-|B|}{\tau+|B|}c$.

Second, consider IC constraints (26). We can re-write them as:

$$\kappa_1(2\theta_i-1)\left(\sum_{\theta_{-i}\in\{0,1\}^{N-1}}\left(\kappa_0+\kappa_1\left(\sum_{j\in B\setminus i}\theta_j+\frac{1}{2}\right)-c\sum_{k=1}^{N}\theta_k-b_i\right)P(\theta_{-i}|\theta_i)\right)\leq 0.$$

Further simplifying,

$$\kappa_1(2\theta_i-1)\sum_{\theta_{B\setminus i}\in\{0,1\}^{|B|-1}}\left(\sum_{\theta_{-B}\in\{0,1\}^{N-|B|}}\left(\begin{array}{c}\kappa_0+\kappa_1\left(\sum_{j\in B\setminus i}\theta_j+\frac{1}{2}\right)\\ -c\sum_{k=1}^{N}\theta_k-b_i\end{array}\right)P(\theta_{-B}|\theta_B)\right)P(\theta_B|\theta_i)\leq 0.$$

Using $\sum_{\theta_{-B}\in\{0,1\}^{N-|B|}}\left(c\sum_{k\in-B}\theta_k\right)P(\theta_{-B}|\theta_B) = \frac{\rho+\sum_{j\in B}\theta_j}{\tau+|B|}(N-|B|)\,c$, we can simplify the inequality to

$$\kappa_1(2\theta_i-1)\sum_{\theta_{B\setminus i}\in\{0,1\}^{|B|-1}}\left(\begin{array}{c}\kappa_0+\left(\kappa_1-c-\frac{N-|B|}{\tau+|B|}c\right)\sum_{j\in B\setminus i}\theta_j+\kappa_1\frac{1}{2}\\ -\left(c+\frac{N-|B|}{\tau+|B|}c\right)\theta_i-\frac{\rho}{\tau+|B|}(N-|B|)\,c-b_i\end{array}\right)P(\theta_B|\theta_i)\leq 0.$$

For $\theta_i = 1$, the IC constraints are equivalent to

$$-b_i\leq\left(c+\frac{N-|B|}{\tau+|B|}c\right)-\kappa_1\frac{1}{2}-\left(\kappa_0-\frac{\rho}{\tau+|B|}(N-|B|)\,c\right)-\left(\kappa_1-c-\frac{N-|B|}{\tau+|B|}c\right)(|B|-1)\,\mathbb{E}\left[\theta_j|\theta_i=1\right].$$

For $\theta_i = 0$, the IC constraints are equivalent to

$$b_i\leq\kappa_1\frac{1}{2}+\kappa_0-\frac{\rho}{\tau+|B|}(N-|B|)\,c+\left(\kappa_1-c-\frac{N-|B|}{\tau+|B|}c\right)\mathbb{E}\left[\theta_j|\theta_i=0\right].$$

Since biases are symmetrically distributed, IC constraints are the least severe when $\kappa_0 = \frac{\rho}{\tau+|B|}(N-|B|)\,c$ and $\kappa_1 = c+\frac{N-|B|}{\tau+|B|}c$.

Therefore, for any fixed committee B, $\kappa_0 = \frac{\rho}{\tau+|B|}(N-|B|)c$ and $\kappa_1 = c + \frac{N-|B|}{\tau+|B|}c$ achieve the optimal unconstrained action and imply the least severe IC constraints. Finally, using Lemma 2 for $b_m = 0$ and the fact that $a(\theta_B) = \kappa_0 + \kappa_1 \sum_{i\in B}\theta_i$ coincides with the manager's action from Lemma 1 for $b_m = 0$, the ex-ante expected payoff of each agent increases with each additional signal truthfully communicated. Therefore, the optimal B includes as many agents that can truthfully communicate information as possible. This is achieved by including all agents with sufficiently low $|b_i|$ so that $|b_i| \le \left(1+\frac{N-|B|}{\tau+|B|}\right)\frac{c}{2}$. Equivalently, $|b_i| \le \beta$, where β is the solution to the fixed-point equation

$$\beta = \left(1 + \frac{N - \sum_{i=1}^{N} 1_{\{|b_i|\le\beta\}}}{\tau + \sum_{i=1}^{N} 1_{\{|b_i|\le\beta\}}}\right)\frac{c}{2}. \tag{27}$$

Since the left-hand side is strictly increasing in $\beta \in (0,\infty)$, taking values from zero to infinity, and the right-hand side is weakly decreasing in β, taking positive values, this equation has a unique solution.

In what follows, let $(\kappa_0^*, \kappa_1^*, B^*)$ denote the solution of this auxiliary problem.

Step 2. We argue that if $\bar{u}$ is below a certain constant, the solution to the auxiliary problem cannot produce a lower expected utility of the median shareholder than the equilibrium of the game for any committee choice. The sufficient condition for this is that for any realization of θ_{-B^*}, the median shareholder is better off under action $a(\theta_{B^*}) = \kappa_0^* + \kappa_1^* \sum_{i\in B^*}\theta_i$ than under the outside option:

$$u_0 - \left(\frac{\rho + \sum_{j\in B^*}\theta_j}{\tau + |B^*|}(N - |B^*|)c - c\sum_{i\in -B^*}\theta_i\right)^2 \ge \bar{u}.$$

It is sufficient to consider corner cases: if $\rho \ge \frac{\tau}{2}$, then $\sum_{j\in B^*}\theta_j = |B^*|$ and $\sum_{i\in -B^*}\theta_i = 0$; if $\rho \le \frac{\tau}{2}$, then $\sum_{j\in B^*}\theta_j = 0$ and $\sum_{i\in -B^*}\theta_i = N - |B^*|$, which imply $\bar{u} \le \bar{u}^*$, where

$$\bar{u}^* \equiv u_0 - \left(\frac{\max\{\rho, \tau-\rho\} + |B^*|}{\tau + |B^*|}(N - |B^*|)c\right)^2. \tag{28}$$

Imposing $\bar{u} \le \bar{u}^*$, we can prove the statement by contradiction. By contradiction, suppose that the equilibrium game produces a higher expected utility of the median shareholder than the solution to the auxiliary problem. Let $\tilde{B}$ be the optimal committee, $\tilde{B}_I \subseteq \tilde{B}$ be the set of committee members that are informed, and $\tilde{B}_T \subseteq \tilde{B}_I$ be the set of informed committee members that communicate signals truthfully. Correspondingly, let $a\left(\theta_{\tilde{B}_T}\right) = \tilde{\kappa}_0 + \tilde{\kappa}_1 \sum_{i\in\tilde{B}_T}\theta_i$ be the proposed action of the manager. Note that since all messages are equally informative about the state and publicly observed, the equilibrium proposed action must depend on $\sum_{i\in\tilde{B}_T}\theta_i$. Also note that since payoffs of all agents are of the form (1), it is without loss of generality to suppose that the action is linear. These follow from the quadratic feature of the payoffs and the fact that the prior is conjugate (Beta). Depending on the voting rule B_V, composition of the rest of committee $\tilde{B}\backslash\tilde{B}_T$, and the realizations of $\left\{\theta_i, i\in\tilde{B}_I\backslash\tilde{B}_T\right\}$, the equilibrium outcome is either an implementation of $a\left(\theta_{\tilde{B}_T}\right)$ or the status-quo. Given $\bar{u}\leq\bar{u}^*$, the utility of the median shareholder is strictly higher under action $\kappa_0^*+\kappa_1^*\sum_{i\in B^*}\theta_i$ than u_0 for any realizations of $\left\{\theta_i, i\in\tilde{B}_I\backslash\tilde{B}_T\right\}$. Hence, giving the ability to implement the status quo does not change the solution to the auxiliary problem. Since a triple $\left(\tilde{\kappa}_0,\tilde{\kappa}_1,\tilde{B}_T\right)$ was not optimal in the auxiliary problem, it therefore cannot yield a higher utility of the median shareholder than $(\kappa_0^*,\kappa_1^*,B^*)$.

Step 3. We prove that the following committee and the unanimity voting rule implement the solution of the auxiliary problem. Consider a committee that consists of all informed agents $i\in B^*$ and one uninformed agent with bias b_u. By Lemma 1, the ideal manager's action, given set R of agents that communicate signals truthfully, is

$$b_m+c\sum_{i\in R}\theta_i+c\left(N-|R|\right)\frac{\rho+\sum_{i\in R}\theta_i}{\tau+|R|}.$$

The uninformed agent finds it optimal to vote for this action over the status-quo if and only if

$$u_0-(b_m-b_u)^2-\frac{\rho(\tau-\rho)}{\tau(\tau+1)}\left(c^2\left(N-|R|\right)+\frac{c^2(N-|R|)^2}{\tau+|R|}\right)\geq\bar{u}\Leftrightarrow$$
$$|b_m-b_u|\leq\sqrt{u_0-\frac{\rho(\tau-\rho)}{\tau(\tau+1)}\left(c^2\left(N-|R|\right)+\frac{c^2(N-|R|)^2}{\tau+|R|}\right)-\bar{u}}.$$

If this inequality does not hold, the manager's best response will be to adjust b_m to $\tilde{b}_m$, closest to b_m, for which this inequality holds. Given this logic, consider an uninformed agent with bias

$$b_u=\begin{cases}-\sqrt{u_0-\frac{\rho(\tau-\rho)}{\tau(\tau+1)}\left(c^2\left(N-|B^*|\right)+\frac{c^2(N-|B^*|)^2}{\tau+|B^*|}\right)-\bar{u}}, & \text{if } b_m>0,\\ \sqrt{u_0-\frac{\rho(\tau-\rho)}{\tau(\tau+1)}\left(c^2\left(N-|B^*|\right)+\frac{c^2(N-|B^*|)^2}{\tau+|B^*|}\right)-\bar{u}}, & \text{if } b_m<0.\end{cases}\tag{29}$$

Then, the equilibrium action proposed by the manager is $\kappa_0^*+\kappa_1^*\sum_{i\in B^*}\theta_i=c\sum_{i\in B^*}\theta_i+c\left(N-|B^*|\right)\frac{\rho+\sum_{i\in B^*}\theta_i}{\tau+|B^*|}$. Indeed, any higher action would be voted down by the uninformed voter, while any lower action would be less valued by the manager. Given this proposed action, the IC constraints for truth-telling are given by (10), and since they hold in the auxiliary problem, they also hold here. Thus, the committee that includes all informed agents $i \in B^*$ and one uninformed agent, who is biased against the manager and whose bias is given by (29), together with the unanimity rule, implement the solution to the auxiliary problem. Hence, this committee and voting rule are optimal.

Online appendix for "Advising the management"

The online appendix presents the analysis of the extensions of the model.

A.1 Endogenous ownership structure

In this extension, we endogenize the decisions of the N potentially informed agents to become shareholders. Specifically, we consider the setup described in Section 5.1, where agent i's utility from holding stake α_i is given by (13). First, all N agents trade in a competitive market; then, the firm forms an optimal advisory committee B; and after that, the game proceeds as in the basic model. We assume that constant u_0 in the payoff specification U_i is sufficiently high, so that the equilibrium share price is positive.[27]

Note that this setup assumes that agents' trading is based on their preferences and prior beliefs regarding the firm's decision, but that agents do not trade again ex-post, after learning their private signals. There are two arguments for this simplifying assumption. One is tractability: the model in which agents both trade on private information and then decide whether to reveal it to the manager is very difficult to analyze. The second argument is more fundamental. As discussed in the literature review, prior research has extensively studied how market trading incorporates agents' private information into real decisions through its impact on *prices*. In contrast, our contribution is to examine how trading incorporates agents' information into real decisions through a different channel, *communication*: trading determines the firm's shareholder base and thus, determines which agents communicate their information to the manager via voting or being on the board. Assuming that agents to not trade based on private information allows us to abstract from the price channel and focus on the more novel communication channel.

Finally, note that another interpretation of this setup is that b_i and ρ_i capture agents' general preferences and beliefs regarding the firm (e.g., how congruent they are with the overall strategic direction the management is pursuing), and are not decision-specific. In this interpretation, the firm's shareholder base S captures the firm's long-term shareholders, and hence it is reasonable to assume that such long-term shareholders' ownership stakes are not affected by more transitory, decision-specific private information.

Analysis. We solve the model by backward induction. The analysis at the communication stage remains unchanged, and hence we only need to consider the trading stage. Using (13), the optimal ownership stake of agent i is

$$\alpha_i(p) = \max\left\{\frac{\mathbb{E}_i[U_i] - p}{\lambda}, 0\right\}. \tag{30}$$

[27] The only role of this assumption is to make the interpretation more intuitive, but none of the results depends on u_0.

As expected, a larger holding cost λ decreases the agent's demand for shares, while higher expected utility $\mathbb{E}_i[U_i]$ from holding each share increases his demand. In particular, as Lemma 2 and expressions (11)-(12) for $\mathbb{E}_i[U_i]$ demonstrate, the shareholder's expected utility is higher when the manager learns more from other shareholders. This implies that an agent's optimal stake in the firm is affected by the firm' overall ownership structure, which will be important for the results that follow.

Market clearing implies $1 = \sum_{i=1}^{N} \alpha_i(p) = \sum_{i \in S} \frac{\mathbb{E}_i[U_i]-p}{\lambda}$, and hence the price is

$$p^* = \frac{1}{|S|}\left(\sum_{i \in S} \mathbb{E}_i[U_i] - \lambda\right). \tag{31}$$

Similarly to the basic model, the implications under endogenous ownership structure are, as we show next, very different depending on whether agents differ in their preferences or beliefs. We consider each of these cases separately.

A.1.1 Heterogeneity in beliefs

Suppose agents have heterogeneous beliefs: $\rho_i \neq \rho_m$ for some i, but $b_i = b_m = b$ for all i. Our first observation is that there always exists an equilibrium in which all informed agents become shareholders and the manager's action reflects all available information. Indeed, according to the proof of Proposition 3, if the shareholder base is $S = \{1, ..., N\}$, and hence the advisory vote includes all informed agents, there exists an equilibrium in which all shareholders report their information truthfully. Intuitively, if the manager learns all the N signals, there is no ex-post disagreement between him and any of the shareholders, which makes truthful communication by all shareholders indeed optimal. Moreover, because of no ex-post disagreement between agents, they all value the firm in the same way and thus acquire equal stakes, $\alpha_i = \frac{1}{N}$. In addition, there can also exist an inefficient equilibrium, in which only a subset of the agents become shareholders, the manager's decision is not based on all the information, and total holding costs are larger. The share price in this inefficient equilibrium is lower than in the efficient equilibrium. Specifically:

Proposition 7. *Suppose agents have heterogeneous beliefs. There always exists an efficient equilibrium in which all N agents become shareholders, acquire equal stakes $\alpha_i = \frac{1}{N}$ in the firm, the optimal committee is the set of all shareholders, and the efficient action $a = b + Z$ is undertaken. However, there can also exist an inefficient equilibrium, in which a strict subset of agents become shareholders, an inefficient action $a \neq b + Z$ is undertaken, and the share price is lower than in the efficient equilibrium.*

The source of equilibrium multiplicity is the following. If only a subset of agents are expected to become shareholders and communicate their information to the manager, there is still ex-post disagreement between the manager and the shareholders. Realizing this at the trading stage, shareholders who have substantially different beliefs from those of the manager (i.e., have large enough $\left|\rho_i - \rho_m\right|$), do not buy shares, making this equilibrium self-fulfilling.

One interesting implication of the above results is for the effect of index funds in the firm's ownership structure. There is an ongoing active debate about the role of passive (index) asset managers in corporate governance. Some market participants have expressed concerns that the presence of passive investors weakens governance and even proposed that they should be restricted from voting, noting that they may lack the adequate incentives to become informed. Others argue that index funds actively engage with their portfolio companies, even more so than investors who can easily exit.[28] An implication of our analysis is that the presence of index funds can make advisory shareholder voting more effective. Indeed, suppose investors expect that a large fraction of the firm's shareholders will consist of index funds, whose stake in the firm will not depend on whether their fund managers agree or disagree with the firm's CEO. In this case, the inefficient equilibrium may again cease to exist due to positive externalities in communication, as long as these passive investors are sufficiently informed and have diverse prior beliefs. This logic highlights a positive effect of passive investors on the voting of *other* shareholders, with a caveat that it relies on passive investors being informed.

A.1.2 Heterogeneity in preferences

Recall that when agents have heterogeneous preferences, communication externalities are negative. Since positive externalities were the key reason for equilibrium multiplicity under heterogeneous beliefs, it is intuitive to expect that equilibrium under heterogeneous preferences is unique. This is indeed the case, as shown in Proposition 8. In this equilibrium, unlike in the efficient equilibrium under heterogeneous beliefs, the shareholder base is generally restricted and consists of agents whose preferences are sufficiently aligned with those of the manager (small $|b_m - b_i|$). This is because under heterogeneous preferences, there are always ex-post disagreements between the manager and shareholders about the optimal course of action, regardless of the amount of information conveyed.

Proposition 8. *Suppose agents have heterogeneous preferences and for at least one agent the*

[28]See "Vanguard, Trian and the problem with 'passive' index funds", *Forbes*, HBS Working Knowledge (Feb 15, 2017) and "The case against passive shareholder voting," *CLS Blue Sky Blog* (August 2, 2017) for negative views on the governance role of passive investors; and "Passive investment, active ownership," *Financial Times* (April 6, 2014) for a positive view.

preference misalignment with the manager is sufficiently large, $|b_m - b_i| > \frac{1}{2}c_i$. In the unique equilibrium, agent i becomes a shareholder if and only if $|b_m - b_i|$ is sufficiently low. The equilibrium number of shareholders increases with λ. There exists cutoff $\lambda^* \in (0, \infty)$, such that:

- *if $\lambda \leq \lambda^*$, then $B^* = S \backslash \{m\}$, i.e., the optimal advisory body includes all non-manager shareholders;*
- *if $\lambda > \lambda^*$, then $B^* \subset S \backslash \{m\}$, i.e., the optimal advisory body is a strict subset of non-manager shareholders.*

As Proposition 8 shows, the key factor that determines the optimal advisory body is the holding cost λ. Intuitively, λ affects how concentrated vs. dispersed the firm's ownership structure is, and hence how close shareholders' preferences are to those of the management. If holding costs are small, the equilibrium features concentrated ownership: investors with similar preferences to those of the manager end up holding large stakes in the firm. Under such concentrated ownership, shareholder voting is effective in its advisory role, in the sense that all shareholders share their information with the manager. Hence, the optimal advisory committee includes all of the firm's shareholders. In contrast, if holding costs are large, concentrated ownership becomes too expensive, so the firm's shareholder base is relatively diverse and includes many shareholders whose preferences are misaligned with those of the manager. In this case, advisory voting is no longer effective: there is no equilibrium in which all of the shareholders truthfully convey their views to the manager. Hence, advice is optimally provided by a board consisting of a subset of the firm's shareholders, rather than through an advisory vote.

A.2 Expertise of the manager

To analyze the effect of the manager's expertise on the board's advising effectiveness, we consider a small extension of the basic model by assuming that the manager knows a subset $\mathcal{M}$ of signals $\{\theta_i, i \neq m\}$ in addition to signal θ_m. If $\mathcal{M} = \emptyset$, then the manager only knows his private signal θ_m, as in the basic model. If $\mathcal{M} \neq \emptyset$, then the manager also knows some signals of the other agents, in addition to his private signal θ_m. We interpret an expansion/contraction of $\mathcal{M}$ (i.e., addition/removal of signals to/from $\mathcal{M}$) as an increase/decrease in the manager's expertise.[29] For simplicity, we assume that the costs of information acquisition are sufficiently small that all directors acquire information and focus on the effects of communication frictions.

[29]An alternative way to model higher managerial expertise is to increase c_m, while normalizing $\sum_{i=1}^{N} c_i = 1$. This model leads to the same result. However, it cannot be used to analyze how firm value changes with the manager's expertise because a change in c_m changes the distribution of state Z in this formulation.

Let us fix all parameters of the model and consider any board B, i.e., a subset of agents $\{1,...,N\}$. We say that board B is efficient at providing advice to the manager if truthful communication by all members of B to the manager is an equilibrium. Without loss of generality, consider boards in which each member has some information that the manager does not already know: $\{\theta_i, i \in B\} \cap \mathcal{M} = \emptyset$.[30] The next proposition shows how the advising effectiveness of the board varies with the manager's information:

Proposition 9 (Manager's expertise). *Consider any board B with $\{\theta_i, i \in B\} \cap \mathcal{M} = \emptyset$.*

1. *If $|b_m - b_i| \leq \frac{1}{2}c_i \ \forall i \in B$ and board B is efficient at providing advice under $\mathcal{M}$, then it is also efficient at providing advice if set $\mathcal{M}$ expands, i.e., as the manager becomes more informed.*
2. *If $\left|\rho_m - \rho_i\right| \leq \frac{1}{2} \ \forall i \in B$ and board B is efficient at providing advice under $\mathcal{M}$, then it is also efficient at prodiving advice if set $\mathcal{M}$ contracts, i.e., as the manager becomes less informed.*

Intuitively, as the manager becomes more informed, there is less information relevant for the decision that neither the manager nor the board knows. This has different effects depending on the nature of communication frictions. When the friction of heterogeneous beliefs is relatively more important, the key consequence is the increase in the manager's congruence because there is less information that the manager and board members can disagree about. As a result, board members have stronger incentives to truthfully reveal their information to the manager, explaining the first statement of the proposition. In contrast, when communication is primarily hampered by conflicts of interest, greater managerial expertise decreases directors' costs of misreporting their information because the manager is expected to react less to each director's message. This explains the second statement of the proposition.

Since Proposition 8 applies to any committee, both optimal and suboptimal ones, we can conclude that the manager's expertise enhances the advisory role of the board if communication is mainly hampered by disagreement between the manager and board members, but impedes its advisory role if communication is mainly hampered by conflicts of interest.

Moreover, the latter effect implies that it can be Pareto improving to appoint a less informed manager. Intuitively, a less informed manager has worse private information, but obtains more information from the board members. The numerical example below illustrates that this second effect can dominate.

Example (Manager's expertise can be harmful).

[30] Clearly, there is no benefit of adding an agent to the advisory board if he has no information that the manager does not already possess.

There is a manager and 100 other agents, divided into two groups. The parameters are: $c_m = 0.3$, $b_m = 0.0475$, $b_i = 0 \ \forall i \neq m$, $\rho_i = \rho = 2$, $\tau = 4$. The first group are the relatively more informed agents: it contains $\bar{N} = 10$ agents with $c_i = \bar{c} = 0.05$. The second group are the relatively less informed agents: it contains $\underline{N} = 90$ agents with $c_i = \underline{c} = 0.2/90$ (for simplicity, $\sum_{i=1}^{N} c_i$ is normalized to one). Thus, the manager's signal θ_m has weight 30% in the state, the sum of all signals of the more informed agents has weight 50% in the state, and the sum of all signals of the less informed agents has weight 20% in the state. If the manager only knows signal θ_m (i.e., $\mathcal{M} = \emptyset$), then the optimal board is comprised of 5 agents from the first group. These agents report their signals to the manager truthfully, and the implied expected payoff of each agent $i \neq m$ is $V = -0.0093$ (the payoff of the manager is higher by b_m^2). In contrast, if the manager also knows one of the signals with $c_i = 0.2/90$ and the board is comprised of 5 agents from the first group, the IC constraint is violated and truthful revelation by all members of this board is not an equilibrium. Instead, the optimal board is comprised of 4 agents from the first group. The implied expected payoff of each agent $i \neq m$ is $V = -0.0109$, which is lower than if the manager is less informed. Thus, a reduction in the manager's information improves the values of all agents by promoting more efficient communication.

In contrast, the fact that a more informed manager improves the advisory role of the board when conflicts of interest play a small role relative to differences in beliefs, implies that in this case, both effects act in the same direction. As the manager becomes more informed, he both makes better decisions due to his own information and can also get better advice from other agents.

A.3 Heterogeneous interpretation of information

In this extension, we analyze the model with heterogeneous interpretations of signals, described in Section 5.3. Specifically, we assume that given a set of signals $\left\{\theta_j, j = 1, ..., N\right\}$, shareholder i believes that the state of the world is equal to (14).

Because the analysis of this model repeats the analysis of the basic model, we leave the details for the proofs and only highlight the novel parts. After the communication stage, if the manager knows subset R of signals, his optimal action is

$$a_m(\theta_R) = \gamma c \theta_m + \sum_{i \in R, i \neq m} c\theta_i + \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|} (N - |R|) \, c. \tag{32}$$

As (32) shows, the manager places a higher weight on his signal θ_m than on any other signal. In contrast, the optimal action from shareholder i's perspective places a higher weight on signal θ_i than on any other signal.

The next proposition characterizes the constraint for truthful communication by agent i and the optimal advisory body:

Proposition 10. *(i) Suppose that the manager learns subset R of signals (which includes his own signal θ_m but not θ_i) and does not know all the other signals, $-R$. Then, agent i reports his signal truthfully if and only if*

$$\begin{aligned} &\text{for } \rho_i \leq \rho_m: \quad \rho_m - \rho_i \leq \tfrac{1}{2}\tfrac{N+\tau}{N-|R|-1} + (\gamma-1)\tfrac{\tau-\rho_i}{\tau+1}\tfrac{\tau+1+|R|}{N-|R|-1}, \\ &\text{for } \rho_i > \rho_m: \quad \rho_i - \rho_m \leq \tfrac{1}{2}\tfrac{N+\tau}{N-|R|-1} + (\gamma-1)\tfrac{\rho_i}{\tau+1}\tfrac{\tau+1+|R|}{N-|R|-1}. \end{aligned} \tag{33}$$

(ii) If $\left|\rho_m - \rho_i\right|$ is sufficiently high for each i, there exists a cutoff on committee size $N^{\gamma}_{\min} \geq 1$, which is weakly decreasing in γ, such that an equilibrium where all members of committee B truthfully communicate to the manager does not exist if $|B| < N^{\gamma}_{\min}$.

(iii) For any $\gamma \geq 1$, the optimal committee is the entire set of agents.

Proposition 10 shows that the overconfidence of each agent in the importance of his signal increases his incentives to report truthfully: the right-hand side of (33) increases in γ. Intuitively, if the agent perceives his signal to be more important than it actually is, he perceives lying to be costlier. Proposition 10 also shows that when agents have heterogeneous beliefs but common objectives, communication has positive externalities, as in the baseline model: the right-hand side of (33) increases as R expands. As a consequence of positive externalities, the optimal committee is again the entire set of informed agents (part (iii) of the proposition).

Overall, the key requirement for positive communication externalities is that communication of other agents to the manager moves the manager's and the agent's beliefs about the state closer to each other, which is consistent with heterogeneous interpretations of signals and does not require complete convergence of beliefs under full information.

A.4 A more general specification of differences in beliefs

Consider the baseline model but suppose that each agent i has a prior belief that φ is distributed according to Beta distribution with parameters $(\rho_i, \tau_i - \rho_i)$. We derive the incentive compatibility constraint for this specification and show that the main results of Section 3 (positive externalities of communication and the optimal committee being the full set of agents) extend to this specification.

Proposition 11. *Suppose that the manager learns subset R of signals (which includes his own signal θ_m but not θ_i) and does not know all the other signals, $-R$. Then agent i reports his* *signal truthfully if and only if*

$$\begin{aligned}&\left|(b_m - b_i)\frac{(\tau_i+1)(\tau_m+|R|+1)}{\sum_{j\in -R\setminus\{i\}} c_j} + \frac{\tau_i-\tau_m}{2} + \rho_m(\tau_i+1) - \rho_i(\tau_m+1)\right| \\ \leq\ & \frac{(\tau_i+1)c_i(\tau_m+|R|+1)}{2\sum_{j\in -R\setminus\{i\}} c_j} + \frac{\tau_i+1}{2} - \frac{\tau_i-\tau_m}{2}. \end{aligned} \tag{34}$$

In particular, when $b_i = b_m\ \forall i$, it reduces to

$$\left|\frac{\tau_i-\tau_m}{2} + \rho_m(\tau_i+1) - \rho_i(\tau_m+1)\right| \leq \frac{(\tau_i+1)c_i(\tau_m+|R|+1)}{2\sum_{j\in -R\setminus\{i\}} c_j} + \frac{\tau_i+1}{2} - \frac{\tau_i-\tau_m}{2}. \tag{35}$$

This inequality is relaxed as R expands and is always satisfied if R includes all signals other than θ_i.

Since the right-hand side of (35) increases as R expands, information transmission has positive externalities as in the baseline model: more information revealed to the manager by some agents encourages other agents to report their information truthfully. For this reason, the argument of Proposition 2 also applies to this extended model. Since the right-hand side of (35) is infinite when R includes all signals other than agent i's signal, the argument of Proposition 3 applies to this extended model.

A.5 Model with common priors and two private signals

In this section, we consider a variation of our model in which agents have common priors, and each agent gets two private signals but can send only one binary message and hence effectively can communicate at most one signal. We show that this model with common priors and two private signals, one of which cannot be communicated, is different from the model with heterogeneous priors. In particular, we characterize the incentive compatibility conditions for truthful communication (of one of the two signals) and show that this model does not feature positive externalities of communication, which is a general property of the model with heterogeneous prior beliefs in the paper.

The reason why the model with two private signals, one of which cannot be communicated, is not equivalent to the model with different prior beliefs is because of the following difference between a private signal and a prior belief. Intuitively, if the decision-maker has a different prior belief from an agent, then the decision-maker believes that her prior is correct, while the agent's prior is incorrect, which creates an effective conflict of interest and harms communication. In contrast, if the agent possesses a signal that she cannot communicate to the decision-maker, the decision-maker believes that the agent's signal is relevant for the decision. Thus, different priors prevent informative communication unless the decision-maker is expected to obtain enough signals from other agents, so that the effect of priors becomes less relevant. In contrast, if the agent has the same prior as the decision-maker, even if he can communicate at most a binary message, this binary message will be informative since some information communicated to the decision-maker is better than none.

In particular, assume that all agents share the same prior $\rho_i = \rho_m = \rho$, but each of them receives two informative private signals. The state of the world is equal to the weighted sum of $2N$ signals:

$$Z = \sum_{i=1}^{N} c_i \theta_i + \sum_{j=N+1}^{2N} c_j \theta_j, \tag{36}$$

where agent i privately learns two signals, θ_i and θ_{i+N}. However, suppose that the agent can only send one binary message $m_i \in \{0,1\}$. We will focus on equilibria in which message m_i is either fully informative about one of the two signals (about θ_i if $c_i > c_{i+N}$, and about θ_{i+N} if $c_i < c_{i+N}$) or fully uninformative. Thus, each agent i is able to communicate at most one of his two signals. Without loss of generality, suppose that $c_i > c_{i+N}$, so that signals from θ_1 to θ_N can be potentially communicated, while signals from $N+1$ to $2N$ cannot be. For simplicity, we consider the special case in which φ is distributed uniformly over $[0,1]$ (i.e., $\rho = 1$ and $\tau = 2$) and ignore costs of information acquisition (i.e., we assume that these costs are infinitesimal).

Consider any informed agent i and suppose that the manager knows the subset $R \subset \{1,...,N\}$ of signals, where R includes the manager's own signal θ_m but not agent i's signal θ_i. The optimal action of the manager is the same as in the baseline model: If agent i reveals his signal truthfully,

$$a_m(\theta_R, \theta_i) \equiv b_m + c_i\theta_i + \sum_{k\in R} c_k\theta_k + \frac{1 + \theta_i + \sum_{k\in R}\theta_k}{3 + |R|} \sum_{j\in -R\backslash\{i\}} c_j, \tag{37}$$

and if agent i misreports, the manager's action is

$$a_m(\theta_R, 1-\theta_i) \equiv b_m + c_i(1-\theta_i) + \sum_{k\in R} c_k\theta_k + \frac{1 + (1-\theta_i) + \sum_{k\in R}\theta_k}{3 + |R|} \sum_{j\in -R\backslash\{i\}} c_j. \tag{38}$$

Agent i communicates signal θ_i truthfully if and only if

$$\sum_{\theta_{-i}} \left[(a_m(\theta_R, \theta_i) - Z - b_i)^2 - (a_m(\theta_R, 1-\theta_i) - Z - b_i)^2 \right] P_i(\theta_{-i}|\theta_i, \theta_{i+N}) \leq 0 \tag{39}$$

for all $(\theta_i, \theta_{i+N}) \in \{0,1\}^2$.

Note that

$$a_m(\theta_R, \theta_i) - a_m(\theta_R, 1-\theta_i) = (2\theta_i - 1)\left(c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{|R|+3}\right).$$

Hence, plugging (37) and (38) into (39), we get that (39) is equivalent to

$$(2\theta_i - 1)\sum_{\theta_{-i}} (a_m(\theta_R, \theta_i) + a_m(\theta_R, 1-\theta_i) - 2Z - 2b_i) P_i(\theta_{-i}|\theta_i, \theta_{i+N}) \leq 0$$

or

$$(2\theta_i - 1)\sum_{\theta_{-i}} P_i(\theta_{-i}|\theta_i, \theta_{i+N}) \begin{pmatrix} 2(b_m - b_i) + c_i(1-2\theta_i) - 2c_{i+N}\theta_{i+N} \\ -2\sum_{j\in -R\setminus\{i,i+N\}} c_j\theta_j + \frac{2\cdot\sum_{k\in R}\theta_k + 3}{|R|+3}\sum_{j\in -R\setminus\{i\}} c_j \end{pmatrix} \leq 0. \quad (40)$$

Next, we use

$$\sum_{\theta_{-i}}\left(\sum_{j\in -R\setminus\{i,i+N\}} c_j\theta_j\right) P_i(\theta_{-i}|\theta_i, \theta_{i+N}, \theta_R) = \frac{1 + \sum_{k\in R}\theta_k + \theta_i + \theta_{i+N}}{|R|+4}\left(\sum_{j\in -R\setminus\{i,i+N\}} c_j\right)$$

and hence,

$$\begin{aligned} \sum_{\theta_{-i}}\left(\sum_{j\in -R\setminus\{i,i+N\}} c_j\theta_j\right) P_i(\theta_{-i}|\theta_i, \theta_{i+N}) &= \frac{1 + \frac{1+\theta_i+\theta_{i+N}}{4}|R| + \theta_i + \theta_{i+N}}{|R|+4}\left(\sum_{j\in -R\setminus\{i,i+N\}} c_j\right) \\ &= \frac{1+\theta_i+\theta_{i+N}}{4}\left(\sum_{j\in -R\setminus\{i,i+N\}} c_j\right). \end{aligned}$$

Hence, (40) simplifies to

$$(2\theta_i - 1)\begin{pmatrix} 2(b_m - b_i) + c_i(1-2\theta_i) - 2c_{i+N}\theta_{i+N} \\ -\frac{1+\theta_i+\theta_{i+N}}{2}\left(\sum_{j\in -R\setminus\{i,i+N\}} c_j\right) + \frac{\frac{1+\theta_i+\theta_{i+N}}{2}|R|+3}{|R|+3}\sum_{j\in -R\setminus\{i\}} c_j \end{pmatrix} \leq 0.$$

There are four possible realizations of (θ_i, θ_{i+N}). For each of these four realizations, we get the following inequalities:

- for $\theta_i = 1, \theta_{i+N} = 1$:

$$2(b_m - b_i) \leq c_i + \frac{\frac{1}{2}|R| + 3}{|R| + 3} c_{i+N} + \frac{3}{2(|R| + 3)} \left(\sum_{j \in -R \setminus \{i, i+N\}} c_j \right)$$

- for $\theta_i = 0, \theta_{i+N} = 0$:

$$2(b_i - b_m) \leq c_i + \frac{\frac{1}{2}|R| + 3}{|R| + 3} c_{i+N} + \frac{3}{2(|R| + 3)} \left(\sum_{j \in -R \setminus \{i, i+N\}} c_j \right)$$

  These two cases can be summarized as:

$$2|b_m - b_i| \leq c_i + \frac{\frac{1}{2}|R| + 3}{|R| + 3} c_{i+N} + \frac{3}{2(|R| + 3)} \left(\sum_{j \in -R \setminus \{i, i+N\}} c_j \right),$$

  which is more difficult to satisfy as R expands.

- for $\theta_i = 1, \theta_{i+N} = 0$:

$$2(b_m - b_i) \leq c_i - c_{i+N}$$

- for $\theta_i = 0, \theta_{i+N} = 1$:

$$2(b_i - b_m) \leq c_i - c_{i+N}$$

  These two cases can be summarized as:

$$2|b_i - b_m| \leq c_i - c_{i+N},$$

  where the right-hand side is positive because $c_i > c_{i+N}$. This inequality is unaffected by the communication strategy of others.

In particular, if all agents have the same preferences ($b_i = b_m$), then regardless of the composition of committee, all committee members communicate truthfully the signal that the communication protocol allows them to communicate. This contrasts with the positive externalities of communication in the model where agents have common preferences but different beliefs: In that model, the agent will not communicate truthfully unless the committee size is sufficiently large.

A.6 Proofs for the Online Appendix

Proof of Proposition 7

We prove the existence of the efficient equilibrium by showing that no agent wants to deviate from the strategies described in the proposition. Consider a subgame that happens after all N agents become shareholders. According to Proposition 3 and its proof, there exists an equilibrium in which all N agents communicate their information truthfully, and hence the optimal committee consists of the set of all agents (excluding the manager). Since all agents communicate their signals truthfully, the manager takes action $a_m = b + Z$ (a special case of Lemma 1). Now, consider the trading game, provided that each agent expects that all agents will become shareholders and the manager will undertake action $a_m = b + Z$. Given this expectation, Lemma 2 implies that the expected per-share payoff of each agent i (excluding the holding cost) is u_0. Hence, from (31), the market-clearing price reduces to $p^* = u_0 - \frac{\lambda}{N}$. From (30), at this price, the fraction of the firm purchased by agent i is $\alpha_i = \frac{u_0 - p^*}{\lambda} = \frac{1}{N}$. This equilibrium is efficient in the sense of maximizing the sum of the seller's proceeds and the utilities of all the agents, because it (1) achieves the most efficient action from all agents' point of view ($a_m = b + Z$), and (2) minimizes the agents' total holding costs by distributing the asset evenly among all agents ($\alpha_i = \frac{1}{N}$).

We prove the possibility of equilibrium multiplicity by construction of an example. Suppose there are two groups of agents, Γ_1 and Γ_2. Agents in the first group have the same prior beliefs as the manager: $\rho_i = \rho_m \ \forall i \in \Gamma_1$ (this group includes the manager). In contrast, agents in the second group have different beliefs than the manager: $\rho_i \neq \rho_m$ with $\left|\rho_m - \rho_i\right| = \delta > 0$. We show that if δ is sufficiently high, there exists an equilibrium in which only agents from group Γ_1 become shareholders and an inefficient action $a \neq b + Z$ is implemented. Consider a subgame that happens after trading, in which only agents from group Γ_1 became shareholders. Since $\rho_i = \rho_m$ for these agents, the IC condition for truthful communication is satisfied for each $i \in \Gamma_1 \backslash \{m\}$. Hence, the optimal committee is the set of all agents excluding the manager, $\Gamma_1 \backslash \{m\}$, and the manager learns all signals in Γ_1. Using Lemma 1, the action that the manager undertakes is

$$a_m(\theta_R) = b + c \sum_{i \in \Gamma_1} \theta_i + \frac{\rho_m + \sum_{i \in \Gamma_1} \theta_i}{\tau + |\Gamma_1|} c \left(N - |\Gamma_1|\right). \tag{41}$$

The expected per-share payoff of each agent $i \in \Gamma_1$ (excluding the holding cost) is

$$u_0 - c^2 \frac{\rho_m \left(\tau - \rho_m\right) \left(N + \tau\right)}{\tau(\tau + 1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|}.$$

Hence, from (31), the market-clearing price is

$$p^* = u_0 - c^2 \frac{\rho_m (\tau - \rho_m)(N+\tau)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} - \frac{\lambda}{|\Gamma_1|}. \tag{42}$$

From (30), at this price, the fraction of the firm purchased by agent $i \in \Gamma_1$ is $\alpha_i = \frac{1}{|\Gamma_1|}$. To show that this is an equilibrium, it remains to show that no agent $j \in \Gamma_2$ is better off deviating to buying shares at price p^* if δ is sufficiently high. Suppose that one agent $j \in \Gamma_2$ deviates to $\alpha_j > 0$. Using Proposition 1, the IC condition on truthful reporting for this agent, (9) with $R = \Gamma_1$, is violated if $\delta > \delta_1$, where

$$\delta_1 \equiv \frac{1}{2}\left[1 + \frac{\tau + |\Gamma_1| + 1}{N - |\Gamma_1| - 1}\right].$$

Hence, for $\delta > \delta_1$, if agent j deviates to $\alpha_j > 0$, the optimal committee and firm's action will remain the same because the agent will not communicate his information to the manager. Therefore, the expected per-share payoff of this agent (excluding the holding cost) is

$$\mathbb{E}_j[U_j] = u_0 - c^2 \frac{\rho_j \left(\tau - \rho_j\right)(N+\tau)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} - c^2 \delta^2 \left(\frac{N - |\Gamma_1|}{\tau + |\Gamma_1|}\right)^2.$$

Using (30), deviation to $\alpha_j > 0$ is unprofitable for agent j if $\mathbb{E}_j[U_j] < p^*$, where p^* is given by (42). Simplifying, a sufficient condition for this deviation to be unprofitable is that $\delta > \delta_2$, where

$$\delta_2 \equiv \frac{\tau + |\Gamma_1|}{N - |\Gamma_1|} \sqrt{\frac{\lambda}{c^2 |\Gamma_1|} + \frac{(N+\tau)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} \rho_m (\tau - \rho_m)}.$$

Hence, this strategy profile is indeed an equilibrium if $\delta > \max\{\delta_1, \delta_2\}$. Comparing the expression (42) and the price $p^* = u_0 - \frac{\lambda}{N}$ in the efficient equilibrium, it automatically follows that the price in the inefficient equilibrium is lower.

Proof of Proposition 8

Suppose agents have heterogeneous preferences: $b_i \neq b_m$ for some i, but $\rho_i = \rho_m = \rho$ for all i. Let S denote the equilibrium set of agents that become shareholders, $B \subseteq S$ be the equilibrium committee chosen out of these shareholders, and $|B|$ denote the equilibrium committee size. Then, the expected per-share payoff of each agent $i \in S$ (excluding the holding cost) is

$$u_0 - (b_m - b_i)^2 - c^2 \frac{\rho(\tau - \rho)(N+\tau)}{\tau(\tau+1)} \frac{N - |B| - 1}{\tau + |B| + 1}.$$

Plugging this expression into (31), we obtain the equilibrium price:

$$p^* = u_0 - c^2 \frac{\rho(\tau - \rho)(N + \tau)}{\tau(\tau + 1)} \frac{N - |B| - 1}{\tau + |B| + 1} - \frac{1}{|S|} \left(\sum_{j \in S} (b_m - b_j)^2 + \lambda \right).$$

Plugging this price into the demand equation (30), we obtain:

$$\alpha_i^* = \max \left\{ \frac{1}{|S|} + \frac{1}{\lambda} \left(\frac{1}{|S|} \sum_{j \in S} (b_m - b_j)^2 - (b_m - b_i)^2 \right), 0 \right\}. \tag{43}$$

Note that this expression does not depend on the composition of the committee. Hence, we can solve for the set of all equilibria in two steps: (1) solve for the equilibrium set of shareholders; (2) find the optimal committee given the solution to the first step.

Consider the first step. Note that

$$\alpha_i^* > 0 \Leftrightarrow \lambda + \sum_{j \in S} (b_m - b_j)^2 - |S| (b_m - b_i)^2 > 0.$$

Suppose, without loss of generality, that agents are ordered in the order of (weakly) increasing $|b_m - b_i|$, with the manager being number 1, and the agent with the highest $|b_m - b_i|$ being number N. Consider the function

$$d(k) \equiv \lambda + \sum_{j \le k} (b_m - b_j)^2 - k(b_m - b_k)^2 > 0. \tag{44}$$

Note that $d(k)$ is monotone decreasing in k. To see this, take the difference:

$$d(k+1) - d(k) = k \left[(b_m - b_k)^2 - (b_m - b_{k+1})^2 \right] \le 0.$$

Therefore, there exists a unique K such that $d(k) > 0$ for all $k \le K$, but $d(K+1) \le 0$. Then, (43) implies that there is a unique equilibrium set of shareholders, which is given by $S = \{1, ..., K\}$, so that $|S| = K$.

Note also that from (44), the equilibrium number of shareholders K is increasing in λ; we denote this function $K(\lambda)$. In particular, if $\lambda \to 0$ (holding costs are negligible), the ownership structure is very concentrated and only agents with exactly the same preferences as the manager become shareholders. Conversely, if λ exceeds $N(b_m - b_N)^2 - \sum_{j=1}^{N} (b_m - b_j)^2$, then all agents become shareholders.

Next, consider the optimal advisory committee B given that $K(\lambda)$ agents become shareholders. This is the committee of the maximum size for which the IC constraint on truthful reporting is satisfied for all its members. The IC constraint for committee member i (using (10) with $R = B \cup \{m\} \backslash \{i\}$) is

$$(\tau + |B| + 1)\,|b_m - b_i| \leq (\tau + N)\,\frac{c}{2}. \tag{45}$$

Since agents are ranked in order of increasing $|b_m - b_i|$ and have the same quality of information, it is without loss of generality to consider committees that include all agents of sufficiently low rank (except the manager). Suppose we take an advisory committee consisting of agents $\{2, ..., k\}$ (recall that agent 1 is the manager). The size of this committee is $k - 1$, so (45) for agent k, who has the largest misalignment of preferences with the manager among all committee members, becomes

$$(\tau + k)\,|b_m - b_k| \leq (\tau + N)\,\frac{c}{2}. \tag{46}$$

Since $|b_m - b_k|$ increases in k, the left-hand side expression $(\tau + k)\,|b_m - b_k|$ is increasing in integer k, taking value of zero for $k = 1$ and value exceeding $(\tau + N)\,\frac{c}{2}$ for $k = N$ (by assumption of the proposition). Therefore, there exists a unique integer $\hat{K} \leq N - 1$, such that among committees of the form $\{2, ..., k\}$, the constraint (45) holds for all agents of this committee if and only if $k \leq \hat{K}$. Since the optimal committee is that of the maximum size subject to truth-telling of all its members, the optimal committee is $\{2, ..., \min(\hat{K}, K(\lambda))\}$ and has size $\min\left(\hat{K}, K(\lambda)\right) - 1$. Thus, the optimal committee includes all non-manager shareholders if and only if $\hat{K} \geq K(\lambda)$. Since $\hat{K}$ does not depend on λ and $K(\lambda)$ is increasing in λ, there exists a cutoff λ^* such that the optimal committee includes all non-manager shareholders if and only if $\lambda \leq \lambda^*$.

Proof of Proposition 9

Rewriting the IC constraint from Proposition 1 and using $(b_m - b_i)(\rho_m - \rho_i) \geq 0$, board B is efficient if and only if $\mathcal{I}_i \geq 0$ for all $i \in B$, where

$$\mathcal{I}_i \equiv \frac{\tau + |B| + |\mathcal{M}| + 2}{\sum_{j \in -B_m} c_j}\left(\frac{1}{2}c_i - |b_m - b_i|\right) + \frac{1}{2} - \left|\rho_m - \rho_i\right|,$$

where $-B_m$ is a set of all signal indices that are not known to the board or the manager. Consider an expansion of $\mathcal{M}$ by one element. If this element belongs to $\{\theta_i, i \in B\}$, then all statements of the proposition are vacuously true, as the IC constraints are unaffected. Thus, consider the case when this element does not belong to $\{\theta_i, i \in B\}$. In this case, an expansion in $\mathcal{M}$ increases $|\mathcal{M}|$ and decreases $\sum_{j \in -B_m} c_j$. Suppose that $|b_m - b_i| \leq \frac{1}{2}c_i \; \forall i \in B$. Then, an expansion in $\mathcal{M}$ increases $\mathcal{I}_i$ for any i. Hence, if $\mathcal{I}_i \geq 0$ for all i for some $\mathcal{M}$, then $\mathcal{I}_i \geq 0 \; \forall i$ for any expansion in set $\mathcal{M}$. This proves the first statement of the proposition.

To prove the second statement, rewrite the IC constraint from Proposition 1 as $\mathcal{J}_i \geq 0$, where

$$\mathcal{J}_i \equiv \frac{\sum_{j\in -B_m} c_j}{\tau + |B| + |\mathcal{M}| + 2}\left(\frac{1}{2} - \left|\rho_m - \rho_i\right|\right) + \frac{1}{2}c_i - |b_m - b_i|.$$

Again consider an expansion of $\mathcal{M}$ by one element that does not belong to $\{\theta_i, i \in B\}$. Suppose that $\left|\rho_m - \rho_i\right| \leq \frac{1}{2}$ $\forall i \in B$. Then, an expansion in $\mathcal{M}$ reduces $\mathcal{J}_i$ for any i, because it increases $|\mathcal{M}|$ and decreases $\sum_{j\in -B_m} c_j$. Hence, if $\mathcal{J}_i \geq 0$ for all i for some $\mathcal{M}$, then $\mathcal{J}_i \geq 0$ $\forall i$ for any contraction in set $\mathcal{M}$.

Proof of Proposition 10

Proof of part (i)**.** Suppose that the manager expects agent i to report his signal truthfully, and consider agent i's decision whether to do so. If agent i reveals his signal truthfully, the manager's action is

$$a_m(\theta_R, \theta_i) \equiv c\theta_i + \gamma c\theta_m + \sum_{k\in R\setminus\{m\}} c\theta_k + \frac{\rho_m + \theta_i + \sum_{k\in R}\theta_k}{\tau + 1 + |R|}(N - |R| - 1)c. \quad (47)$$

In contrast, if agent i misreports, the manager's action is

$$a_m(\theta_R, 1-\theta_i) \equiv c(1-\theta_i) + \gamma c\theta_m + \sum_{k\in R\setminus\{m\}} c\theta_k + \frac{\rho_m + (1-\theta_i) + \sum_{k\in R}\theta_k}{\tau + 1 + |R|}(N - |R| - 1)c. \quad (48)$$

Truthful reporting is optimal if and only if

$$\sum_{\theta_{-i}\in\{0,1\}^{N-1}} \left[(a_m(\theta_R, \theta_i) - Z_i)^2 - (a_m(\theta_R, 1-\theta_i) - Z_i)^2\right] P_i(\theta_{-i}|\theta_i) \leq 0 \quad (49)$$

for each $\theta_i \in \{0,1\}$, where $Z_i = \gamma c\theta_i + c\sum_{j\neq i}\theta_j$. Simplifying, (49) reduces to

$$(2\theta_i - 1)\left[c(1 - 2\gamma\theta_i) + 2(\gamma - 1)c\frac{\rho_i + \theta_i}{\tau + 1} + \frac{2(\rho_m - \rho_i) + 1 - 2\theta_i}{\tau + 1 + |R|}(N - |R| - 1)c\right] \leq 0.$$

If $\theta_i = 1$, we have:

$$\rho_m - \rho_i \leq \frac{1}{2}\frac{N + \tau}{N - |R| - 1} + (\gamma - 1)\frac{\tau - \rho_i}{\tau + 1}\frac{\tau + 1 + |R|}{N - |R| - 1},$$

which is trivially satisfied for $\rho_m \le \rho_i$, but may be violated if $\rho_m > \rho_i$. If $\theta_i = 0$, we have:

$$\rho_i - \rho_m \le \frac{1}{2}\frac{N+\tau}{N-|R|-1} + (\gamma - 1)\frac{\rho_i}{\tau+1}\frac{\tau+1+|R|}{N-|R|-1},$$

which is trivially satisfied for $\rho_m \ge \rho_i$, but may be violated if $\rho_m < \rho_i$. Combining the two cases proves part (i) of the proposition.

Proof of part (ii)**.** For any agent i with sufficiently high $\left|\rho_i - \rho_m\right|$, (33) is violated for a sufficiently small committee, unless γ is too high. Specifically, the condition for this is:

$$\begin{aligned} \rho_m - \rho_i &> \frac{1}{2}\frac{N+\tau}{N-2} + (\gamma-1)\frac{\tau-\rho_i}{\tau+1}\frac{\tau+2}{N-2} \\ \text{or } \rho_i - \rho_m &> \frac{1}{2}\frac{N+\tau}{N-2} + (\gamma-1)\frac{\rho_i}{\tau+1}\frac{\tau+2}{N-2} \end{aligned}$$

for any i. In this case, there exists no committee consisting of a single agent that can communicate information truthfully (R in the case of a single-member committee is simply the manager's own signal). Since (33) relaxes as R expands, there exists $N^{\gamma}_{\min} \ge 1$ such that there is no committee consisting of $N^{\gamma}_{\min}$ or fewer agents for which truthful communication of information to the manager is an equilibrium. For any R, (33) relaxes as γ increases. Therefore, $N^{\gamma}_{\min}$ is (weakly) decreasing in γ.

Proof of part (iii)**.** Consider the committee consisting of all agents, $B = \{1,...,N\}\backslash\{m\}$. Since the right-hand sides of inequalities (33) equals infinity in this case ($|R| = N-1$), there is an equilibrium in which all agents in this advisory body communicate information truthfully to the manager. The argument identical to the proof of Lemma 2 implies that every agent is better off if the manager is more informed ($\mathbb{E}_i[U_i|R]$ increases as R expands). Therefore, $B = \{1,...,N\}\backslash\{m\}$ is the optimal committee.

Proof of Proposition 11

The proof largely repeats the proof of Proposition 1 in the paper. Using $a_m(\theta_R, \theta_i)$ and $a_m(\theta_R, 1-\theta_i)$, the IC becomes

$$\sum_{\theta_R,\theta_{-R}} P_i(\theta_R,\theta_{-R}|\theta_i)\left[c_i(2\theta_i - 1) + \left(\sum_{j\in -R\backslash\{i\}} c_j\right)\tfrac{2\theta_i - 1}{\tau_m+|R|+1}\right]\times \\ \left[2(b_m - b_i) + c_i(1-2\theta_i) - 2\sum_{k\in -R\backslash\{i\}} c_k\theta_k + \left(\sum_{k\in -R\backslash\{i\}} c_k\right)\tfrac{2(\rho_m + \sum_{k\in R}\theta_k)+1}{\tau_m+|R|+1}\right] \ge 0.$$

Note that $P_i(\theta_R, \theta_{-R}|\theta_i) = P_i(\theta_{-R}|\theta_R, \theta_i)P_i(\theta_R|\theta_i)$. Since $\left[c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau_m+|R|+1}\right] > 0$, this is equivalent to

$$-(2\theta_i - 1)\times\left[2(b_m - b_i) + c_i(1-2\theta_i) - 2\frac{\rho_i+\theta_i}{\tau_i+1}\sum_{k\in -R\setminus\{i\}} c_k + (\sum_{k\in -R\setminus\{i\}} c_k)\frac{2\rho_m + 2|R|\frac{\rho_i+\theta_i}{\tau_i+1}+1}{\tau_m+|R|+1}\right] \geq 0$$

or equivalently,

$$-(2\theta_i - 1)\times\left[2(b_m - b_i) + c_i(1-2\theta_i) + (\sum_{k\in -R\setminus\{i\}} c_k)\left[\frac{2\rho_m(\tau_i+1)+\tau_i+1-2(\rho_i+\theta_i)(\tau_m+1)}{(\tau_i+1)(\tau_m+|R|+1)}\right]\right] \geq 0$$

Considering two cases ($\theta_i = 1$ and $\theta_i = 0$) and simplifying the expressions (similar to the proof of Proposition 1), we obtain (34). It is easy to see that (34) is equivalent to the condition in Proposition 1 of the paper when $\tau_i = \tau$ for all i.