When does introducing verifiable communication choices improve welfare?*

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Abstract

This paper studies when introducing verifiable communication choices between agents in a cheap-talk benchmark setting, with social tie, is beneficial to welfare. In our model agents have *two* ways to communicate their private information: either through a costly *verifiable information* (hard) link or through a low-cost *cheap talk* (soft) link. We identify that the appearance of hard links in the pure cheap talk setting has two *opposing* effects on welfare: (i) a positive effect stems from the information improvement and (ii) a negative effect arises from crowding out soft communication with costly verifiable communication. Surprisingly, the final welfare outcome of the two opposing forces depends on the cost structure. If only one party bears the cost of a hard link, then the positive (*informational*) effect always dominates the negative (*crowding out*) effect, and thus introducing hard links is beneficial to welfare. In contrast, if the cost of a hard link is *shared* by both parties, then allowing for verifiable communication can be detrimental to welfare. We also derive several testable implications about introducing hard links in corporate governance, and demonstrate the robustness of our findings in face of heterogenous costs, general signal structures, as well as the case where cost is endogenizied via negotiation about how to split the costs.

Keywords: Hard and soft communication, Welfare, Incentive, Networks, Cost structure.

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1 Introduction

In many multi-division corporations a manager of a division is required to make his judgements about the uncertain state of the world and picks the most appropriate actions that influence other divisions as well. Oftentimes, these decisions are made based not only on private information of managers, but also on the information collected from other managers (agents) during preliminary discussions.[1] As a result, quality of communication between managers is critical for efficient and successful decision making in corporations. What communication policies are efficient to improve the quality of discussion between managers? And, how do these policies affect welfare?

The literature that studies possible communication policies can generally be classified by the type of information transmission technology into two categories. One approach follows Grossman (1981) and Milgrom (1981) in assuming a verifiable (or hard) type of information, i.e., information that can be withheld by the manager (player) but not lied about (*hard talk*). The second approach complies with Crawford and Sobel (1982) and Green and Stokey (2007) in considering an unverifiable (or soft) type of information, i.e., information that can be arbitrarily misreported by the manager at no cost (*soft*, *cheap talk*). The overwhelming majority of theoretical studies share the assumption that communication mode, either cheap talk or hard talk, is a feature of the environment, and hence is a fixed characteristic of communication stage.[2] While a predetermined communication mode might be natural in some cases, in practice, the participants not only choose with whom to communicate, but also get to determine the way in which the information is transferred.[3]

[1]See e.g., Malenko (2014), Harris and Raviv (2008), Stevenson and Radin (2009) and Hart (2003).

[2]Exceptions include Kartik, Ottaviani and Squintani (2007) and Kartik (2009) who consider the case when it is costly to misreport the information and show that the equilibrium outcome involves separation with inflated language. Dessein and Santos (2006) assume that the information transmission takes the form of a noisy hard talk and allow the organization to choose the level of communication precision. Dewatripont and Tirole (2005), Calvó-Armengol, de Martí and Prat (2011) and Persson (2011) depart from the predetermination of communication mode by considering the setting where the trustworthiness of information transmission is endogenously defined by the communication efforts of the parties.

[3]In particular, communication of the same piece of private information can take different forms, from sending one-sentence emails (soft talk) to prolonged discussions (hard talk). With the extreme options, it is natural to assume that the email communication is cheap and the discussion is costly, because sending a one-sentence email requires considerably less time and effort than participating in long discussions. Moreover, one would expect that a one-sentence email gives a decision-relevant summary of the sending party's private information without providing supplementary materials and explanations that will allow the receiving party to verify the message. Such limitation leaves the composition of the message entirely up to the sender's discretion. At the same time, prolonged discussions can give an opportunity for the reporting party to present the supporting data and provide the necessary justifications, so that the receiving manager is able to uncover the decision-relevant piece of information himself. From this point of view, sending a one-sentence email is closer to cheap talk communication, while prolonged meetings resemble verifiable

The goal of this paper is to study when introducing verifiable communication choices in a pure cheap-talk benchmark is beneficial to welfare. A major twist in our model, and a point of departure from the existing literature, is that agents have **two ways** to communicate their private information **over a communication network**: either through a **costly verifiable information (hard link)** or through a **low-cost cheap talk (soft link)**.

We particularly ask the following questions: How does communication mode (hard vs. soft) affect the quality of discussion between agents as well as welfare? Can verifiable (hard) communication technology choices, paradoxically, be detrimental to welfare? If introducing verifiable communication does have unintended effects then what types of policy regulations can improve welfare in such environments? In this paper we provide answers to these questions. Moreover, we study whether existing corporate policies mitigate or exacerbate the problem of ineffective communication, and specify which policies are optimal.

1.1 Environment

We consider decision-making under imperfect information by managers (players) with conflicts of interest in multi-division corporations. In such environments, managers have to decide on strategies for their respective divisions.[4] Moreover, the effect of chosen strategies on the firm's well-being depends on the economic environment, which is usually uncertain. However, each manager holds some private information about the characteristics of the economic environment. Corporation divisions might be endowed with different goals and interests depending on the divisions, which implies preference divergence among the managers.[5] Nevertheless, different divisions are parts of one corporation, meaning that the strategy chosen by some manager has an effect on other divisions' payoffs as well as corporation's welfare (shareholder value). Total corporation's welfare (or simply welfare) in our formulation also refers to shareholder value.

Several empirical studies show that preliminary discussion between managers is *not* necessarily all-to-all. Instead managers communicate based on their social tie, prior relationships, geography, etc. (e.g., Stevenson and Radin (2009) and references therein). We model this by a communication network. That is, before making decisions, managers can simultaneously transmit their private signals to each other according to the communi-

information transmission; and often it is up to the participants which type of communication to engage in.

[4]For example, managers are potentially located in different locations (e.g., countries), each being responsible for the corporation's performance in the manager's respective division.

[5]Conflicts of interest between managers may arise in corporate control transactions, for example, due to managers' ownership, geography, affiliation, social tie, etc. (see e.g. Adams, Hermalin and Weisbach (2010)).

cation network, which is set prior to the signals' realization. The network has a general structure and is described by a directed graph of hard and soft links. A manager can send messages to other manager that he has links to in compliance with the link type: if the link is soft, then reporting takes the form of cheap talk, while if the link is hard, then communication is non-strategic and verifiable to the other party.

While soft links are very cheap to sustain, hard links are costly. The way the cost of a hard link is split between the involved parties is defined by a specific cost structure that is the same for all pairs of managers. That is, the costs of outgoing and incoming hard links are fixed across managers. While considering a general cost structure where the cost is split arbitrarily between the parties, we distinguish between the following two cases:

- The first case is one in which only one party, either the sender or the receiver, bears the cost of a hard link. The structure with only the sender paying the cost naturally arises when it takes considerable effort to provide supporting data and to develop argumentation, while it is very easy for the receiver to uncover the underlying signal after being presented with the collected materials. Similarly, the cost structure where only the receiver incurs the cost corresponds to a situation in which it is much easier to formulate the message and provide the material, than it is to decode the underlying signal.

- The second case is one in which both parties bear strictly positive costs of a hard link and represents situations in which both the sender and the receiver are required to put in considerable effort to transmit and understand the information (e.g., Dewatripont and Tirole (2005)).

After finishing the communication stage, managers simultaneously choose actions that influence each other's payoffs as well as welfare. To study the model, we define an *equilibrium* to be an extension of pure strategies Perfect Bayesian Equilibrium (PBE). Namely, communication and action strategies form the usual pure strategies PBE, holding the communication network fixed. At the same time, the communication network must be such that no manager prefers to delete a link, and no two managers prefer to form a hard link in the absence of truthful communication, given rationally updated communication and decision making strategies. The resulting communication pattern is described by a directed truthful network, in which every link corresponds to truthful communication through a soft or hard link.

1.2 Overview of the results

The first—rather intuitive—result of the paper describes the effect of introducing the possibility of forming hard links on the scope of information transmission and the quality of discussion between managers. More precisely, start from the pure cheap talk setting, i.e., verifiable information transmission is prohibitively costly and all communication is performed through soft links. Then introduce feasible hard links, i.e., lower the cost of hard links such that they can emerge in the equilibrium truthful network. As a result, there is a new equilibrium that generates more intensive information transmission. Using the introduced notation, it means that a new equilibrium with hard links generates a truthful network with weakly *larger* in-degrees for all managers. As a result, this result shows that introducing verifiable communication choices (hard links) always improves the *quality of discussion* between managers. But, does that also benefit welfare?

The second—and main—result concerns the welfare aspect of introducing the feasible verifiable information transmission channel. The fact that there are almost always multiple equilibria presents some difficulties in comparing the welfare across different settings. We resolve the multiplicity issue by introducing the notion of pairwise stability—a natural equilibrium refinement. This notion is an analogue to the pairwise stability condition commonly used in network theory, but adapted to the defined equilibrium concept. More precisely, an equilibrium is called *pairwise stable*, if no two managers can find an incentive compatible way to improve interaction between them by communicating truthfully through a soft link instead of a costly hard link or by not communicating at all, holding other communication and decision making strategies fixed.

The welfare outcome of introducing feasible hard links in the initial pure cheap talk setting is composed of two effects: the positive effect of an **information improvement** and the negative effect of **crowding out cheap talk communication** with costly verifiable information transmission. The information improvement effect is the substance of the informational result described above: availability of verifiable information transmission leads to greater and more evenly distributed in-degrees. The crowding out effect is at the heart of the paper. As hard links become available, some managers who could not truthfully communicate via cheap talk can find it beneficial to pay the cost and transfer the private information through hard links. The appearance of hard links increases managers' in-degrees which can render truth-telling through some soft links no longer incentive compatible.[6] Those soft links should be replaced by hard links for the managers to remain credible. Thus, even though managers get more truthful messages, the number

[6]Clearly, the vulnerability of a soft link increases with the divergence in preferences.

of soft links might decrease, implying that the cheap talk communication is crowded out by the costly verifiable information transmission.

We find that, the final welfare outcome of the two opposing forces depends on the cost structure. In the case where only one party bears the cost of a hard link, there always exists a pairwise stable equilibrium with feasible hard links that generates greater total welfare than in the pure cheap talk case. Interestingly, the positive welfare result no longer holds when the cost of a hard link is shared between the managers. In particular, we analyze communication between two communities composed of agents with the same preferences (e.g., similar geographical locations). In this setting, we find that introducing feasible hard links does not alter soft intra-group communication—managers with the same preferences can always be credible to each other through a cheap talk. At the same time, newly created hard links can make cross-group cheap talk communication no longer credible, wipe out all cross-group soft links, and replace them with costly hard links. As a result, the welfare can decrease relative to the pure cheap talk case.

As another extreme, we consider the case of a diverse group of managers who have equidistant preference biases, and show that adding the possibility of forming hard links can decrease the welfare only when the number of managers is 3.

1.2.1 Extension and Robustness

Finally, we study three natural extensions of the model. In the first extension, we allow the managers to negotiate how to split the cost of a hard link between them. Surprisingly, we find that endogenizing of the cost shares does not imply aggregate efficiency and introducing hard links can still lead to lower welfare—which reinforces the main result of the paper.

In the second extension, we allow the costs of hard links differ across the pairs of managers. This might happen for various reasons, e.g., managers working at the same location might face a lower cost of verifiable information transmission that takes the form of personal meetings, than managers from different locations. We illustrate that when the difference in costs is substantial, introducing hard links likely results in *localization* of communication—pairs of managers with a low cost communicate with each other via hard links. At the same time, the amount of information accumulated by every manager can remain the same. This implies a decrease in the welfare compared to the pure cheap talk case, even when only one party bears the cost.

Finally, we show that our main conclusions do not depend on the deliberate additive signal structure and hold for natural *non-additive* signal structures as well. This makes the message of the paper more general and applicable to greater number of real-life situations.

Plan. The paper is organized as follows. Section 2 presents the model and discusses the solution concept. Section 3 provides the incentive for truthful reporting via a soft link, the benefit of forming a hard link, and their implications for the equilibrium communication networks. The informational result of introducing verifiable information transmission is presented in Section 4, and the welfare results are analyzed in Section 5. The extensions of the model with endogenous cost shares and heterogeneous hard links costs are considered in Section 6. Policy implications are in Section 7. Section 8 concludes the paper. Finally, Appendix A provides some additional details on characterization of the pairwise stable equilibria, and Appendix B and Appendix C contain all the proofs omitted in the main text.

1.3 Related literature

A major twist in our model, and a point of departure from the existing literature, is to analyze **welfare** when agents have **two ways** to communicate their private information over a **communication network**. Hence, this paper develops the strategic communication literature by endogenizing the choice between costless cheap talk (soft talk) and costly verifiable communication (hard talk) in communication networks. The paper also contributes to the theoretical literature in corporate governance in multi-division corporations.

The communication feature of our model contributes to the literature on information transmission. The recent literature has a variety of focuses: optimal design of contests (e.g., Bimpikis, Ehsani and Mostagir (2015)), design of crowdfunding campaigns (e.g., Alaei, Malekian and Mostagir (2016)), inspection and information disclosure (e.g., Papanastasiou, Bimpikis and Savva (2018)), information diffusion in networks (e.g., Acemoglu, Ozdaglar and ParandehGheibi (2010), Candogan and Drakopoulos (2019)), among others. Also, this paper is related to the growing literature on studying decision making with externalities: for example, strategic information exchange (e.g., Sadler (Forthcoming), Lobel and Sadler (2015)), optimal static pricing under presence of local externalities (e.g., Candogan, Bimpikis and Ozdaglar (2012), Jadbabaie and Kakhbod (2019)), experimentation with technology innovation (e.g., Acemoglu, Bimpikis and Ozdaglar (2011)) and information diffusion due to word of mouth effect (e.g., Ajorlou, Jadbabaie and Kakhbod (2018)).

Our results on how the structure of a hard link cost influences communication patterns contribute to the literature on organizational design with verifiable information (e.g., Bolton and Dewatripont (1994), Hart and Moore (2005), Radner (1992), Radner (1993), Sah and Stiglitz (1986), Van Zandt and Radner (2001), Harris and Raviv (2008), Malenko (2014), Malenko (2018)), cheap talk (e.g., Alonso, Dessein and Matouschek (2008), Caillaud and Tirole (2007), Rantakari (2008), Harris and Raviv (2008), Hagenbach and Koessler (2010), Galeotti, Ghiglino and Squintani (2013), Grenadier, Malenko and Malenko (2016), Kakhbod et al. (2018), and noisy hard talk, where the information is transmitted perfectly with some probability less than 1 (e.g., Dessein and Santos (2006)).[7] In contrast to these works, in our paper communication choice is *two-dimensional*, i.e., agents have two ways to communicate their private information: either through a costly verifiable information (hard) link or through a low-cost cheap talk (soft) link. Importantly, we show that the interplay between theses two dimensions has a crucial impact on welfare.

Also related are studies that focus on questions of coordination and adaptation with verifiable information transmission in communication networks. In particular, Chwe (2000) studies a collective action problem with preliminary communication regarding participation activity in a deterministic exogenous network. Calvó-Armengol and de Martí (2007) and Calvó-Armengol and de Martí (2009) analyze how the communication pattern affects individual behavior and aggregate welfare in a setting in which the players not only want to coordinate their actions, but also adapt to an unknown state of the world. Instead, Calvó-Armengol, de Martí and Prat (2011) consider local uncertainty regarding the state and study information transmission patterns that arise when the agents are allowed to alter the communication precision.

Our analysis of the communication patterns arising in equilibria contributes to the literature of strategic network formation, which includes Jackson and Wolinsky (1996), Bala and Goyal (2000), Goyal (2007) and Jackson (2008), and the studies that consider questions of learning in networks (e.g., Acemoglu et al. (2011)).

2 Model

Let the set of managers be $N = \{1, \ldots, n\}$ with $n \geq 2$, where each manager (player) i has the preference bias b_i. The underlying *economic environment* is summarized by θ that is un-

[7] While in this paper we focus on private communication, there are studies that compare private with public information transmission, e.g. Farrell and Gibbons (1989), Goltsman and Pavlov (2011) and private and public histories, e.g., Kakhbod and Song (2020).

known to the managers. Each manager i's prior of θ is characterized by Beta distribution with parameters (α, β) and density of $f(\theta) = \frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$. The preference profile $\{b_1, ..., b_n\}$ and the managers' priors are publicly known. There are D different *aspects* $s_1, ..., s_D$ that determine the *state of the world* as

$$S = \sum_{d=1}^{D} s_d. \tag{1}$$

Conditional on the underlying economic environment θ, the aspects $\{s_d\}_{d=1}^{D}$ are independent and identically distributed, and $s_d = 1$ with probability θ and $s_d = 0$ with complementary probability $1-\theta$. The total number of aspects is greater than the number of managers, $D \geq n$, and each manager i is privately informed of the aspect s_i (i.e., manager i receives the private signal s_i). Thus, all managers cumulatively get to know the first n of D aspects. Note that if $D = n$, then the managers jointly hold all the relevant information regarding the state of the world.

In Section 6.3 we will show that our main conclusions do not depend on the additive signal structure, and we extend the analysis to non-additive signal structures as well.

The communication network is set prior to the signals' realization and is described by a directed graph $g \in \{0, s, h\}^{n\times n}$, where $g_{ij} = s$ if and only if a soft link ij is present, meaning that i reports to j via a cheap talk channel; $g_{ij} = h$ if and only if a hard link ij is present, meaning that i communicates his signal to j via a verifiable information channel; $g_{ij} = 0$ if and only if there is no link from manager i to manager j.[8] It is assumed that hard links are costly and the way the cost of a hard link $C \geq 0$ is split between the managers depends on the specific cost structure. Once the specific cost structure is fixed, it is the same for all managers. For the main part of the paper, we assume cost C of a hard link to be the same for all pairs of managers; the case in which cost can vary with the managers' identities is considered in the Extensions section. At the same time, soft links are cheap to sustain: each involved party bears just an infinitesimal cost $\varepsilon > 0$.

While the managers (players) are aware of each other's existence and preference biases, *the type of links* in the communication network g is not common knowledge. Rather, each manager i knows only the structure of his respective neighborhood. That is, manager i observes the set of managers to whom he has soft links, denoted as $N_i^s(g) = \{j \in N :$

[8]The assumption that the communication network is defined before the realization of the signals, for instance, can be justified by the necessity of forming the communication schedule beforehand. Another case where this assumption is appropriate corresponds to repeated interactions, where each period there is a new draw of the state θ, then signals arrive and decisions are made, while the communication stage is a part of the routine schedule that stays constant across periods.

$g_{ij} = s\}$; and the set of managers to whom he has hard links, denoted as $N_i^h(g) = \{j \in N : g_{ij} = h\}$. Similarly, manager i observes the set of managers who have soft links directed to him, $N_i^{-1,s}(g) = \{j \in N : g_{ji} = s\}$; and the set of managers who have hard links directed to him, $N_i^{-1,h}(g) = \{j \in N : g_{ji} = h\}$.

Cost structure. If there is a hard link ij from manager i (the sender) to manager j (the receiver), then manager i bears a share $\lambda \in [0,1]$ and manager j bears a share $1-\lambda$ of the total link cost C. We distinguish between two cost structures:

(i) *Only one party bears the cost of a hard link, C.* $\lambda = 1$ corresponds to a case in which the sender has to exert some effort to create a message, which then allows the receiver to easily extract the signal. On the other hand, $\lambda = 0$ represents a case in which it is costless for the sender to elaborate the message, but then the receiver has to make a costly effort in order to understand it and reveal the underlying signal.

(ii) *The sender and the receiver share the cost of a hard link, C,* with the weights $\lambda \in (0,1)$ and $(1-\lambda)$, respectively. This happens when manager i, as a sender, needs to exert some effort to elaborate the message about the signal, while manager j, as a receiver, needs to make an effort in order to understand the message and learn what the underlying signal is.

Communication. Each manager i sends private messages to the managers that he has links to according to the respective links' nature in the communication network g: if a link ij is hard, then the message sent to manager j truthfully reveals the signal, $m_{ij}^g = s_i$; if a link ij is soft, then any message $m_{ij}^g \in \{0,1\}$ can be sent. It is assumed that the messages are sent simultaneously and are observed only by the sending and the receiving parties. Note, that communication via hard links is non-strategic—the message perfectly reveals the signal.[9] In contrast, communication through soft links is in the form of cheap talk, and hence is strategic. A *communication strategy* of manager i with the private signal s_i defines a vector

$$\mu_i^g(s_i) = \left\{\{\mu_{ij}^g(s_i)\}_{j\in N_i^s(g)}, \{\mu_{ij}^g(s_i)\}_{j\in N_i^h(g)}\right\},$$

where $\{\mu_{ij}^g(s_i)\}_{j\in N_i^s(g)} \in \{0,1\}^{|N_i^s(g)|}$ and $\mu_{ij}^g(s_i) = s_i$ for every $j \in N_i^h(g)$. A communication strategy profile is denoted by $\mu^g = \{\mu_1^g, \ldots, \mu_n^g\}$. The messages actually sent by manager i

[9]Note that, due to the unraveling argument, the perfect revelation of the underlying signal is also the outcome in the usual verifiable information setting with the option to withhold information. Indeed, assume that the message space is $\{0,1,\{0,1\}\}$. If manager i reports to manager j with the bias $b_j > b_i$ via a hard link, then manager i would always choose to reveal the signal 0. This, in turn, leads to the unique full revelation outcome.

are denoted by vector $\widehat{m}_i^g$, while the profile of all sent messages is $\widehat{m}^g = \{\widehat{m}_1^g, \ldots, \widehat{m}_n^g\}$. The superscript g signifies the dependence of the communication strategies on the network structure. We use the same superscript g for strategies of different managers to simplify the notation, however, one needs to keep in mind that every manager i conditions his communication strategy only on the available information about the communication network—the structure of manager i's neighborhood.

Decision making. After the communication stage, each manager i chooses an action $y_i^g \in \mathbb{R}$. Denote the set of all managers who have a link to i as $N_i^{-1}(g) = \{j \in N : g_{ji} = s \text{ or } g_{ji} = h\}$. Because the information set of manager i consists of his own signal, s_i, and the messages he gets from $N_i^{-1}(g)$, $\widehat{m}^g_{N_i^{-1}(g),i}$, the *action strategy* of manager i is a function $y_i^g : \{0,1\} \times \{0,1\}^{|N_i^{-1}(g)|} \to \mathbb{R}$. Let $y^g = \{y_1^g, \ldots, y_n^g\}$ denote the action strategy profile.[10] Conditional on the state of the world S, if the chosen action profile is $\hat{y}^g = \{\hat{y}_1^g, \ldots, \hat{y}_n^g\}$, then the realized payoff (utility) of manager i is

$$u_i(\hat{y}^g|S) = -\sum_{j=1}^{n} (\hat{y}_j^g - S - b_i)^2 - \mathbf{Cost}_{i,h}\Big(\{g_{ij}\}_{j \in N_i^h(g)}, \{g_{ji}\}_{j \in N_i^{-1,h}(g)}\Big),$$

manager i's payoff depends on how close his own action and the actions of other managers are to manager i's ideal action, $S + b_i$. Also, the last term captures the cost payed by i to receive (from $j \in N_i^{-1,h}(g)$) or send (to $j \in N_i^h(g)$) information via hard links (verifiable communications) to her neighbors. We note that the cost function $\mathbf{Cost}_{i,h}(\cdots)$ does not explicitly depend on y_i^g.

Time notation. For further analysis, it is useful to introduce the following time notation to distinguish among periods with different scopes of information available to the managers: *"ex-ante"* to denote the stage prior to when the signals are realized (such that the only information about the state of the world each manager has is the common prior), *"interim"* to refer to the time period after the signals' realization but prior to communication (such that each manager knows the common prior and his private signal), and finally, *"ex-post"* - for the period after communication has occurred but before the actions are taken (such that each manager knows the common prior, his private signal, and reported messages).

[10] Superscript g emphasizes the dependence of the action strategies on the communication network: each manager i knows the structure and the type of links in his neighborhood.

Solution concept. We solve the model using the concept of pure strategies Perfect Bayesian Equilibrium (PBE). The restriction to pure strategies simplifies the analysis and implies that cheap talk communication can take two forms: *truthful*, where the message reflects the signal perfectly, or *uninformative*, where, for any signal s_i, manager i sends the same message, either 0 or 1. In the latter case, we assume that when a manager gets a message which is off the equilibrium path, he ignores it and does not update his belief. Because of this simplification, equilibrium beliefs are defined in a straightforward way: any message received through a hard link or in truthful communication through a soft link induces perfect knowledge about the underlying signal, while any message received in uninformative communication through a soft link leaves the prior belief about the underlying signal unchanged.

Holding the communication network g fixed, it is natural to determine the communication and action strategy profile $(\mu^g, y^g) = (\{\mu_i^g\}_{i \in N}, \{y_i^g\}_{i \in N})$ by using the standard PBE solution concept. However, conditional on a particular choice of g and equilibrium (μ^g, y^g), some soft and costly hard links in g might be ex-ante undesired by at least one party involved in the link (i.e., a sender or a receiver with respect to this link). In particular, all soft links with uninformative communication are ex-ante unprofitable to both parties.[11] On the other hand, it might as well be the case that the two managers, i and j, would prefer to have a costly hard link ij in order to be able to directly transmit the signal s_i to j, rather than having no link or having a soft link with uninformative communication, while holding all other strategies fixed. One of the underlying ideas for the model is that managers can to some extent manage the links themselves and might object to existence of soft links with uninformative communication and to some hard links. To account for this, we define an equilibrium as a communication network g coupled with a strategy profile (μ^g, y^g) such that (i) the pair (μ^g, y^g) forms a PBE given g, (ii) no manager would strictly prefer to break some incoming or outgoing link from an ex-ante perspective, and (iii) no two managers, i and j, would strictly prefer to form a costly hard link $g_{ij} = h$ from an ex-ante perspective, holding other communication and action strategies fixed.[12]

[11] Note that no manager will object to a cheap soft link with truthful communication. Indeed, as it is shown in Lemma 1, destroying the soft link with truthful communication will strictly harm the ex-ante expected payoffs of both parties involved in the link (provided that the cost ε is infinitesimal).

[12] So far we don't consider a similar desire to form a soft link, because communication through a soft link is strategic, and the mere existence of a soft link does not guarantee informativeness of the communication pattern. Adding a soft link with truthful communication requires more coordination at the communication stage, otherwise the communication might be completely uninformative. The possibility of adding a soft link and coordinating on the respective communication pattern is considered later in the context of pairwise stability.

To formally state the equilibrium definition, it is useful to denote by $\mathbf{E}u_l(g, \mu^g, y^g)$ the ex-ante expected utility of manager l, where (μ^g, y^g) are communication and strategy profiles given some communication network g.[13] Let $g(g_{ij} = 0)$ be the communication network with the same set of links as in g, except that there is no link from i to j; let $\mu^{g(g_{ij}=0)}$ be the profile of communication strategies which coincides with μ^g everywhere, except that now there is no communication from i to j; and let $y^{g(g_{ij}=0)}$ be the same action profile as y^g for all managers but j, while manager j's action is now optimally defined conditional on the lower number of truthful messages. Similarly, let $g(g_{ij} = h)$ denote the communication network with the same set of links as in g, except that there is a hard link from i to j. Then $\mu^{g(g_{ij}=h)}$ is the profile of communication strategies which coincides with μ^g for all links but g_{ij}, through which the communication is truthful. Finally, $y^{g(g_{ij}=h)}$ is the same action profile as y^g for all managers but j, while manager j's action is now optimally defined given the new information structure. Using this notation, below is the formal equilibrium definition:

Definition. *Equilibrium* $\{g, (\mu^g, y^g)\}$ consists of a communication network g and a strategy profile $(\mu^g, y^g) = (\{\mu_i^g\}_{i \in N}, \{y_i^g\}_{i \in N})$, such that the following properties hold:

(i) The pair (μ^g, y^g) forms a PBE given the communication network g.

(ii) For any i and j such that $g_{ij} = h$ or $g_{ij} = s$:

$$\mathbf{E}u_l(g, \mu^g, y^g) \geq \mathbf{E}u_l\left(g(g_{ij} = 0), \mu^{g(g_{ij}=0)}, y^{g(g_{ij}=0)}\right), \text{ for } l = i, j.$$

(iii) For any i and j such that $g_{ij} = 0$ or $g_{ij} = s$:

$$\mathbf{E}u_l(g, \mu^g, y^g) \geq \mathbf{E}u_l\left(g(g_{ij} = h), \mu^{g(g_{ij}=h)}, y^{g(g_{ij}=h)}\right), \text{ for either } l = i \text{ or } l = j.$$

Remark. In any equilibrium $\{g, (\mu^g, y^g)\}$, communication network g is *truthful*, i.e., all links of g represent truthful revelation of private signals. Different equilibria correspond to different communication networks.

[13]Note that the ex-ante expected utility $\mathbf{E}u_l(g, \mu^g, y^g)$ takes into account all link costs that accrue to manager l.

3 Analysis

3.1 Choice of action

To derive the optimal choice of action, fix the communication network g and consider manager i who learned his private signal s_i and received the messages $\widehat{m}^g_{N_i^{-1}(g),i}$ from his neighbors $N_i^{-1}(g)$. Manager i then chooses an action $y_i^g(s_i,\widehat{m}^g_{N_i^{-1}(g),i})$ to maximize his ex-post expected payoff,

$$\mathbf{E}\left(-\sum_{j=1}^{n}(y_j^g - S - b_i)^2 - \mathbf{Cost}_{i,h}\Big(\{g_{ij}\}_{j\in N_i^h(g)},\{g_{ji}\}_{j\in N_i^{-1,h}(g)}\Big)\,\middle|\, s_i,\widehat{m}^g_{N_i^{-1}(g),i}\right),$$

This means that the manager optimally picks

$$\begin{aligned} y_i^g(s_i,\widehat{m}_{N_i^{-1}(g),i}) &= \arg\max_{y_i^g}\left\{\mathbf{E}\left(-(y_i^g - S - b_i)^2\,\middle|\, s_i,\widehat{m}^g_{N_i^{-1}(g),i}\right)\right\} \\ &= b_i + \mathbf{E}\left(S\,|\,s_i,\widehat{m}^g_{N_i^{-1}(g),i}\right). \end{aligned} \tag{2}$$

Because the communication is assumed to be either truthful or uninformative, the information set $\left(s_i,\widehat{m}^g_{N_i^{-1}(g),i}\right)$ can be equivalently represented as the set of revealed signals. Specifically, assume that manager i gets to know k signals summarized in a set s_R. The unknown $D-k$ signals are denoted as a set s_{-R}. Using this notation, i's optimal action can be written as

$$y_i^g(s_R) = b_i + \sum_{s_d\in s_R} s_d + \mathbf{E}\left(\sum_{s_d\in s_{-R}} s_d\,\middle|\, s_R\right) = b_i + \sum_{s_d\in s_R} s_d + (D-k)\mathbf{E}(s|s_R),$$

where the second equality holds because the aspects are identically distributed. Thus, the optimal action is the sum of the preference bias, the known aspects, and the prediction of the unknown part of the state.

3.2 Equilibrium networks

Assume that the strategy profile (μ^g, y^g) is such that the communication network g is truthful: communication through each link of g leads to a perfect signal revelation. Define $k_j(g)$ to be the number of other managers who report truthfully to j via either channel, and refer to it as the *in-degree* of manager j.

Benefits of signal revelation. For the analysis of equilibrium networks, it is useful to study the ex-ante expected additional benefit that accrues to some manager $i \in N$ from an extra signal revealed to manager j. In order to do this, assume that k_j managers report truthfully to manager j. This means that, together with his own private signal, manager j gets to know a set of $k_j + 1$ signals. Denote the set of known $k_j + 1$ aspects as $s_R \in \{0,1\}^{k_j+1}$ and the set of other aspects, as $s_{-R} \in \{0,1\}^{D-k_j-1}$. Based on the information s_R, j chooses action $y_j(s_R) = b_j + \mathbf{E}(S|s_R)$. Consider the ex-ante expected input from manager j into i's utility. Given the quadratic loss utility function, the input is comprised of an expected residual variance of the unknown part of the state S and a square of the preference divergence:

$$-\mathbf{E}\left[\mathbf{Var}\left(\sum_{s_d \in s_{-R}} s_d | s_R\right)\right] - (b_j - b_i)^2 = -\Phi(k_j) - (b_j - b_i)^2.$$

Assume now that an extra signal is revealed to manager j by some manager $l \in N$. Then j bases his decision on $k_j + 2$ signals (s_R, s_l). The ex-ante expected input from j into i's utility becomes $-\Phi(k_j+1) - (b_j - b_i)^2$. As a result, the ex-ante expected additional benefit that accrues to manager i from an additional truthful link directed to j is in reducing the expected residual variance of the state S:

$$\Phi(k_j) - \Phi(k_j + 1) = \mathbf{E}\left[\mathbf{Var}\left(\sum_{s_d \in s_{-R}} s_d | s_R\right)\right] - \mathbf{E}\left[\mathbf{Var}\left(\sum_{s_d \in s_{-R}/s_l} s_d | s_R, s_l\right)\right] > 0. \quad (3)$$

Note first, that the extra benefit is positive for all values of k_j, because every additional signal improves the information available to manager j, and hence reduces the expected residual variance of S. Second, the additional benefit of manager i given by (3) doesn't depend on the preference divergence, $b_i - b_j$. Intuitively, manager j chooses an action equal to the expected state S plus his preference bias b_j, and the only way in which extra signal impacts this action is through the precision of the expected value of S. Thus, each manager $i \in N$, including manager j himself, enjoys the same ex-ante expected extra benefit of (3) from manager j having an extra truthful link directed to him.[14] Finally, as shown in the following lemma, $\Phi(k_j)$ is a decreasing and convex function of k_j. Therefore, the additional benefit (3) is a decreasing function of the number of signals that manager j already gets, k_j. Intuitively, the better the information of manager j is, the lower is the

[14]Clearly, the result that manager j obtains the same extra benefit as other managers is due to the specific form of the utility function, namely, that the terms corresponding to different managers' actions enter the sum with the same coefficients.

marginal impact of an additional signal in improving the residual variance, implying a lower ex-ante expected gain in the payoff.

These results are summarized in the following lemma:

Lemma 1. *Fix a truthful network g and consider manager j with the in-degree $k_j < n-1$. If j learns one extra signal s_i, then each manager $l \in N$ derives the ex-ante expected benefit of $\Phi(k_j) - \Phi(k_j+1) > 0$ that decreases with k_j, where*

$$\Phi(k_j) = \frac{\alpha\beta(\alpha+\beta+D)(D-k_j-1)}{\alpha+\beta(\alpha+\beta+1)(\alpha+\beta+k_j+1)}.$$

Proof. See Appendix B. □

Communication through soft links. While Lemma 1 ensures that the truthful communication via a soft link is always desirable from the ex-ante perspective, there is a credibility issue at the interim stage. To study the incentive of truthful reporting via a soft link, consider the case in which manager i has a soft link to manager j. Suppose that j gets to know k_j signals: 1 signal manager j gets himself and $(k_j - 1)$ signals he infers from other managers' messages, excluding i. Denote a vector of these k_j signals as s_R. Assume that manager j believes i's message, i.e., j puts probability 1 on that $s_i = m_{ij}$. If i reveals his private signal, $m_{ij} = s_i$, then manager j optimally picks an action $y_j(s_R, s_i)$; if i misreports and sends $m_{ij} = 1 - s_i$, then j chooses $y_j(s_R, 1-s_i)$. manager i reveals his signal whenever it results in a greater interim expected payoff:

$$\sum_{s_R \in \{0,1\}^{k_j}, s_{-R} \in \{0,1\}^{D-k_j-1}} -\left[(y_j(s_R,s_i) - S - b_i)^2 - (y_j(s_R,1-s_i) - S - b_i)^2\right] P(s_R, s_{-R}|s_i) \geq 0.$$

As is shown in the proof of Proposition 1, this incentive compatibility constraint of truth-telling can be rewritten as

$$|b_j - b_i| \leq \frac{\alpha+\beta+D}{2(\alpha+\beta+k_j+1)}, \tag{4}$$

which means that manager i can report truthfully to manager j via a soft link as long as manager j doesn't get to know too many signals, given their preference divergence. Formally speaking, while this constraint (4) is always satisfied when $|b_j - b_i| \leq 1/2$, it might fail to hold when the preference divergence is significant, $|b_j - b_i| > 1/2$. In the latter case, manager i can be credible so long as manager j doesn't get to know too many signals relative to the divergence in their preferences.

To see the intuition behind this, assume that $b_j > b_i$ meaning that the ideal action for manager j is greater than the ideal action for manager i. The effect of an additional signal on j's action decreases with the number of signals that j gets to know, k_j. Thus, for sufficiently high k_j, the effect of i's message on j's action is so small that i would prefer to lie when $s_i = 1$ and report $m_{ij} = 0$ in order to shift j's action closer towards i's preferred one. On the other hand, when j's in-degree is quite low, the effect of an additional signal on j's action is quite large, in which case misreporting when $s_i = 1$ may shift j's action downward too much, making it undesirable.

The incentive compatibility constraint (4) relates to the negative externality effect under conflicting preferences of Kakhbod et al. (2018) and the congestion effect of Galeotti, Ghiglino and Squintani (2013). We also refer to this effect as the negative externality effect of information transmission in communication networks—greater information has a negative effect on further information accumulation by discouraging other managers to report truthfully.[15]

Incentives to form/delete hard links. Unlike soft-link communication, information transmission via a costly hard link is not affected by credibility concerns. The decisions regarding the existence of particular hard links are made before the private signals are realized. Thus, in order to study the incentives to form (maintain/not destroy) a hard link ij, we consider the net expected value of ij from the ex-ante perspective using the prior distribution of θ. Fix a truthful network g and assume that k_j other managers apart from i report truthfully to manager j, i.e., $k_j = |N_j^{-1}(g)/\{i\}|$. By Lemma 1, the ex-ante expected additional benefit that accrues to manager i and manager j from having a hard link ij is solely in reducing the residual uncertainty regarding the state of the world, $\Phi(k_j) - \Phi(k_j + 1)$.

Consider a general case of the cost structure, where C is distributed between the sender and the receiver with the shares $\lambda \in [0, 1]$ and $1 - \lambda$, respectively. Both managers, i and j, would like a hard link ij to be a part of the communication network only if the cost paid by each of them is lower than the expected benefit from having the hard link:

$$\max\{\lambda, 1 - \lambda\}C \leq \Phi(k_j) - \Phi(k_j + 1). \tag{5}$$

[15] Another natural way to think about the negative externality effect is to view the privately observed aspects as *substitutes*. Indeed, if manager i reports to j truthfully, then some other manager l might not be credible in communicating to manager j. If, on the other hand, manager i does not report to j, then manager l might be able to transmit his private information truthfully. This means that in such a communication process, private signals of i and l act as substitutes.

Given the properties of the additional benefit outlined in Lemma 1, a few things can be noted. First, while the willingness to truthfully communicate the signal via a soft link decreases with the preference divergence, the incentive to form (or maintain) a hard link is independent of the preference biases. Second, similar to the condition of credible soft-link communication, the incentive to form a hard link represents a negative externality effect. Indeed, because the benefit of the hard link decreases with k_j, the managers will be reluctant to form a hard link when k_j is sufficiently high. Finally, the incentive condition to form (or maintain) a hard link to manager j (5) is the same across every two settings characterized by (C_1, λ_1) and (C_2, λ_2) such that $\max\{\lambda_1, 1-\lambda_1\}C_1 = \max\{\lambda_2, 1-\lambda_2\}C_2$. This also means that the two settings share the same set of equilibria. In particular, the equilibria are the same across the following 3 cases: (i) only the sender bears the cost of the hard link C, (ii) only the receiver bears the cost C, and (iii) the cost of the hard link $2C$ is split equally between the sender and the receiver.

Equilibrium characterization. To fix ideas, we assume that whenever a manager is indifferent, the choice is made in favor of forming (or maintaining) a hard link. This assumption along with the incentive conditions (4) and (5) lead to the following equilibrium characterization:

Proposition 1. *Consider a triple $\{g, (\mu^g, y^g)\}$ and assume that each element of y^g satisfies the optimality condition (2). Then $\{g, (\mu^g, y^g)\}$ constitutes an equilibrium if and only if the communication network g is truthful and satisfies the following conditions: for any manager j with an in-degree $k_j = k_j(g)$ and any manager i,*

$$\begin{aligned} g_{ij} = s &\quad \textit{only if } \ |b_j - b_i| \leq \frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)}, \\ g_{ij} = h &\quad \textit{only if } \ \max\{\lambda, 1-\lambda\}C \leq \Phi(k_j - 1) - \Phi(k_j), \\ g_{ij} = 0 &\quad \textit{only if } \ \max\{\lambda, 1-\lambda\}C > \Phi(k_j) - \Phi(k_j + 1). \end{aligned}$$

Proof. See Appendix B. □

Intuitively, for manager j with an in-degree k_j to get truthful messages through the soft links from managers $N_j^{-1,s}(g) = \{i \in N : g_{ij} = s\}$ in equilibrium, the incentive compatibility constraint (4) must be satisfied for every $i \in N_j^{-1,s}(g)$. Further, managers $N_j^{-1,h}(g) = \{i \in N : g_{ij} = h\}$ reveal their signals to j in a verifiable way whenever no manager objects to any existing hard link, i.e., the incentive condition (5) is satisfied. Finally, for managers $N_j^{-1,0}(g) = \{i \in N : g_{ij} = 0\}$ not to report truthfully to j, it must be the case that there is no manager $i \in N_j^{-1,0}(g)$, such that i and j prefer to create a new hard link ij.

More can be noted about the equilibrium networks for extreme levels of the cost C. On the one hand, a sufficiently high cost C precludes the existence of verifiable information transmission in equilibrium. To find the threshold value of C, above which no hard links can be sustained, recall that the benefit from a hard link is a decreasing function of the manager's in-degree. Consequently, the expected additional benefit can not exceed $\Phi(0) - \Phi(1)$. This means that no equilibrium network can have hard links whenever the cost $C > (\Phi(0) - \Phi(1)) / \max\{\lambda, 1-\lambda\}$.[16] On the other hand, if the cost C is sufficiently small, then any equilibrium network is necessarily complete. Indeed, if $C \leq \Phi(n-2) - \Phi(n-1)$, then for any two managers, i and j, a hard link ij is preferred to no link independently of $k_j \leq n-2$. Hence, in any equilibrium communication network, the in-degree of each manager must be $n-1$, i.e., every individual obtains all the information.

4 Informational result

In this section we start with the case in which only soft communication is available and show how the introduction of a verifiable communication technology expands the information accumulated by each manager—the first (and the intuitive) result of the paper.

The setting with only cheap communication can be viewed as a setup with a prohibitively high hard link cost. In particular, assume that

$$C = C_0 > (\Phi(0) - \Phi(1)) / \max\{\lambda, 1-\lambda\}$$

and consider some equilibrium $\left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\}$. Let the in-degrees of managers in the equilibrium network be $k_1 = k_1(g(C_0)), \ldots, k_n = k_n(g(C_0))$. We will refer to such equilibria with only cheap communication as the *pure cheap talk* or the *pure soft-link* equilibria. Now suppose that the cost of a hard link is decreased, so that hard links can be a part of some equilibrium network. If for every manager $j \in N$, $C_1 \geq \frac{\Phi(k_j)-\Phi(k_j+1)}{\max\{\lambda,1-\lambda\}}$, then the considered pure cheap talk equilibrium still remains an equilibrium. If, however, $C_1 < \frac{\Phi(k_j)-\Phi(k_j+1)}{\max\{\lambda,1-\lambda\}}$ for some j, then, by Proposition 1, $g(C_0)$ fails to be an equilibrium network. The question of interest is whether instead there is an equilibrium $\left\{g(C_1), (\mu^{g(C_1)}, y^{g(C_1)})\right\}$, such that the information accumulated by each manager is improved, i.e., the in-degrees $k_1' = k_1(g(C_1)), \ldots, k_n' = k_n(g(C_1))$ exceed the in-degrees $k_1, \ldots, k_n$, respectively. The following theorem provides a positive answer: indeed, when communication via hard links becomes feasible, there exists an equilibrium with hard links that is weakly information superior to the pure soft-link equilibrium.

[16] Clearly, if $C > 2(\Phi(0) - \Phi(1))$ then no equilibrium network can have hard links, independently of α.

Theorem 1. *Take any cost* $C_0 > (\Phi(0) - \Phi(1)) / \max\{\lambda, 1-\lambda\}$ *and consider some pure cheap talk equilibrium* $\left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\}$*. Let the in-degrees in the equilibrium network* $g(C_0)$ *be* $k_j = k_j(g(C_0))$*,* $j \in N$*. Then for any cost* $C_1 \leq (\Phi(0) - \Phi(1)) / \max\{\lambda, 1-\lambda\}$ *there exists an equilibrium* $\left\{g(C_1), (\mu^{g(C_1)}, y^{g(C_1)})\right\}$ *in which the managers have weakly greater in-degrees:*

$$k'_j = k_j(g(C_1)) \geq k_j \text{ for any } i \in N.$$

Proof. See Appendix B. □

While the complete proof is presented in Appendix B, here we illustrate the intuition for the result with a particular C_1. Consider a pure soft-link equilibrium that corresponds to the cost C_0 and renumber the managers such that their in-degrees in the equilibrium network $g(C_0)$ are increasing in their respective number: $k_1 \leq k_2 \leq \ldots \leq k_n$. Assume now, that the cost is set to the level of C_1 such that

$$\max\{\lambda, 1-\lambda\}C_1 \in (\Phi(k_j+1) - \Phi(k_j+2), \Phi(k_j) - \Phi(k_j+1)]$$

for some $j \in N$, where $k_j < k_{j+1}$. By Proposition 1, the in-degree of every manager must be at least k_j+1, because otherwise, there exists a pair of managers who would prefer to form a new hard link. In particular, set $g(C_1)$ such that each manager $i = 1, \ldots, j$ has exactly $k_j + 1$ incoming hard links and no incoming soft links. For other managers $j+1, \ldots, n$, suppose that the only links directed towards them in $g(C_1)$ are the soft links from the pure cheap talk equilibrium network $g(C_0)$. Clearly, truthful communication through these soft links is still incentive compatible because the in-degrees of the managers are the same as in the pure cheap talk equilibrium. Further, no manager would like to destroy a hard link directed towards manager $i = 1, \ldots, j$, because $\max\{\lambda, 1-\lambda\}C_1 \leq \Phi(k_j) - \Phi(k_j+1)$. Also, no two managers would want to deviate and create hard links directed towards managers $j+1, \ldots, n$, because $\max\{\lambda, 1-\lambda\}C_1 > \Phi(k_j+1) - \Phi(k_j+2) \geq \Phi(k_i+1) - \Phi(k_i+2)$ for any $i = j+1, \ldots, n$. Thus, the construction results in an equilibrium network $g(C_1)$ with the in-degrees

$$\underbrace{k'_1 = \ldots = k'_j = k_j + 1}_{\text{Hard-link communication}}, \underbrace{k'_{j+1} = k_{j+1}, \ldots, k'_n = k_n}_{\text{Soft-link communication}},$$

that are greater than or equal to the corresponding in-degrees in $g(C_0)$.

5 Welfare result

Throughout the section we consider a non-trivial case of a strictly positive hard link cost: $C > 0$.[17] Because there might be multiple equilibria, we focus on those that satisfy the natural refinement of pairwise stability—no two managers can profitably deviate by changing the communication pattern between them. As Section 4 shows, availability of hard links leads to greater in-degrees which, by Lemma 1, positively affects the total welfare. However, in what follows, we demonstrate that, apart from the positive informational effect, there is a negative crowding out effect: the appearance of costly hard links crowds out costless soft communication which, in turn, harms the welfare. This section presents the second (and the main) result of the paper, namely, introducing feasible hard links can be both beneficial and detrimental to the total welfare, depending on the setting and the cost structure.

To gain a better understanding of the welfare implications, we first illustrate the interaction of the two effects in an example with three managers. Afterwards, we move to a more general setting and demonstrate that if only one party bears the cost of a hard link, then introducing hard links is welfare beneficial. Later on, we consider the case in which the cost of a hard link is shared between the parties and derive conditions for the welfare decrease in natural settings of the two communities and the diverse group of managers.

5.1 Pairwise stable equilibria

There may exist multiple equilibria, and as a natural refinement, we adapt the common notion of pairwise stability from the networks literature (e.g., Bala and Goyal (2000), Goyal (2007), Jackson (2008), Jackson and Wolinsky (1996)). In particular, call an equilibrium $\{g, (\mu^g, y^g)\}$ *pairwise stable*, if no pair of managers can change the communication pattern between them to improve their ex-ante expected utilities, while satisfying the interim incentive compatibility constraints of truth-telling, holding other strategies fixed. More formally:

Definition. An equilibrium $\{g, (\mu^g, y^g)\}$ is *pairwise stable* if

(i) For any $i, j \in \{1, .., n\}$, $g_{ij} = 0$ only if, holding other strategies fixed, i cannot credibly report to j via a soft link, assuming that j believes i's message, and communication via a hard link is not desired by at least one party.

[17] Case $C = 0$ is trivial because each equilibrium network is necessarily complete, and managers face zero costs of sustaining it. This implies the highest possible levels of individual and welfare.

(ii) For any $i, j \in \{1,..,n\}$, $g_{ij} = h$ only if, holding other strategies fixed, i cannot credibly report to j via a soft link, assuming that j believes i's message.

Characterization of pairwise stable equilibria. In order to prove the existence and study the properties of pairwise stable equilibria, we first define the notion of a *maximal equilibrium* as an equilibrium that generates the maximal vector of in-degrees among all equilibria:

Definition. Equilibrium $\{g, (\mu^g, y^g)\}$ with the in-degrees $k_1 = k_1(g), ..., k_n = k_n(g)$ is *maximal* if for any other equilibrium with the in-degrees $k'_1, ..., k'_n$:

$$k_i \geq k'_i,\ i = 1, ..., n.$$

In turn, the in-degrees $k_1, ..., k_n$ are called *maximal in-degrees*.

The following lemma states that the set of pairwise stable equilibria is non-empty and is a subset of maximal equilibria.

Lemma 2. *There exist a maximal and a pairwise stable equilibrium. Any pairwise stable equilibrium is maximal.*

Proof. See Appendix B. □

Let the manager welfare denote the ex-ante expected manager payoff and the welfare stand for the sum of all ex-ante expected manager payoffs. Because the pairwise stability incorporates efficient communication in terms of its cost and informativeness, there exists a pairwise stable equilibrium whose welfare is (weakly) higher than in any other equilibrium.[18] For example, such a pairwise stable equilibrium can be constructed in the following way (we refer to this construction further in the text as well). For each $i \in N$ perform the following procedure: order other managers $j \in N/\{i\}$ in the increasing absolute values of their preference divergence from i, $|b_j - b_i|$; let this order be $i_1, ..., i_{n-1}$. Consider the maximal in-degree of manager i, k_i. If $k_i = 0$, then nobody can report truthfully to i in equilibrium. If $k_i > 0$, then take the closest manager i_1: if truth-telling through a soft link $i_1 i$ is incentive compatible for i_1, given that i gets $k_i - 1$ other truthful messages, then let i_1 report truthfully to i via a soft link. Otherwise, set $g_{i_1 i} = h$. Repeat this procedure for other $k_i - 1$ closest to i managers to set the links of particular type with truthful communication through them. In the proof of Lemma 2, we show that this

[18] This statement cannot be extended to a per-individual basis because costly hard links can be distributed differently in various maximal and pairwise stable equilibria.

construction leads to a pairwise stable equilibrium, in which each manager gets truthful messages through the soft links from the managers sufficiently close in their preferences, truthful messages through the hard links from the less close managers, and no messages from the more distinct managers. This equilibrium generates the greatest total welfare, because, first, the equilibrium construction guarantees the minimal possible number of the costly hard links across all equilibria. And second, the equilibrium network provides the maximal level of individual informativeness which positively influences welfare, because, by Lemma 1, the ex-ante expected individual payoffs increase with the in-degree of each manager.

An additional comment regarding the welfare can be made in the pure cheap talk setting: all maximal equilibria generate the same ex-ante expected individual payoffs (and, as a consequence, the same welfare) that are the greatest across all equilibria. Indeed, provided that the cost of a soft link ε is infinitesimal, the ex-ante expected payoff of each individual depends only on the vector of in-degrees; hence, the expected individual payoffs are the same across all maximal equilibria. These results are presented in the lemma below.

Lemma 3. *There exists a pairwise stable equilibrium that generates the greatest welfare across all equilibria. In the pure cheap talk setting, all maximal equilibria generate the same ex-ante expected individual payoffs that are the greatest across all equilibria.*

Interested readers are referred to Appendix A for an additional discussion of pairwise stable equilibria.

5.2 Example with three managers

Let $\alpha = \beta = 1$, i.e., the prior distribution of θ is uniform on the interval $[0,1]$. Consider three managers with the preference biases $b_1 = 0$, $b_2 = b$, $b_3 = 2b$, where $\frac{2+D}{10} < b \leq \frac{2+D}{8}$. Such preference structure implies that truth-telling through a soft link ij is incentive compatible for manager i if and only if $|b_i - b_j| = b$ and nobody else reports to j truthfully. If hard links are prohibitively costly, $\max\{\lambda, 1-\lambda\}C > \Phi(0) - \Phi(1)$, then there are two pairwise stable pure cheap talk equilibria that generate the following truthful networks (see Figure 1, where dashed lines depict soft links):

(i) $g_{21} = g_{23} = g_{12} = s$, $g_{13} = g_{31} = g_{32} = 0$,

(ii) $g_{21} = g_{23} = g_{32} = s$, $g_{13} = g_{31} = g_{12} = 0$.

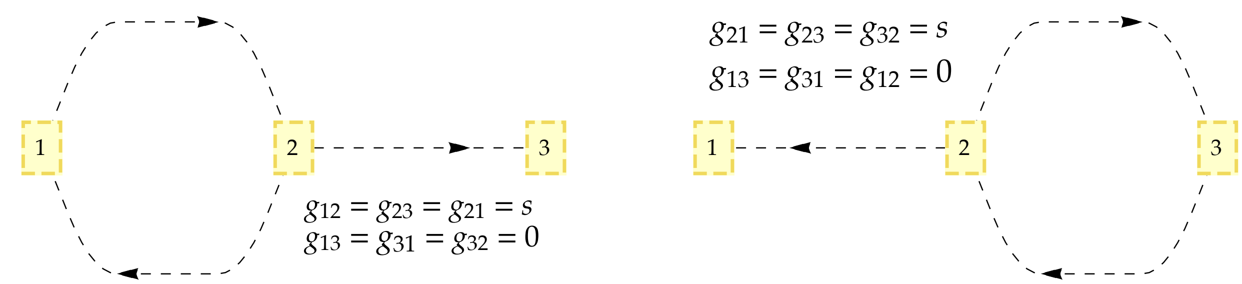

Figure 1: Communication networks (i) (left graph) and (ii) (right graph) of pairwise stable pure cheap talk equilibria.

Note, that managers have the same in-degrees of 1 and the same ex-ante expected payoffs across the pairwise stable equilibria. Consider the equilibrium corresponding to the soft-link truthful network (i), denoted by g^s. Using Lemma 1 and ignoring the infinitesimal costs of the soft links, the ex-ante expected payoff of manager i, denoted as $\mathbf{E}u_i(g^s)$, is:

$$\mathbf{E}u_i(g^s) = -\sum_{j=1}^{3}[\Phi(1) + (b_j - b_i)^2] = -3\Phi(1) - B_i, \;\; i = 1,2,3,$$

where B_i depends only on divergence in preferences, $B_i = \sum_{j=1}^{3}(b_j - b_i)^2$.

Suppose that the cost of a hard link satisfies $\max\{\lambda, 1-\lambda\}C \in (\Phi(1)-\Phi(2), \Phi(0)-\Phi(1)]$, i.e., managers i and j prefer to form a hard link ij if the in-degree of manager j is 0. Clearly, in this case, the set of the pairwise stable equilibria is the same as in the pure cheap talk setting above.

Now assume that the cost is decreased further: $\max\{\lambda, 1-\lambda\}C \in (0, \Phi(1) - \Phi(2)]$, i.e., i and j prefer to introduce a hard link ij if the in-degree of manager j is $k_j \leq 1$. Consequently, the in-degree of any manager in any equilibrium network must be 2. Because manager i cannot be credible in cheap talk communication to manager j with $k_j = 2$, no soft links can be a part of an equilibrium network. As a result, the only equilibrium (which is also pairwise stable) has a complete communication network consisting of hard links, which we denote by g^h (see Figure 2, where solid lines depict hard links).

This analysis shows how soft links are substituted with costly hard links, as the cost decreases.[19] Regarding the welfare effect, there are two forces. On the one hand, crowding out of cheap communication by costly verifiable communication is welfare decreasing. On the other hand, managers accumulate more information, which is welfare increasing. Below we show how the resulting impact depends on the cost structure and the

[19] Consider, for example, the effect of introducing a hard link 31 in the soft-link network g^s. Once the hard link 31 is formed, then manager 2 is no longer credible in reporting to manager 1 via a soft link. This forces manager 2 and manager 1 to substitute a soft link 21 with a costly hard link.

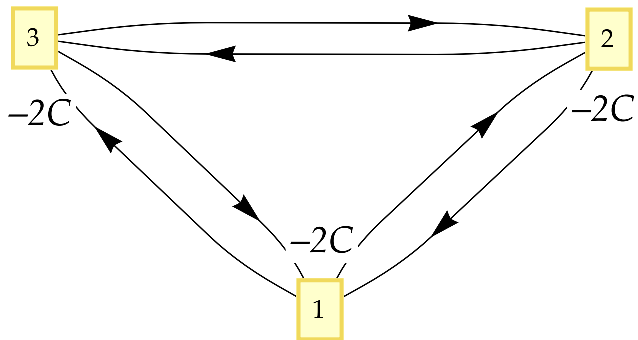

Figure 2: Equilibrium network when $\max\{\lambda, 1-\lambda\}C \in (0, \Phi(1) - \Phi(2)]$.

cost level.

In the communication network g^h, each manager has the in-degree of 2 and supports 4 hard links—2 incoming and 2 outgoing—which implies that each manager faces the cost of $2C$ (see Figure 2). The ex-ante expected payoff of manager i then is

$$\mathbf{E}u_i(g^h, C) = -3\Phi(2) - 2C - B_i, \;\; i = 1, 2, 3.$$

Only one party bears the cost of a hard link. If only one party faces the cost of a hard link, then the difference in the ex-ante expected payoffs corresponding to g^h and g^s is

$$\mathbf{E}u_i(g^h, C) - \mathbf{E}u_i(g^s) = 2\left(\frac{3}{2}(\Phi(1) - \Phi(2)) - C\right) > 0, \;\; i = 1, 2, 3,$$

because $C \leq \Phi(1) - \Phi(2)$. This means that the positive informational effect dominates the negative crowding-out effect, and the pairwise stable equilibrium with the hard links generates higher ex-ante expected individual payoffs (as well as higher welfare) than the pure cheap talk pairwise stable equilibrium. Clearly, the ex-ante expected individual payoffs (and the welfare) in a pairwise stable equilibrium weakly increase as C decreases: they are flat when $C > \Phi(1) - \Phi(2)$ (the pure cheap talk equilibrium) and strictly increase when $C \leq \Phi(1) - \Phi(2)$ (the hard-link equilibrium).

The sender and the receiver share the cost of a hard link. When the cost of a hard link is divided between the parties, introducing costly hard links might harm the ex-ante expected individual payoffs. In particular, if $\lambda \in (\frac{1}{3}, \frac{2}{3})$, the difference $\mathbf{E}u_i(g^h, C) - \mathbf{E}u_i(g^s)$ is strictly negative whenever $C \in \left(\frac{3(\Phi(1)-\Phi(2))}{2}, \frac{\Phi(1)-\Phi(2)}{\max\{\lambda, 1-\lambda\}}\right]$. In this case, the crowding-out effect dominates the information improvement effect, and the unique hard-link equilibrium generates strictly lower ex-ante expected individual payoffs compared to the pairwise stable soft-link equilibrium. Intuitively, compared to the previous case in which one

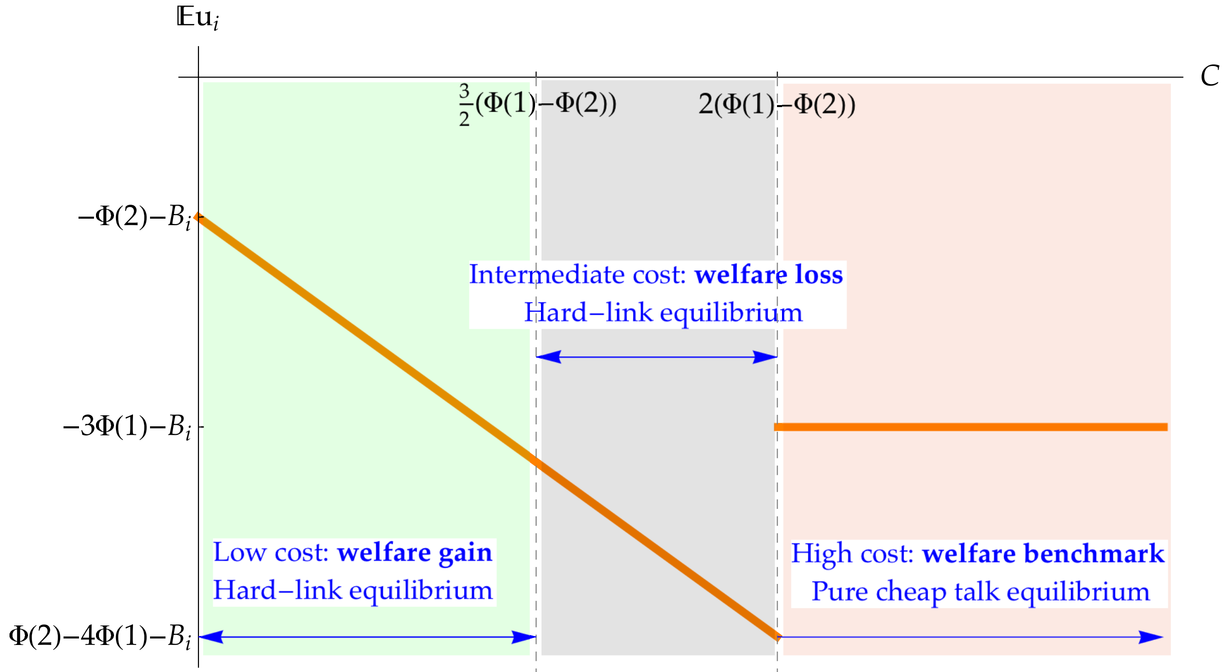

Figure 3: Ex-ante expected payoff of manager i, $\mathbf{E}u_i$, in a pairwise stable equilibrium when $\lambda = \frac{1}{2}$.

party internalizes the entire cost of a hard link, the parties now share the cost of a hard link while enjoying the same benefit. This leads to the formation of too many hard links relative to the cost C.[20] If, however, the cost is sufficiently low, $C \in \left(0, \frac{3(\Phi(1)-\Phi(2))}{2}\right]$, then the hard-link equilibrium generates greater individual ex-ante expected payoffs than any pairwise stable soft-link equilibrium. As a result, the ex-ante expected individual payoffs (and the welfare) are non-monotonic in the cost C: Figure 3 depicts $\mathbf{E}u_i$ in a pairwise stable equilibrium when the cost C is divided equally between the parties, $\lambda = \frac{1}{2}$.

5.3 One party bears the cost of a hard link

Consider the setting in which the cost of a hard link C accrues to only one party, either the one sending the message or receiving it. The following theorem states that allowing for an additional means of communication via hard links can only improve the total welfare in a pairwise stable equilibrium.

Theorem 2. *Take any cost $C_0 > \Phi(0)-\Phi(1)$ and consider some pure cheap talk pairwise stable equilibrium with the communication network $g(C_0)$ and the welfare $W(g(C_0))$. Then for any cost $C_1 \leq \Phi(0)-\Phi(1)$, there exists a pairwise stable equilibrium with the communication*

[20] Note that no hard links are present in a pairwise stable equilibrium when only one party bears the cost $C \geq \frac{3(\Phi(1)-\Phi(2))}{2}$.

network $g(C_1)$ such that the total welfare $W(g(C_1), C_1) \geq W(g(C_0))$.[21]

Proof. See Appendix B. □

To understand what drives the result, renumber the managers such that their in-degrees in $g(C_0)$ are increasing, $k_1 \leq k_2 \leq ... \leq k_n$, and consider a particular case of the cost $C_1 \in (\Phi(k+1) - \Phi(k+2), \Phi(k) - \Phi(k+1)]$ for some k, $k_1 \leq k < k_n$. Clearly, there exists manager j such that $k_j < k+1 \leq k_{j+1}$. This means that under the cost of C_1, there are incentives to form hard links towards the first j managers, whose in-degrees are less than $k+1$, until their in-degrees become equal to $k+1$. Thus, the set of new maximal in-degrees is:

$$k'_1 = ... = k'_j = k+1, k'_{j+1} = k_{j+1}, ..., k'_n = k_n.$$

Note that because of the negative externality effect, the new hard links towards managers $1, ..., j$ might crowd out some soft links. In the worst case, all soft links directed to managers $1, ..., j$ are substituted by the hard links. This corresponds to a maximal equilibrium with the communication network $g(C_1)$, in which all links directed towards managers $1, ..., j$ are hard, while all links directed to managers $j+1, ..., n$ are soft and the same as in $g(C_0)$. To compare the welfare in $g(C_0)$ and $g(C_1)$, consider the total cost of the newly introduced hard links and the additional ex-ante expected payoff arising from information improvement. The total cost of the hard links amounts to

$$j(k+1)C_1 \leq j(k+1)(\Phi(k) - \Phi(k+1)).$$

The gain in the welfare compared to the pure cheap talk case is

$$n \sum_{i=1}^{j} (\Phi(k_i) - \Phi(k+1)) \geq n \cdot j(\Phi(k) - \Phi(k+1)).$$

The lower bound for the welfare gain strictly exceeds the upper bound for the cost, because $n - 1 \geq k_n > k$. Intuitively, while $k+1$ hard links are used to increase the in-degree of manager $l \in \{1, ..., j\}$ by at least 1, all n managers enjoy the additional benefit arising from a greater accuracy of l's action. Hence, for this maximal equilibrium, $W(g(C_1), C_1) > W(g(C_0))$. Next, Lemma 3 insures that there exists a pairwise stable equilibrium that achieves the welfare of at least $W(g(C_1), C_1)$, which implies the result.

From this example, it becomes apparent that while the welfare in a pairwise stable equilibrium goes up, individual welfare might go down. In particular, assume that the

[21] Notational comment: the cost enters only the welfare $W(g(C_1), C_1)$ and is omitted from $W(g(C_0))$ to signify that, under the cost of C_0, a pairwise stable equilibrium has no hard links.

receiver bears the cost of a hard link and the maximal equilibrium considered above with the communication network $g(C_1)$ is actually pairwise stable.[22] Then it might be the case that manager j would prefer the pure cheap talk equilibrium with the communication network $g(C_0)$ to the pairwise stable equilibrium with $g(C_1)$ because the expected gain in his payoff is dominated by the high cost of maintaining $k+1$ links. This point is emphasized in the following remark.

Remark. The positive shareholder (total) welfare result of Theorem 2 does not extend to a per-manager basis. Specifically, a pairwise stable equilibrium with hard links might generate a lower welfare for some manager compared to the cheap talk case, if he ends up sustaining too many costly hard links relative to the individual gains from the information improvement.

5.4 Two parties bear the cost of a hard link

Assume that the cost of a hard link C is distributed between the sender and the receiver with the shares λ and $1-\lambda$, respectively. A previously considered example with 3 managers revealed that the welfare might go down when the cost C is decreased, so that hard links appear in a pairwise stable equilibrium. In this section, we follow Galeotti, Ghiglino and Squintani (2013) and Hagenbach and Koessler (2010) in considering natural cases of two communities and a diverse group of managers, and describe conditions under which the welfare decreases (or increases) when verifiable information transmission becomes feasible.

The welfare outcome depends on which effect—the positive information improvement or the negative crowding out—dominates. Clearly, if no crowding out of cheap communication occurs, then the welfare necessarily goes up, because the benefit of an extra hard link enters the payoffs of more than 2 managers, which strictly outweighs the total cost of the link, C. A more general version of this intuitive result, which is useful for further analysis, is presented in the following lemma:

Lemma 4. *Consider two cost levels, $C_0 \geq C_1$. For each C_i, fix some equilibrium and consider the corresponding communication network $g(C_i) = g^s(C_i) \cup g^h(C_i)$, where $g^s(C_i)$ is the set of soft links of $g(C_i)$ and $g^h(C_i)$ is the set of hard links of $g(C_i)$. If $g^s(C_0) \subseteq g^s(C_1)$ and $g^h(C_0) \subseteq g^h(C_1)$, then $W(g(C_1), C_1) \geq W(g(C_0), C_0)$, where $W(g(C_i), C_i)$ is the welfare corresponding to the equilibrium network $g(C_i)$ and the cost C_i. Moreover, the same welfare implications hold on a per-individual basis.*

[22] The examples of such pairwise stable equilibria are presented in the two communities setting (with $n_1 = 1$) of the next subsection.

Proof. See Appendix B. □

Excessive formation of hard links. The result that the welfare can decrease when verifiable information transmission becomes feasible hinges on the excessive formation of hard links (and hence, extensive crowding out of soft links) if both parties bear the cost. When $\lambda \in (0,1)$, the individuals fail to account for the total cost of the hard link, which results in stronger incentives to form hard links than in the case in which one party faces the full cost.[23] For instance, assume that the cost C is such that hard links don't appear when $\lambda \in \{0,1\}$, but necessarily emerge for some $\lambda \in (0,1)$. Then as the examples in this paper illustrate, having the cost of a hard link shared between the parties can be both beneficial and detrimental to the welfare compared to the case in which one party bears the full cost.

Two communities. The set of managers consists of two groups, N_1 and N_2, with sizes n_1 and n_2, respectively, where $1 \leq n_1 < n_2$ and the total number of managers is $n = n_1 + n_2$. Each member of group N_1 has a preference bias normalized to 0, while each member of N_2 has a bias b.

In any pairwise stable equilibrium, there is complete communication via cheap talk inside each group, because the incentive compatibility constraint of truth-telling is always satisfied for the managers who share the same preferences. In addition, all managers in the same group receive the same number of truthful messages, because exactly the same managers can report truthfully to them through links of the same type.

We introduce notation similar to Galeotti, Ghiglino and Squintani (2013): Denote by k_i the maximal in-degree of an arbitrary manager in group N_i. Further, $k_i = k_{ii} + k_{ij}$, where $k_{ii} = n_i - 1$ reflects the level of *intra-group communication*—the number of truthful messages that a manager from N_i receives from the members of the same group, and k_{ij} stands for the level of *cross-group communication*—the number of truthful messages that a manager from N_i receives from the members of the opposite community N_j.

Consider the pure cheap talk case that corresponds to a prohibitively high cost $C_0 > \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$. As mentioned above, in the communication network of any pairwise stable equilibrium the level of intra-group communication is $k_{ii} = n_i - 1$. Regarding cross-group communication, note that if members of a smaller group N_1 report truthfully to some members of N_2, then by the negative externality effect, the pairwise stability implies that members of a larger group N_2 report truthfully to some members of N_1. Thus, depending

[23]Indeed, the incentives to form hard links are determined by the maximum individual cost of a hard link $\max\{\lambda, 1-\lambda\}C$ that is lower than the total cost C.

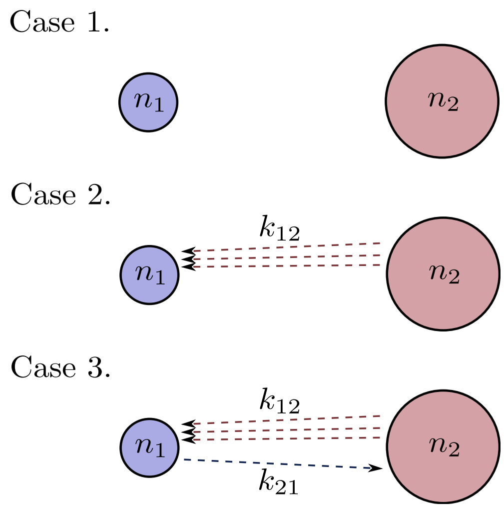

Figure 4: Communication networks of pairwise stable pure cheap talk equilibria.

on the parameters, cross-group communication can take one of the following three forms (see Figure 4):

1. No cross-group communication, i.e., $k_{21} = k_{12} = 0$.
2. Communication from group N_2 to group N_1, i.e., $k_{12} > 0$, $k_{21} = 0$.
3. Cross-group communication, i.e., $k_{12} > 0$, $k_{21} > 0$.

Assume now, that the cost is reduced to $C_1 > 0$, so that hard links become feasible. While complete soft-link intra-group communication is still a part of the truthful network of any pairwise stable equilibrium, cross-group communication can change due to the emergence of cross-group hard links. Because there are only two types of managers, pairwise stability requires that members of the same group receive identical numbers of truthful cheap talk messages and verifiable information messages across different equilibrium networks. This, in turn, means that the welfare is the same across all pairwise stable equilibria. Whether feasibility of hard links results in a welfare gain or a welfare loss depends on how the informational benefit from hard links compares with the loss from crowding out soft links.

In particular, it can be easily seen that introducing feasible hard links in case 1 necessarily improves the welfare because no soft links are crowded out in a pairwise stable equilibrium. As the proof of Theorem 3 demonstrates, the same positive welfare result holds in case 3, even though introducing hard links crowds out some soft cross-group communication. The reason is that the feasibility of hard links in case 3 increases the in-degrees of all managers, which outweighs the cost of hard links and the crowding out effect. The negative welfare result arises in case 2 for some parameters, the reason being that the feasibility of hard links increases the in-degrees of only group N_1 members, which is dominated by the negative crowding out effect. The following theorem describes the necessary and sufficient conditions for the welfare decrease.

Theorem 3. *Take any cost* $C_0 > \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$ *and consider some pure cheap talk pairwise stable equilibrium with the communication network* $g(C_0)$ *and the welfare* $W(g(C_0))$*. Introduce the possibility to form hard links with the cost* C_1 *and consider some pairwise stable equilibrium with the communication network* $g(C_1)$ *and the welfare* $W(g(C_1),C_1)$*. There exists a non-degenerate set* $(\underline{C}_1,\overline{C}_1)$ *of* C_1 *such that* $W(g(C_1),C_1) < W(g(C_0))$ *if and only if the preferential difference* b *satisfies*

$$b \in \left(\frac{\alpha+\beta+D}{2(\alpha+\beta+n_1+k+1)}, \frac{\alpha+\beta+D}{2(\alpha+\beta+n_1+k)}\right],$$

for some k*, where* $\max\{\lambda,1-\lambda\}n-1 < k \leq n_2-n_1-1$*. Otherwise, for all* C_1*:* $W(g(C_1),C_1) \geq W(g(C_0))$*.*

Proof. See Appendix B. □

To gain intuition for why introducing feasible hard links can lead to a welfare decrease in case 2, note that for the negative welfare result to occur, it must be that the two communities are sufficiently unbalanced in their size: $n_1 < \frac{n_2}{3}$. In addition, the in-degree of each N_1 member in a pure cheap talk equilibrium must be sufficiently high: $k_1 > \max\{\lambda,1-\lambda\}n-1 \geq \frac{n}{2}-1$. Assume that hard links become feasible with the cost C_1 being such that the in-degrees of managers in group N_1 increase by just 1, while no hard links from N_1 to N_2 appear. Since in the pure cheap talk equilibrium members of N_1 already receive relatively many signals, increasing their informativeness by 1 signal leads to a moderate additional individual benefit. Given that the size of community N_1 is relatively small, $n_1 < \frac{n}{4}$, this sums up to a moderate increase in the total welfare. At the same time, all cross-group soft links are crowded out and substituted by hard links, which amounts to a considerable cost, given that the level of cross-group communication was $k_{12} > \max\{\lambda,1-\lambda\}n-1 \geq \frac{n}{2}-1$. As a result, the net welfare effect is negative.

The structure of pairwise stable equilibria allows to make several observations regarding communication patterns. In particular, managers with similar preferences can easily communicate with each other via cheap talk, which leads to complete soft intra-group communication that is robust to introducing feasible hard links. On the contrary, soft-link cross-group communication is less intensive, with the information flow being greater towards the smaller group, and is vulnerable to the appearance of verifiable communication (soft cross-group communication can be easily crowded out with costly verifiable information transmission).[24] This allows one to expect the mode of communication between managers with different characteristics to be more substantial and proof-oriented than between similar managers.

Diverse group of managers. In this setting, managers $1,2,...,n$ have equidistant biases that satisfy $b_1 = 0$ and $b_{i+1} - b_i = b > 0$, $i = 1,...,n-1$. Assume that for some $C > 0$ the maximal in-degrees are $k_1,...,k_n$, and construct a pairwise stable equilibrium that generates the greatest welfare in a way described in Lemma 3. In the communication network of this equilibrium, manager i gets truthful messages through soft links from managers with sufficiently close preferences, truthful messages via hard links from more distinct managers, and no messages from those who are further away in their preferences. In case of prohibitively costly hard links, $\max\{\lambda, 1-\lambda\}C_0 > \Phi(0) - \Phi(1)$, truthful reporting to manager i boils down to the k_i closest managers revealing their information to i truthfully via soft links.

As was demonstrated in the example of three managers, for some parameters, introducing feasible hard links necessarily harms the welfare in the pairwise stable equilibrium. The following theorem states that this is no longer the case for bigger groups: if $n \geq 4$, then for any difference in preferences b and any cost $C_1 < \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$, there is a pairwise stable equilibrium that generates a greater total welfare than in the pure cheap talk case.

Theorem 4. *Let* $C_0 > \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$ *and consider some pure cheap talk pairwise stable equilibrium with the communication network* $g(C_0)$ *and the welfare* $W(g(C_0))$*. Introduce feasible hard links with the cost of* $C_1 \leq \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$ *and consider a pairwise stable equilibrium with the communication network* $g(C_1)$ *that generates the greatest welfare* $W(g(C_1),C_1)$*. There exists a non-degenerate set* $(\underline{C}_1,\overline{C}_1)$ *of* C_1 *such that* $W(g(C_1),C_1) < W(g(C_0))$ *if and only if* $n = 3$, $b \in \left(\frac{2+D}{10}, \frac{2+D}{8}\right]$ *and* $\lambda \in (\frac{1}{3}, \frac{2}{3})$*. Otherwise, for all* C_1*:* $W(g(C_1),C_1) \geq W(g(C_0))$*.*

Proof. See Appendix B. □

[24] These observations expand the new perspective to the study of homophily presented in Galeotti, Ghiglino and Squintani (2013).

6 Robustness and Extensions

In this section we extend our model in several other directions and analyze the robustness of our main findings. In Section 6.1 we demonstrate that the main result holds when the cost is endogenizied via negotiation about how to split the costs. In Section 6.2 we consider heterogenous costs. Finally, in Section 6.3 we show that our main conclusions can go beyond the additive signal structures.

6.1 Endogenous costs of hard links

One of the possible extensions of the model is to allow the parties to negotiate the way they split the cost of a hard link C each time they create a link (e.g., how to share the traveling cost). Surprisingly, such endogeneity of share λ, although implying pairwise efficiency, does not imply aggregate efficiency, and introducing hard links can still lead to lower shareholder (total) welfare.

To see this, consider managers i and j such that manager i is not credible in reporting to manager j (with the in-degree k_j) via cheap talk. The aggregate benefit of managers i and j from creating a hard link ij is $2(\Phi(k_j)-\Phi(k_j+1))$, while the total cost of a hard link is C. Thus, as long as the benefit exceeds the cost, $2(\Phi(k_j)-\Phi(k_j+1)) > C$, the managers can split the cost of a hard link so that they both strictly benefit from its creation. In particular, the managers can always share C equally between each other and enjoy the additional benefit of $\Phi(k_j)-\Phi(k_j+1)-C/2$ each.

In general, the managers can find a way to profitably split the cost C and introduce the hard link ij if and only if the hard link ij is desired by the managers when the cost C is split equally between the parties, $\lambda = 1/2$. This observation implies that communication patterns (and hence, the welfare) of the pairwise stable equilibria are the same across the two settings: (i) the share λ is endogenous, and (ii) $\lambda = 1/2$. Because in the latter case of $\lambda = 1/2$ introducing hard links can deteriorate the welfare, it can do so in the case of endogenous λ as well.

6.2 Heterogeneous costs of hard links

While in some situations a natural assumption about the hard links is that the cost value is similar across managers, it might be not quite adequate in others. Thus, a straightforward extension of the model would be to allow the cost of a hard link to depend on the managers' identities. As one example of cost heterogeneity, consider managers of an international corporation who might be located in different cities or countries, and who need to communicate with each other before making individual decisions for their respective divisions. A hard link might correspond to a personal meeting, while a soft link corresponds to a phone call or an email. In this case, it is relatively easy for two managers from the same location to meet, while a personal meeting of two managers from different countries entails additional time and spending.

Another example where the cost heterogeneity naturally arises, concerns existence of social ties between decision makers. Some managers might be friends, which means that it is easier for them to transmit information truthfully, because of the high psychological cost of lying to a friend. Other managers might have no such social relationships, and the absence of psychological obligation of truth-telling requires them to spend more time collecting supporting materials, arguing, and proving their private information.

In this section, we highlight some insights stemming from the cost heterogeneity assumption in an extreme setting, where, for some pairs of managers, it is easy to communicate in verifiable way, while others face a considerable cost. More formally, the cost depends on the pair of managers, i and j, and is the same independent of the link direction: the cost of a hard link ij is equal to the cost of a hard link ji, $C_{ij} = C_{ji} = C$. The cost can be either prohibitively high, $C_0 > \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$, or feasibly low, $C_1 \leq \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$. For simplicity, we maintain the assumption that the cost structure that is specified by the share $\lambda \in [0,1]$ is the same across all pairs of managers. In some sense, the setting with heterogeneous costs is in between the two cases of homogeneous cost values C_0 and C_1. In particular, switching from the case of homogeneous cost C_0 to heterogeneous costs can be viewed as lowering the cost to the level of C_1 for *some* pairs; while switching to homogeneous cost C_1 corresponds to lowering the cost values to C_1 for *all* pairs.

Consider a particular case where C_1 is sufficiently small: $C_1 \leq \frac{\Phi(n-2)-\Phi(n-1)}{\max\{\lambda,1-\lambda\}}$. Clearly, in any equilibrium, managers in pairs with the cost C_1 must communicate truthfully with each other. This *localization* of communication might change the truthful communication network compared to the pure cheap talk case, while not necessarily leading to an information improvement. Indeed, consider the previously studied example of three managers, where the prior distribution of θ is uniform $[0,1]$ and the preference biases are $b_1 = 0$, $b_2 = b$, $b_3 = 2b$, such that $\frac{2+D}{10} < b \leq \frac{2+D}{8}$. Consider the case of prohibitively costly hard links—homogeneous cost C_0. A considerable difference in preferences prevents managers 1 and 3 from truthful communication with each other via cheap talk. The two pairwise stable pure cheap talk equilibria have the in-degrees $k_1 = k_2 = k_3 = 1$ and the communication networks described in Figure 1.

Assume now that the cost of a hard link for managers 1 and 3 is decreased to the level of C_1, implying the following heterogenous costs setting: $C_{12} = C_{23} = C_0$, $C_{13} = C_1$.

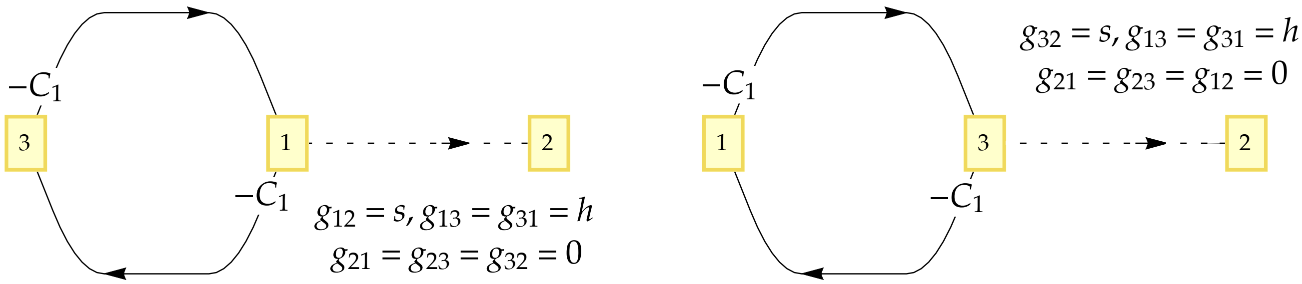

Figure 5: Communication networks (i) (left graph) and (ii) (right graph) of pairwise stable equilibria: heterogenous costs.

In this case, managers 1 and 3 must communicate with each other via hard links, thus, the only two pairwise stable equilibria have the following communication networks (see Figure 5):

(i) $g_{12} = s$, $g_{13} = g_{31} = h$, $g_{21} = g_{23} = g_{32} = 0$.

(ii) $g_{32} = s$, $g_{13} = g_{31} = h$, $g_{21} = g_{23} = g_{12} = 0$.

Note that the in-degrees of the managers are the same as in the pure cheap talk case, $k'_i = 1$, $i = 1, 2, 3$, while some links are hard.

The fact that introducing heterogeneous costs creates localization of communication that might fail to generate an informational gain implies that the welfare can decrease even when only one party bears the cost of a hard link. This contrasts with the case of homogeneous cost, in which introducing feasible hard links necessarily leads to a welfare improvement when only one party faces the cost. Thus, heterogeneous costs can result in a welfare lower than in both cases of homogeneous cost, C_0 and C_1.

6.3 Non-additive signal structure

Our modeling assumption about the additive (aggregate) nature of the state of the world (see (1)) is natural in environments where manager decisions depend on summary characteristics. For example, the total expected payoff of a project is comprised from realizations of certain project components s_d; the total financial or time budget depends on individual budgets s_d of various divisions.

To establish the robustness of the results, we show that the exact additive form is *not* necessary for the findings in this paper. In Appendix C we show that all the results hold for a natural non-additive signal structure as well. This makes the message of the paper more general and applicable to greater number of real-life situations.

7 Policy implications and Empirical predictions

In this section we discuss the results and empirical predictions, and study how to implement optimal corporate policies that not only improve the quality of discussion, but also benefit shareholders in corporations.

7.1 Prediction 1:

Hard links are formed towards managers who receive relatively small number of truthful signals via soft links. The network implications of our results predict that when and where managers (in equilibrium) are likely to use verifiable communications (hard links) to communicate their private information. As shown in Section 3.2, while the truthful revelation of a signal is always beneficial to both parties from the ex-ante perspective, there is a credibility issue in cheap talk communication at the interim stage. In particular, there exists a negative externality effect, i.e., the willingness of manager i to report truthfully to manager j decreases with the preference divergence and with increasing the number of truthful messages reported to manager j—the in-degree of manager j, k_j. In the case of verifiable information transmission, the incentive to form a hard link ij does *not* depend on the preference divergence; rather, it depends only on the in-degree of manager j and is strictly decreasing in k_j. The straightforward equilibrium implication for the network structure is that hard links must be directed towards managers who receive relatively small numbers of truthful messages via soft links.

To test this prediction there are several potential proxies an empiricist may use for preference divergence among managers (e.g., division size or age, ownership, geography, affiliation, social tie, etc.). Prediction 1 states that managers with similar preferences (e.g., homogenous and strong social tie between all of them) can easily communicate with each other via costless cheap talk, which leads to complete communication network that is robust to introducing feasible hard links. In contrast, however, when there is drastic divergence between managers' preferences (e.g., due to heterogenous properties of divisions, geography, or polarized nature of social tie between managers), a manager whose bias is far from other managers most likely use costly verifiable communication choices to communicate with others.

7.2 Prediction 2:

All else equal, introducing verifiable communication choices improves the quality of discussion between the managers (Positive information improvement effect). As shown in Section 4, our first—and intuitive—policy result is that introducing reliable communication choices improves the quality of discussion between the managers. This policy follows from a positive informational effect arising from introducing a feasible verifiable communication channel, which is independent of the cost structure (see Theorem 1). That is, if a pure cheap talk equilibrium fails to exist when hard links become feasible, then there exists an equilibrium with hard links, in which every manager accumulates a weakly greater number of signals.[25]

To test this prediction one needs to make reliable communications choices feasible, say by sufficiently reducing its cost.[26] A potential proxy that an empiricist may use to measure verifiable communication costs are managers' costs of communicating outside meetings (e.g. Hart (2003)). These costs are likely to be lower if managers (or divisions) are geographically closer to each other or if their social and professional ties are stronger (Stevenson and Radin (2009)). Moreover, busy managers—for example, those who hold several managerships—are less likely to find time to communicate via costly and time consuming reliable communications. Prediction 2 is thus consistent with the empirical work in Fich and Shivdasani (2006) which shows that managers' communications are less informative when the cost of verifiable communication is high, e.g., managers are busy and hold several managerships.

7.3 Prediction 3:

Soft links will be replaced by costly hard links (Negative crowding out effect). Our theory predicts that the above policy, however, may not be beneficial to shareholders.

As shown in Section 5 the appearance of hard links in the pure cheap talk setting has two effects on the expected welfare: a positive effect stems from the information improvement and a negative effect arises from crowding out soft communication with costly verifiable communication. The crowding out effect—that is at the heart of the paper—arises because newly formed hard links increase managers' in-degrees, which by the negative externality effect, destroys the credibility of communication through some soft links. As a result, for the managers to transfer their signals truthfully, those soft links should be replaced by costly hard links. The policy implications of this result is important. This result identifies that corporate policies that promote verifiable communication choices between managers, paradoxically, may *not* be optimal for shareholders. Interestingly,

[25]In other words, for any pure cheap talk equilibrium there is an equilibrium with feasible hard links, in which every manager accumulates weakly greater amount of information.

[26]Such policies include: encouraging personal discussions by appropriate financing or making traveling a pleasant experience when divisions are far from each other.

this crucially depends on the cost structure, summarized as follows.

7.4 Prediction 4:

All else equal, welfare (shareholder value) increases when only one party bears the cost of a hard link. As shown in Section 5.3, the positive informational effect always dominates the negative crowding out effect when only one party bears the cost of a hard link. In other words, when only one party (between any two communicating managers) bears the cost of a hard link, then verifiable communication not only improves the quality of discussion but also increases the total welfare as well as welfare in the corporation. Our theory therefore predicts that communication policies promoting any of the following two simple schemes via verifiable channels are optimal cost structures:

(i) **Encoding policy:** Processing cost only at the *encoding* stage. Here, the sender pays the cost that naturally arises when it takes considerable effort to provide supporting data and to develop argumentation, while it is very easy for the receiver to uncover the underlying signal after being presented with the collected materials.

(ii) **Decoding policy:** Processing cost only at the *decoding* stage. Here, the receiver pays the cost that naturally arises when it takes considerable effort to process data and to verify the information content of the received data, while it is very easy for the sender to send its signal knowing that the receiver is equipped with enough tools to verify the signal.

7.5 Prediction 5:

All else equal, welfare (shareholder value) can decrease when the cost of a hard link is *shared*. As shown in Section 5, our theory predicts that the above positive welfare result no longer holds when cost of a hard link is *shared* between the managers. This result suggests that corporate policies that promote verifiable communications in which both parties (involved in a communication) *share* the cost of its verification can reduce the total welfare and thus be detrimental to shareholders.[27] Instead, it is socially beneficial to replace those policies with policies that *only one party* pays the cost of verification (e.g., the encoding and decoding polices introduced above).

[27] Examples of such sharing cost policies are: Costly long meetings between managers; Sharing data processing costs both for sender and receiver.

7.6 Prediction 6:

Endogenous Negotiation and Heterogenous Costs. Our results are robust to endogenous negotiation polices. In one extension, we allow the parties to negotiate how to split the cost of a hard link between them. As shown in Section 6.1 such endogeneity of the cost shares does not necessarily lead to aggregate efficiency; introducing hard links can still decrease the total welfare and thus be detrimental to welfare. Therefore, policies that endogenize the cost sharing of verifiable communications between managers (e.g., via negotiation) may *not* be optimal for shareholders.

In another extension we allow the costs of hard links to differ (for example due to social tie) across the pairs of managers. As shown in Section 6.2, when the cost difference is substantial, the availability of hard links is likely to result in the localization of communication with respect to the low cost of hard links, with the information accumulated by every manager being the same. As a result, expected (total welfare) welfare can decrease *even when only one party bears the cost*. The policy implications of this result is important. *In fact, our theory predicts that to maximize welfare by introducing verifiable communications, not only one party should pay the cost of communications, but also cost should be* homogeneous *among the parties. As otherwise, with heterogenous costs (and substantial differences) welfare may decrease even when only one party bears the cost.* This policy provides a rationale for certain rules and principles in corporations aimed on equalizing the costs of verifiable communications (hard links) across different pairs of managers.[28]

Heterogenous costs can also be due to social ties between managers (Stevenson and Radin (2009)). Regarding homogenizing this type of heterogenous costs, the corresponding corporate policy that an empiricist can consider may be oriented at increasing conformity of social ties between managers. For example, on the one extreme, the corporate culture might imply regular corporate parties or retreats that help building and improving friendly relationships (Adams, Hermalin and Weisbach (2010)). In terms of the model, this means moving towards the homogenous cost of low value. On the other extreme is a weak type of the corporate culture, where informal meetings are not encouraged, which, in turn, precludes developing of social ties between managers and leads to the setting with the homogeneous cost of high value.

[28]Thus, a corporate policy might be not only to cover, for example, the nominal traveling costs, but also to make traveling process more pleasant by means of purchasing business class tickets, reserving accommodations in five-star hotels and providing nice travel compensation. The goal of this policy is to *equalize* personal costs of private meetings across locations, and hence, mitigate localization of communication.

8 Conclusion

We develop a model of corporate governance in which every manager needs to take an action that matches his preferences given the state of the world and that action affects the payoffs of all other managers in multi-division corporations. Before deciding upon the action, managers can choose how to reveal their private information about the state. The major twist in our model, and a point of departure from existing literature, is that there are *two* ways in which the information can be communicated: either through a costly *verifiable information* (hard) channel or through a low-cost *cheap talk* (soft) channel.

We identify that the appearance of hard links in the pure cheap talk setting has two *opposing* effects on welfare: (i) a positive effect stems from the information improvement and (ii) a negative effect arises from crowding out soft communication with costly verifiable communication. As a result, we show that lowering the cost of hard links, paradoxically, can *hurt* shareholders. Surprisingly, this result crucially depends on the cost structure. If only one party bears the cost of a hard link, then the positive (*informational*) effect always dominates the negative (*crowding out*) effect, and thus introducing hard links increases the total expected welfare. In contrast, if the cost of a hard link is *shared* by both parties, then allowing for verifiable communication can *decrease* the total welfare.

We also derive several testable welfare implications about introducing verifiable communications in corporate governance, and demonstrate the robustness of our findings in face of heterogenous costs as well as the case where cost is endogenizied via negotiation about how to split the costs, and extend the analysis to the case where signal structure is non-additive.

Finally, we highlight corporate policies that not only improve quality of discussion between the managers, but also are beneficial to welfare (shareholder value). We discuss several environments in which these implications can be tested/implemented.

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A Appendix: Pairwise stable equilibria

An equilibrium $\{g,(\mu^g,y^g)\}$ is *pairwise stable*, if no pair of managers can change the communication pattern between them to improve their ex-ante expected utilities, while satisfying the interim incentive compatibility constraints of truth-telling, holding other strategies fixed. To better understand how the communication pattern can be improved, consider three possible cases of information transmission from manager i to some manager j with in-degree k_j in g:

1. Manager i reports truthfully to manager j through a soft link: $g_{ij} = s$. There is no way to improve communication: making communication uninformative will result in the ex-ante expected loss of $\Phi(k_j - 1) - \Phi(k_j)$, while switching to verifiable communication will involve a cost.

2. Manager i reports truthfully to manager j through a hard link: $g_{ij} = h$. Because the hard link ij is a part of the equilibrium, it is preferred to no link by both managers, and deleting this link will not improve the ex-ante expected payoffs. It may, however, be possible to reach an improvement by substituting the costly hard link ij with a soft link ij and inducing truthful communication through it. This option can be realized if manager i can credibly communicate given that j believes i's message. Otherwise, it is not possible to change the communication pattern in a profitable direction.

3. Manager i does not report informatively to manager j, $g_{ij} = 0$. Here it may be possible to improve by creating a soft link ij with truthful communication, if it is interim incentive compatible. If not, then the second-best option is creating a hard link ij. If the latter option is undesired by at least one manager, then there exists no possibility to improve.

Given these alternatives, pairwise stability can be formally defined as follows:

Definition. An equilibrium $\{g,(\mu^g,y^g)\}$ is *pairwise stable* if

(i) For any $i,j \in \{1,..,n\}$, $g_{ij} = 0$ only if, holding other strategies fixed, i cannot credibly report to j via a soft link, assuming that j believes i's message, and communication via a hard link is not desired by at least one party.

(ii) For any $i,j \in \{1,..,n\}$, $g_{ij} = h$ only if, holding other strategies fixed, i cannot credibly report to j via a soft link, assuming that j believes i's message.

Symmetry. An immediate property of a pairwise stable equilibrium is that any two managers i and j with the same preference biases, $b_i = b_j$, must be treated in symmetric way, i.e., they communicate truthfully with each other via cheap talk and receive the same number of truthful messages from other managers. Moreover, if some other manager l truthfully reports to both, i and j, then it must be the case that l uses the same type of information transmission channel.

Maximality and pairwise stability. There might be multiple maximal equilibria generating the same vector of maximal in-degrees, among which some maximal equilibria might not be pairwise stable.

Example. Here we present an example of a maximal equilibrium which is not pairwise stable. Let the prior distribution of θ be uniform on $[0,1]$. Consider 3 managers with the preference biases $b_1 = b_2 = 0$, $b_3 \in (\frac{2+D}{10}, \frac{2+D}{8}]$. Assume that hard links are prohibitively costly. Then there are several maximal pure cheap talk equilibria that generate the in-degrees $k_1 = k_2 = k_3 = 1$. The examples of such communication networks are: (1) $g_{12} = g_{21} = g_{13} = s$, $g_{23} = g_{31} = g_{32} = 0$, and (2) $g_{12} = g_{23} = g_{31} = s$, $g_{13} = g_{32} = g_{21} = 0$. It is easy to see that the first communication network corresponds to a pairwise stable equilibrium. In contrast, the second communication network can not correspond to a pairwise stable equilibrium, because managers 1 and 2, who agree in their preferences, would deviate and induce truthful communication through the soft link 21.

Verifiable information and maximal in-degrees. The informational result, coupled with the fact that the incentive to form a hard link decreases with the manager's in-degree, ensure that allowing for verifiable information transmission results in weakly greater, and more evenly distributed, maximal in-degrees.

Strong stability. As with pairwise stability, one can adapt from the networks literature the notion of *strong stability* to this framework: coordinated change of communication pattern in a group of managers cannot strictly improve the welfare of some members, while weakly improving the welfare of others. Then a pairwise stable equilibrium generates the greatest welfare if and only if it is strongly stable.[29]

[29] This statement is non-trivial, because there exist pairwise stable equilibria that do not generate the greatest welfare. For example, consider 3 managers with the preference biases $b_1 = 0$, $b_2 = b$, $b_3 = 2b$, where $b \in (\frac{2+D}{10}, \frac{2+D}{8}]$, and assume that θ is uniform on $[0,1]$ (the same setup as in Subsection 5.2). If the cost of a hard link C satisfies $\max\{\lambda, 1-\lambda\}C \in (\Phi(1)-\Phi(2), \Phi(0)-\Phi(1)]$, then the greatest welfare is generated, for example, in a pairwise stable equilibrium with the following communication network: $g_{21} = g_{23} = g_{12} = s$, $g_{13} = g_{31} = g_{32} = 0$. However, there exists a pairwise stable equilibrium with the communication network $g_{21} = g_{12} = s$, $g_{13} = h$, $g_{23} = g_{31} = g_{32} = 0$ that achieves a strictly lower welfare.

Proof. The "only if" part of the statement is straightforward. Concerning the "if" part, note that if a strongly stable equilibrium—which is necessarily maximal—didn't generate the greatest welfare, then there must be a hard link that can be severed and a soft link (with the truthful reporting through it) that can be introduced. This contradicts the property of strong stability. □

B Appendix: Proofs

Proof of Lemma 1

Fix a truthful network g and consider manager j with the in-degree $k_j < n-1$. manager j learns information $s_R \in \{0,1\}^{k_j+1}$ (k_j signals coming from other managers excluding manager i, 1 signal j receives himself) and optimally chooses action $y_j(s_R) = b_j + \sum_{s_d \in s_R} s_d + \mathbf{E}(\sum_{s_d \in (s_i, s_{-R})} s_d | s_R)$. An ex-ante expected input from manager j into i's utility is then

$$
\begin{aligned}
& -\sum_{s_R \in \{0,1\}^{k_j+1}} \sum_{(s_i,s_{-R}) \in \{0,1\}^{D-k_j-1}} (y_j(s_R) - S - b_i)^2 P(s_R, s_i, s_{-R}) \\
= & -\sum_{s_R} \sum_{(s_i,s_{-R})} \left[b_j - b_i + \mathbf{E}\left(\sum_{s_d \in (s_i,s_{-R})} s_d | s_R \right) - \sum_{s_d \in (s_i,s_{-R})} s_d \right]^2 P(s_R, s_i, s_{-R}) \\
= & -(b_j - b_i)^2 - 2(b_j - b_i) \sum_{s_R} \sum_{(s_i,s_{-R})} \left[\mathbf{E}\left(\sum_{s_d \in (s_i,s_{-R})} s_d | s_R \right) - \sum_{s_d \in (s_i,s_{-R})} s_d \right] P(s_R, s_i, s_{-R}) \\
& -\sum_{s_R} \sum_{(s_i,s_{-R})} \left[\mathbf{E}\left(\sum_{s_d \in (s_i,s_{-R})} s_d | s_R \right) - \sum_{s_d \in (s_i,s_{-R})} s_d \right]^2 P(s_R, s_i, s_{-R}).
\end{aligned}
$$

The second term in this sum is zero, because:

$$
\begin{aligned}
& \sum_{s_R} \sum_{(s_i,s_{-R})} \left[\mathbf{E}\left(\sum_{s_d \in s_R} s_d | s_R \right) - \sum_{s_d \in (s_i,s_{-R})} s_d \right] P(s_R, s_i, s_{-R}) \\
= & \sum_{s_R} \sum_{(s_i,s_{-R})} \left[\mathbf{E}\left(\sum_{s_d \in (s_i,s_{-R})} s_d | s_R \right) - \sum_{s_d \in (s_i,s_{-R})} s_d \right] P(s_i, s_{-R} | s_R) P(s_R) \\
= & \sum_{s_R} \left[\mathbf{E}\left(\sum_{s_d \in (s_i,s_{-R})} s_d | s_R \right) - \mathbf{E}\left(\sum_{s_d \in (s_i,s_{-R})} s_d | s_R \right) \right] P(s_R) \\
= & \; 0.
\end{aligned}
$$

The third term is an expected residual variance $\Phi(k_j) = \mathbf{E}\Big[\mathbf{Var}\Big(\sum_{s_d \in (s_i, s_{-R})} s_d | s_R\Big)\Big]$ and can be rewritten as

$$\begin{aligned}
& \sum_{s_R}\left[\mathbf{E}\left(\sum_{s_d \in (s_i, s_{-R})} s_d | s_R\right)\right]^2 P(s_R) + \sum_{s_R}\sum_{(s_i, s_{-R})}\left[\sum_{s_d \in (s_i, s_{-R})} s_d\right]^2 P(s_R, s_i, s_{-R}) \\
& -2\sum_{s_R}\mathbf{E}\left(\sum_{s_d \in s_R} s_d | s_R\right)\sum_{(s_i, s_{-R})}\left(\sum_{s_d \in (s_i, s_{-R})} s_d\right) P(s_i, s_{-R}|s_R) P(s_R) \\
= & \underbrace{-\sum_{s_R}\left[\mathbf{E}\left(\sum_{s_d \in (s_i, s_{-R})} s_d | s_R\right)\right]^2 P(s_R)}_{A} + \underbrace{\sum_{(s_i, s_{-R})}\left[\sum_{s_d \in (s_i, s_{-R})} s_d\right]^2 P(s_i, s_{-R})}_{B}.
\end{aligned}$$

Calculate the first term A, denoting l to be the number of 1s in s_R:

$$\begin{aligned}
A &= -\sum_{s_R}\Big[(D - k_j - 1)\mathbf{E}(\theta | s_R)\Big]^2 P(s_R) = -\frac{(D - k_j - 1)^2}{(\alpha + \beta + k_j + 1)^2}\sum_{s_R}(\alpha + l)^2 P(s_R) \\
&= -\frac{(D - k_j - 1)^2}{(\alpha + \beta + k_j + 1)^2}\left[\alpha^2 + 2\alpha\sum_{s_R} l P(s_R) + \sum_{s_R} l^2 P(s_R)\right].
\end{aligned}$$

In this expression

$$\sum_{s_R} l P(s_R) = (k_j + 1)\mathbf{E}(s_1) = \frac{(k_j + 1)\alpha}{\alpha + \beta},$$

$$\sum_{s_R} l^2 P(s_R) = \int_0^1\left(\sum_{s_R} l^2 P(s_R | \theta)\right) f(\theta) d\theta.$$

Since l is equal to the sum of signals in s_R, signals s_j are identically distributed and independent conditionally on θ, the term inside the integral can be rewritten as

$$\begin{aligned}
\sum_{s_R} l^2 P(s_R | \theta) &= \mathbf{E}(l^2 | \theta) = \mathbf{Var}(l | \theta) + (\mathbf{E}(l | \theta))^2 \\
&= (k_j + 1)\theta(1 - \theta) + (k_j + 1)^2\theta^2.
\end{aligned}$$

Taking the integral,

$$\begin{aligned}
\sum_{s_R} l^2 P(s_R) &= \frac{k_j + 1}{B(\alpha, \beta)} B(\alpha + 1, \beta + 1) + \frac{(k_j + 1)^2}{B(\alpha, \beta)} B(\alpha + 2, \beta) \\
&= \frac{k_j + 1}{\alpha + \beta(\alpha + \beta + 1)}[\alpha\beta + (k_j + 1)\alpha(\alpha + 1)].
\end{aligned}$$

Substituting these to A yields

$$\begin{aligned} A &= -\frac{(D-k_j-1)^2}{(\alpha+\beta+k_j+1)^2}\left[\alpha^2+2\alpha\frac{(k_j+1)\alpha}{\alpha+\beta}+\frac{(k_j+1)(\alpha\beta+(k_j+1)\alpha(\alpha+1))}{\alpha+\beta(\alpha+\beta+1)}\right] \\ &= -\frac{(D-k_j-1)^2\alpha}{(\alpha+\beta+k_j+1)\alpha+\beta(\alpha+\beta+1)}\left[\alpha(\alpha+\beta+1)+(k_j+1)(\alpha+1)\right]. \end{aligned}$$

Now consider the second term B, assuming that the number of 1s in (s_i, s_{-R}) is $\tilde{l}$:

$$B = \sum_{(s_i,s_{-R})} \tilde{l}^2 P(s_i, s_{-R}) = \frac{(D-k_j-1)}{\alpha+\beta(\alpha+\beta+1)}[\alpha\beta+(D-k_j-1)\alpha(\alpha+1)].$$

Then after some algebraic transformations, $\Phi(k_j) = A + B$ becomes:

$$\Phi(k_j) = \frac{\alpha\beta(\alpha+\beta+D)(D-k_j-1)}{(\alpha+\beta)(\alpha+\beta+1)(\alpha+\beta+k_j+1)}.$$

Finally, since the ex-ante expected input from manager j into i's payoff is $-\Phi(k_j) - (b_j - b_i)^2$, then the benefit from improving j's information by one additional signal is $\Phi(k_j) - \Phi(k_j + 1)$. Because $\Phi(k_j)$ is a positive, decreasing and convex function of k_j, the benefit exceeds 0 and decreases with k_j. **QED.**

Proof of Proposition 1

The proof follows from the arguments in the main text. However, for completeness we formally show it in the following.

The binary nature of the signal ensures that $\mathbf{E}(s|s_R) = \mathbf{E}(\theta|s_R)$. Assume that l signals in s_R are 1. A straightforward algebra implies that manager i's posterior is Beta distribution with parameters $(\alpha + l, \beta + k - l)$:

$$f(\theta|l,k) = \frac{1}{B(\alpha+l,\beta+k-l)}\theta^{\alpha+l-1}(1-\theta)^{\beta+k-l-1}.$$

The expected value of θ then is $\mathbf{E}_i(\theta|l,k) = \frac{\alpha+l}{\alpha+\beta+k}$ and the optimal action becomes

$$y_i^g(s_R) = b_i + \sum_{s_d \in s_R} s_d + (D-k)\frac{\alpha+l}{\alpha+\beta+k}. \quad \text{(B.1)}$$

The necessary conditions for $g_{ij} = h$ and $g_{ij} = 0$ follow directly from the incentive condition to form/maintain a hard link (5). To derive the necessary condition for $g_{ij} = s$—the incentive compatibility constraint of truthful reporting through a soft link ij—consider manager j and let s_R be the set of k_j signals that manager j gets to know apart from manager i. Specifically, $k_j - 1$ signals from his other communication neighbors $N_j^{-1}(g)/\{i\}$ and his own private signal s_j. Assume that manager j believes that i reports truthfully. Then if i reports truthfully, j optimally chooses $y_j(s_R, s_i)$; if manager i misreports and sends $m_{ij} = 1 - s_i$, j picks the action $y_j(s_R, 1 - s_i)$. manager i reports truthfully his signal if and only if it generates a greater interim expected payoff to i

compared to misreporting:

$$\sum_{s_R, s_{-R} \in \{0,1\}^{D-1}} -\left[(y_j(s_R, s_i) - S - b_i)^2 - (y_j(s_R, 1-s_i) - S - b_i)^2\right] P(s_R, s_{-R}|s_i) \geq 0.$$

This condition can be rewritten as

$$-\sum_{s_R, s_{-R}} \left[(y_j(s_R, s_i) - y_j(s_R, 1-s_i))(y_j(s_R, s_i) + y_j(s_R, 1-s_i) - 2S - 2b_i)\right] P(s_R, s_{-R}|s_i) \geq 0.$$

Assume that there are l signals 1 in s_R and recall that the actions $y_j(s_R, s_i)$ and $y_j(s_R, 1-s_i)$ are given by (B.1), then the condition for truth-telling becomes

$$-\sum_{s_R, s_{-R}} P(s_R, s_{-R}|s_i) \left[2s_i - 1 + (D - k_j - 1)\left(\frac{\alpha + l + s_i}{\alpha + \beta + k_j + 1} - \frac{\alpha + l + 1 - s_i}{\alpha + \beta + k_j + 1}\right)\right] \times$$

$$\times \left[2b_j - 2b_i + 1 - 2s_i + (D - k_j - 1)\left(\frac{\alpha + l + s_i}{\alpha + \beta + k_j + 1} + \frac{\alpha + l + 1 - s_i}{\alpha + \beta + k_j + 1}\right) - 2\sum_{s_d \in s_{-R}} s_d\right] \geq 0.$$

Using $P(s_R, s_{-R}|s_i) = P(s_{-R}|s_i, s_R)P(s_R|s_i)$, this can be simplified to

$$-(2s_i - 1)\frac{\alpha + \beta + D}{\alpha + \beta + k_j + 1} \times$$

$$\times \sum_{s_R}\left[2(b_j - b_i) + 1 - 2s_i + (D - k_j - 1)\frac{2\alpha + 2l + 1}{\alpha + \beta + k_j + 1} - 2A(s_i, s_R)\right] P(s_R|s_i) \geq 0,$$

where

$$\begin{aligned} A(s_i, s_R) &= \sum_{s_{-R}}\left(\sum_{s_d \in s_{-R}} s_d\right) P(s_{-R}|s_i, s_R) = \mathbf{E}\left(\sum_{s_d \in s_{-R}} s_d | s_i, s_R\right) \\ &= (D - k_j - 1)\mathbf{E}\left(\theta | s_i, s_R\right) = (D - k_j - 1)\frac{\alpha + l + s_i}{\alpha + \beta + k_j + 1}. \end{aligned}$$

After accounting for that and canceling the positive term $\frac{\alpha+\beta+D}{\alpha+\beta+k_j+1}$, the truth-telling condition becomes:

$$-(2s_i - 1)\sum_{s_R}\left[2(b_j - b_i) + 1 - 2s_i + (D - k_j - 1)\frac{1 - 2s_i}{\alpha + \beta + k_j + 1}\right] P(s_R|s_i) \geq 0,$$

$$-(2s_i - 1)\left[2(b_j - b_i) + (1 - 2s_i)\frac{\alpha + \beta + D}{\alpha + \beta + k_j + 1}\right] \geq 0.$$

If $s_i = 1$, the truth-telling condition becomes

$$b_j - b_i \leq \frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)}.$$

If $s_i = 0$, the truth-telling condition becomes

$$b_j - b_i \geq -\frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)}.$$

As a result,

$$|b_j - b_i| \leq \frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)},$$

which completes the proof of Proposition 1. **QED.**

Proof of Theorem 1

Consider a pure soft-link equilibrium $\left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\}$. Renumber the managers such that their in-degrees in the communication network $g(C_0)$ are increasing in their respective number: $k_1 \leq k_2 \leq ... \leq k_n$.

If $k_1 = n-1$, then the communication network $g(C_0)$ is complete. When the cost drops to the level of C_1, there exists an equilibrium

$$\left\{g(C_1), (\mu^{g(C_1)}, y^{g(C_1)})\right\} = \left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\}$$

that generates the same complete truthful network. Indeed, given a complete soft-link network $g(C_1)$, the strategy profile $(\mu^{g(C_0)}, y^{g(C_0)})$ forms a PBE. Moreover, no two managers would like to substitute a soft link with a costly hard link, because that will result in ex-ante expected utility decrease. Consequently, in this case $k_1' = k_1, ..., k_n' = k_n$.

Assume now that $k_1 < n-1$, i.e., the equilibrium network $g(C_0)$ is not complete and there are managers who don't have all the signals reported to them. Consider three cases:

Case 1. Sufficiently large cost: $\max\{\lambda, 1-\lambda\}C_1 > (\Phi(k_1) - \Phi(k_1+1))$. In this case, there exists an equilibrium with the same communication network $g(C_0)$. Indeed,

$$\left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\}$$

remains an equilibrium when the cost is C_1, because the pair $(\mu^{g(C_0)}, y^{g(C_0)})$ forms a PBE given $g(C_0)$ and no two managers would want to add a hard link or substitute a soft link with a hard link, because the cost is too high. Thus, it is possible to define

$$\left\{g(C_1), (\mu^{g(C_1)}, y^{g(C_1)})\right\} = \left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\},$$

hence, in this case $k_j' = k_j$ for all $j \in N$.

Case 2. Intermediate cost: $\max\{\lambda, 1-\lambda\}C_1 \in (\Phi(k+1) - \Phi(k+2), \Phi(k) - \Phi(k+1)]$ for some k, $k_1 \leq k \leq n-2$. Then by Proposition 1, it must be the case that in the equilibrium network every manager has at least $(k+1)$ links directed to him. In particular, this means that manager 1 with k_1 and several other managers with in-degrees $< k+1$ will have a strict improvement in their information sets.

Consider the two subcases: (i) If $k+1 \geq k_n$, then each manager gets at least $k+1 \geq k_n$ signals, which proves the result. (ii) If $k+1 < k_n$, then there is manager j such that $k_j < k+1 \leq k_{j+1}$. Theorem 1 implies that $k_l' \geq k+1 > k_l$, $l = 1, ..., j$, in any equilibrium network $g(C_1)$. Concerning other managers, there exists an equilibrium network with $k_l' = k_l$ for $l = j+1, ..., n$. To see this, suppose that the only links in $g(C_1)$ directed towards managers $l = j+1, ..., n$ are the soft links from the pure cheap talk equilibrium network $g(C_0)$. Clearly, truthful communication along these links is still incentive compatible. Further, because $\max\{\lambda, 1-\lambda\}C_1 > \Phi(k+1) - \Phi(k+2)$, then no two managers want to deviate and create hard links directed towards managers $l = j+1, ..., n$. Thus, in the considered equilibrium network the in-degrees of managers $l = 1, ..., j$ are strictly greater than in the pure cheap talk equilibrium, while the in-degrees of the other managers are the same, which confirms the statement of the theorem.

Case 3. Sufficiently low cost: $\max\{\lambda, 1-\lambda\}C_1 \in [0, \Phi(k_{n-1}) - \Phi(k_n)]$. By Proposition 1, in any equilibrium network $g(C_1)$ every manager has exactly $n-1$ links directed to him, which immediately implies that $n - 1 = k'_j \geq k_j$ for all $j \in N$. **QED.**

Proof of Lemma 2

We split the proof into three steps:

Step 1: Existence of a maximal equilibrium. Because the number of managers and strategies is finite, the number of the pure strategy equilibria is also finite. Thus, there exists a well-defined set of numbers, $k_1, ..., k_n$, where k_i is the highest in-degree of manager i that can appear in some equilibrium: for any equilibrium network g', $k_i \geq k'_i = k_i(g')$. Note that the in-degrees k_i and k_j, $i \neq j$, in principle, might be achieved in different equilibrium networks. To prove an existence of a maximal equilibrium, we need to show that $k_1, ..., k_n$ might be achieved in the same equilibrium, i.e., that there exists an equilibrium network g such that $k_i = k_i(g)$ for all i. In order to do this, we construct the equilibrium in the following way: for each $i \in N$ consider an equilibrium where k_i is achieved and let those (and only those) managers who report to i truthfully in that equilibrium to report truthfully to i through the same links (soft or hard) in the constructed equilibrium. Recall that the incentives to form the hard links depend only on the receiver's in-degree, while the incentives of truthful communication through the soft links depend also on the managers' biases. Hence, it is still incentive compatible for those managers to report truthfully to i through the respective soft links and to sustain the corresponding hard links. Thus, this is, indeed, an equilibrium, and, by construction, it is maximal.

Step 2: Maximality of a pairwise stable equilibrium. Consider some pairwise stable equilibrium and assume that it is not maximal. Then there exists manager i whose in-degree in the equilibrium network is lower than his maximal in-degree. Fix some maximal equilibrium; then it must be the case that there is some manager j who reports truthfully to i (through either a soft or a hard link) in this maximal equilibrium, but not in the pairwise stable equilibrium. But then it is profitable for i and j to deviate and induce truthful communication from j to i, which contradicts the pairwise stability. Hence, every pairwise stable equilibrium must be maximal.

Step 3: Existence of a pairwise stable equilibrium. We illustrate this statement by constructing one of (possibly multiple) pairwise stable equilibria. For each $i \in N$ perform the following procedure: order other managers $j \in N/\{i\}$ in the increasing absolute values of their preference divergence from i, $|b_j - b_i|$; let this order be $i_1, ..., i_{n-1}$. Consider the maximal in-degree of manager i, k_i. If $k_i = 0$, then nobody can report truthfully to i in equilibrium. If $k_i > 0$, then take the closest manager i_1. If truth-telling through a soft link $i_1 i$ is incentive compatible for i_1, given that i gets $k_i - 1$ other truthful messages, then let i_1 report truthfully to i via a soft link. Otherwise, set $g_{i_1 i} = h$. Repeat this procedure for other $k_i - 1$ closest to i managers to set the links of particular type with truthful communication through them. Note, that for each of these closest k_i managers, truthful communication through the corresponding link is desired, because truth-telling through a soft link is easier to sustain for closer preferences and the expected benefit of a hard link does not depend on a preference divergence. Since k_i is the maximal possible in-degree of manager i, managers $i_{k_i+1}, ..., i_{n-1}$ cannot communicate to i truthfully via either channel.

The described procedure leads to an equilibrium where each manager gets truthful messages through the soft links from the managers sufficiently close in their preferences, truthful messages through the hard links from the less close managers, and gets no messages from the more distinct managers. To see that this equilibrium satisfies pairwise stability, first, note that by construction no two managers i and j such that $g_{ji} = h$, could deviate to truthful communication through a soft link. Second, no two managers i and j such that $g_{ji} = 0$, could implement truthful communication through either a soft or a hard link ji. Indeed, $g_{ji} = 0$ means that there are k_i other managers closer to manager i in their preferences than manager j. If it were possible for i and j to deviate and implement truthful communication of j's signal through a soft link, given that k_i other managers report truthfully to i, then truthful communication through soft links from each of the closest k_i managers is also incentive compatible (closer biases relax the incentive condition of the truth-telling). But then it means that the in-degree of k_i+1 can be sustained in the equilibrium network, which contradicts that k_i is maximal. On the other hand, if it were desirable for i and j to deviate and create a hard link ji, then truthful communication through the hard links from each of the closest k_i managers must be desirable for them compared to no communication as well. Again, this means that the in-degree of $k_i + 1$ can be sustained in equilibrium, which contradicts that k_i is maximal. As a result, the constructed equilibrium satisfies pairwise stability. **QED.**

Proof of Theorem 2

Reorder the managers in the pure cheap talk pairwise stable equilibrium correspondingly to their in-degrees in $g(C_0)$: $k_1 \le k_2 \le ... \le k_n$. Because any pairwise stable equilibrium is maximal, the in-degrees $k_1,...,k_n$ are maximal when the cost is C_0.

If $k_1 = n-1$, then the truthful network $g(C_0)$ is complete, hence $g(C_1)$ must be also complete and consist of only soft links. Evidently, the welfare is the same across the two cases: $W(g(C_1),C_1) = W(g(C_0))$.

Assume now that $k_1 < n-1$ and consider two cases with regard to the cost C_1:

Case 1. Sufficiently large cost: $C_1 > \Phi(k_1) - \Phi(k_1+1)$. In this case the in-degrees $k_1,...,k_n$ are still maximal and the pure cheap-talk pairwise stable equilibrium still remains a pairwise stable equilibrium. Thus, $g(C_1) = g(C_0)$ and $W(g(C_1),C_1) = W(g(C_0))$.

Case 2. Moderate cost: $C_1 \in (\Phi(k+1) - \Phi(k+2), \Phi(k) - \Phi(k+1)]$ for some k, $k \ge k_1$. Then by Theorem 1, it must be the case that in any equilibrium network every manager has at least $\min\{k+1, n-1\}$ links directed to him.

If $k < k_n$, then there is manager j such that $k_j < k+1 \le k_{j+1}$. Then the set of maximal in-degrees becomes:

$$k'_1 = ... = k'_j = k+1,\ k'_{j+1} = k_{j+1},\ ...,\ k'_n = k_n.$$

There exists a maximal equilibrium with the following communication network: all links directed towards managers $1,...,j$ are hard, all links directed towards managers $j+1,...,n$ are soft and the same as in $g(C_0)$. The upper bound for the total cost of hard links is:

$$j(k+1)C_1 \le j(k+1)(\Phi(k) - \Phi(k+1)).$$

The gain in the welfare compared to the pure cheap talk case is

$$n\sum_{i=1}^{j}(\Phi(k_i) - \Phi(k+1)) \ge n \cdot j(\Phi(k) - \Phi(k+1)).$$

The lower bound for the welfare gain strictly exceeds the upper bound for the cost, because $n-1 \geq k_n > k$, meaning that the constructed maximal equilibrium generates the welfare greater than $W(g(C_0))$. Lemma 3 ensures that there is a pairwise stable equilibrium with the communication network $g(C_1)$ generating the welfare greater than in the constructed maximal equilibrium. Hence, for this pairwise stable equilibrium, $W(g(C_1), C_1) > W(g(C_0))$.

If $k \geq k_n$, then the set of maximal in-degrees becomes $k'_1 = ... = k'_n = \min\{k+1, n-1\}$. Analyze two possibilities:

(a) In case where $k_n < n-1$, consider a maximal equilibrium in which all links are hard. The upper bound for the total cost of the hard links is

$$n \min\{k+1, n-1\} C_1 \leq n \min\{k+1, n-1\}(\Phi(k) - \Phi(k+1)).$$

The gain in the welfare compared to the pure cheap talk case is

$$n \sum_{i=1}^{n} (\Phi(k_i) - \Phi(k+1)) \geq n \cdot n(\Phi(k) - \Phi(k+1)).$$

The lower bound for the welfare gain strictly exceeds the upper bound for the cost, because $n > \min\{k+1, n-1\}$. The considered maximal equilibrium has the greatest level of the total cost and any other pairwise stable equilibrium generates at least the same welfare. Hence, for any pairwise stable equilibrium with the communication network $g(C_1)$: $W(g(C_1) < C_1) > W(g(C_0))$.

(b) In case where $k_n = n-1$, there exists j such that

$$k_1 \leq k_2 \leq ... \leq k_j < n-1 = k_{j+1} = ... = k_n.$$

The set of maximal in-degrees becomes $k'_1 = ... = k'_n = n-1$. Consider a maximal equilibrium in which managers $1, ..., j$ receive signals through only hard links, while managers $j+1, ..., n$ receive messages through only soft links which are part of $g(C_0)$. The upper bound for the total cost of hard links is

$$j(n-1)C_1 \leq j(n-1)(\Phi(k) - \Phi(k+1)),$$

while the gain in the welfare compared to the pure cheap talk case is

$$n \sum_{i=1}^{j} (\Phi(k_i) - \Phi(n-1)) \geq n \cdot j(\Phi(n-2) - \Phi(n-1)).$$

Since $k \geq k_n = n-1$, $\Phi(k) - \Phi(k+1) < \Phi(n-2) - \Phi(n-1)$ and the welfare gain outweighs the cost. Thus, $W(g(C_1), C_1) > W(g(C_0))$.

QED.

Proof of Lemma 4

Since $g^x(C_0) \subseteq g^x(C_1)$ for $x = s, h$, the in-degrees in $g(C_1)$ are larger: $k_i(g(C_1)) \geq k_i(g(C_0))$ for all $i \in N$. To see that $W(g(C_1), C_1) \geq W(g(C_0), C_0)$, perform the following procedure:

First, fix the truthful communication network to be $g(C_0)$ and set the cost level at C_1. Then the welfare $W(g(C_0),C_1)$ is weakly greater than $W(g(C_0),C_0)$.

Second, add links from $g(C_1)/g(C_0)$ one by one and trace the welfare changes. In what follows we show that at each step of adding a link, the ex-ante expected payoff of every individual (and hence, the welfare) increases. Consider some step at which the in-degrees of the managers are $k_1,\dots,k_n$, where $k_i(g(C_0)) \le k_i \le k_i(g(C_1))$ for all $i \in N$ and $k_j < k_j(g(C_1))$ for at least one manager j. If a soft link ij from the remaining soft links $g^s(C_1)/g^s(C_0)$ is added, then the welfare of every manager goes up by $\Phi(k_j) - \Phi(k_j+1) > 0$. If a hard link ij from the remaining hard links $g^h(C_1)/g^h(C_0)$ is added, then the expected payoff of every manager $l \neq i,j$ goes up by $\Phi(k_j) - \Phi(k_j+1) > 0$, while the payoffs of managers i and j increase by at least

$$\Phi(k_j) - \Phi(k_j+1) - \max\{\lambda, 1-\lambda\}C_1 \ge 0,$$

because by Theorem 1

$$\max\{\lambda,1-\lambda\}C_1 \le \Phi(k_j(g(C_1))-1) - \Phi(k_j(g(C_1))) \le \Phi(k_j) - \Phi(k_j+1).$$

QED.

Proof of Theorem 3

Consider the setting in which only soft links are available. Condition $n_2 > n_1$ and the negative externality effect ensure that cross-group truthful communication in any pairwise stable equilibrium might be one of the following 3 kinds:

1. No cross-group communication, i.e. $k_{21} = k_{12} = 0$.
2. Communication from group N_2 to group N_1, i.e. $k_{12} > 0$, $k_{21} = 0$.
3. Cross-group communication, i.e. $k_{12} > 0$, $k_{21} > 0$.

In what follows, we examine each case separately and show that introducing hard links is welfare increasing in cases 1 and 3, and is welfare decreasing for some parameters in case 2.

Case 1 corresponds to

$$b > \max\left(\frac{\alpha+\beta+D}{2(\alpha+\beta+n_1)}, \frac{\alpha+\beta+D}{2(\alpha+\beta+n_2)}\right) = \frac{\alpha+\beta+D}{2(\alpha+\beta+n_1)}.$$

If costly hard links become feasible, then intra-group soft-link communication remains unchanged. Regarding cross-group communication, if the cost C_1 is such that $\max\{\lambda,1-\lambda\}C_1 > \Phi(n_1-1) - \Phi(n_1)$, then cross-group communication remains empty and

$$W(g(C_1),C_1) = W(g(C_0)).$$

If $\max\{\lambda,1-\lambda\}C_1 \le \Phi(n_1-1) - \Phi(n_1)$, then some cross-group communication via hard links appears. Because the set of links in this equilibrium includes the set of links from the cheap talk case (no crowding out occurs), by Lemma 4, the equilibrium with hard links generates a greater total (and individual) welfare, $W(g(C_1),C_1) \ge W(g(C_0))$.

Case 2 applies when the preference bias b satisfies

$$b \in \left(\frac{\alpha+\beta+D}{2(\alpha+\beta+n_1+k_{12}+1)}, \frac{\alpha+\beta+D}{2(\alpha+\beta+n_1+k_{12})}\right], \tag{B.2}$$

where $n_1 + k_{12} \le n_2$. Consider separately two possibilities: $n_1 + k_{12} = n_2$ and $n_1 + k_{12} < n_2$.

First, consider $n_1 + k_{12} = n_2$, which means that $k_1 = k_2 = n_2 - 1$. The welfare in the pure cheap talk equilibrium is

$$W(g(C_0)) = -n(n_1\Phi(k_1) + n_2\Phi(k_2)) - B = -n^2\Phi(n_2 - 1) - B,$$

where the term B depends only on preference bias b, $B = 2\sum_{i\in N_1}\sum_{j\in N_2} b^2 = 2n_1n_2b^2$.

If the cost C_1 is such that $\max\{\lambda, 1-\lambda\}C_1 > \Phi(n_1 - 1 + k_{12}) - \Phi(n_1 + k_{12})$, then the maximal in-degrees remain the same and each pairwise stable equilibrium involves only soft-link communication, meaning that $W(g(C_1), C_1) = W(g(C_0))$. Now let the cost be $\widehat{C}$ such that $\max\{\lambda, 1-\lambda\}\widehat{C} = \Phi(n_1 - 1 + k_{12}) - \Phi(n_1 + k_{12}) = \Phi(n_2 - 1) - \Phi(n_2)$, which means that in a new pairwise stable equilibrium $k'_{21} = 1$, $k'_{12} = k_{12} + 1$, and hence, $k'_1 = k'_2 = n_2$.[30] Given the preference divergence, members of the opposite communities can not report to each other truthfully via soft links, thus, all cross-group soft links are substituted out by costly hard links leading to the following welfare:

$$\begin{aligned} W(g(\widehat{C}), \widehat{C}) &= -n(n_1\Phi(k'_1) + n_2\Phi(k'_2)) - B - \frac{1}{\max\{\lambda, 1-\lambda\}}(\Phi(n_2 - 1) - \Phi(n_2))[n_1k'_{12} + n_2] \\ &= -n^2\Phi(n_2) - B - \frac{1}{\max\{\lambda, 1-\lambda\}}(\Phi(n_2 - 1) - \Phi(n_2))[n_1(k_{12} + 1) + n_2]. \end{aligned}$$

Thus, the difference between the levels of total welfare is

$$W(g(C_0)) - W(g(\widehat{C}), \widehat{C}) = (\Phi(n_2 - 1) - \Phi(n_2))\left[-n^2 + \frac{1}{\max\{\lambda, 1-\lambda\}}(n_1(k_{12} + 1) + n_2)\right] \le 0,$$

because

$$\begin{aligned} -n^2 + \frac{1}{\max\{\lambda, 1-\lambda\}}(n_1(k_{12} + 1) + n_2) &\le -n^2 + 2(n_1(k_{12} + 1) + n_2) \\ &= -(n_1 + n_2)^2 + 2n_1(n_2 - n_1 + 1) + 2n_2 \\ &= -3n_1^2 - n_2^2 + 2n_1 + 2n_2 \le 0, \end{aligned}$$

given that $1 \le n_1 < n_2$. For any lower cost level $C_1 < \frac{\Phi(n_2-1)-\Phi(n_2)}{\max\{\lambda,1-\lambda\}} = \widehat{C}$, cross-group communication is also carried out through only hard links. Compared to the considered case of $\widehat{C}$, no soft links are severed and w.l.o.g. it can be assumed that all hard links are retained and some new hard links are added. Then Lemma 4 implies that the welfare increases: $W(t(C_1), C_1) \ge W(g(\widehat{C}), \widehat{C}) \ge W(g(C_0))$. As a result, any pairwise stable equilibrium with hard links generates a greater welfare than the pure cheap talk equilibrium.

Second, consider $n_1 + k_{12} + 1 \le n_2$. In this case $k_1 = n_1 - 1 + k_{12}$, $k_2 = n_2 - 1$ and the total welfare in the pure cheap talk case is

$$W(g(C_0)) = -n(n_1\Phi(n_1 - 1 + k_{12}) + n_2\Phi(n_2 - 1)) - B.$$

If the cost $C_1 > \frac{1}{\max\{\lambda,1-\lambda\}}(\Phi(n_1 - 1 + k_{12}) - \Phi(n_1 + k_{12}))$, then the maximal in-degrees remain the same and the pairwise stable equilibria are pure soft-link, hence, $W(g(C_1), C_1) = W(g(C_0))$. Let the cost be $\widehat{C} = \frac{1}{\max\{\lambda,1-\lambda\}}(\Phi(n_1 - 1 + k_{12}) - \Phi(n_1 + k_{12}))$. Then cross-group communication

[30] Recall the assumption that whenever the manager is indifferent, the choice is made in favor of a hard link creation.

becomes $k'_{12} = k_{12} + 1$, while k'_{21} remains 0; all cross-group links are hard. The new maximal in-degrees are $k'_1 = n_1 + k_{12}$ and $k'_2 = k_2 = n_2 - 1$, implying the following welfare

$$W(g(\widehat{C}), \widehat{C}) = - n(n_1\Phi(n_1 + k_{12}) + n_2\Phi(n_2 - 1)) - B - \frac{1}{\max\{\lambda, 1-\lambda\}}(\Phi(n_1 - 1 + k_{12}) - \Phi(n_1 + k_{12}))[n_1(k_{12} + 1)].$$

The difference in the levels of the welfare is then

$$W(g(C_0)) - W(g(\widehat{C}), \widehat{C}) = n_1(\Phi(n_1 - 1 + k_{12}) - \Phi(n_1 + k_{12}))\left[-n + \frac{1}{\max\{\lambda, 1-\lambda\}}(k_{12} + 1)\right],$$

which is strictly greater than 0 if and only if $k_{12} > \max\{\lambda, 1-\lambda\}n - 1$. Since the inequality is strict, there exists $\underline{C}_1 < \widehat{C}$ such that the welfare difference remains strictly positive for $C_1 \in (\underline{C}_1, \overline{C}_1)$, where $\overline{C}_1 = \widehat{C}$. Thus, given the preference divergence (C.2), condition $\max\{\lambda, 1-\lambda\}n - 1 < k_{12} \leq n_2 - n_1 - 1$ is sufficient for existence of the cost that leads to a lower welfare. The necessity follows from the fact that for any $C_1 < \widehat{C}$ Lemma 4 implies that $W(g(C_1), C_1) \geq W(g(\widehat{C}), \widehat{C})$.

Finally, **Case 3** corresponds to a sufficiently small preference divergence:

$$b \leq \frac{\alpha + \beta + D}{2(\alpha + \beta + n_2 + 1)}.$$

Non-zero cross-group communication in both directions implies that the maximal in-degrees are the same for members of both communities, $k_1 = k_2$. If the truthful cheap talk network $g(C_0)$ is complete ($k_1 = k_2 = n - 1$), which corresponds to $b \leq \frac{\alpha+\beta+D}{2(\alpha+\beta+n)}$, then introducing feasible hard links does not alter this pairwise stable equilibrium and $W(g(C_1), C_1) = W(g(C_0))$.

Case where $g(C_0)$ is not complete ($k_1 = k_2 < n - 1$) corresponds to the preference divergence

$$b \in \left(\frac{\alpha + \beta + D}{2(\alpha + \beta + n_2 + k_{21} + 1)}, \frac{\alpha + \beta + D}{2(\alpha + \beta + n_2 + k_{21})}\right],$$

where $k_{21} < n_1$. The welfare of the pure cheap talk pairwise stable equilibrium is

$$W(g(C_0)) = -n^2\Phi(k_1).$$

If the cost $C_1 > \frac{1}{\max\{\lambda, 1-\lambda\}}(\Phi(k_1) - \Phi(k_1 + 1))$, then the maximal in-degrees are the same as in the cheap talk case and any pairwise stable equilibrium involves only soft communication, meaning that $W(g(C_1), C_1) = W(g(C_0))$. Consider

$$\widehat{C} = \frac{1}{\max\{\lambda, 1-\lambda\}}(\Phi(k_1) - \Phi(k_1 + 1)).$$

The new maximal in-degrees become $k'_1 = k'_2 = k_1 + 1$ and all cross-group communication in any pairwise stable equilibrium is performed via hard links, such that each member of community N_1 gets $k_{12} + 1$ hard links from members of N_2 and, similarly, each member of N_2 gets $k_{21} + 1$ hard links from members of N_1. The welfare is

$$W(g(\widehat{C}), \widehat{C}) = -n^2\Phi(k_1 + 1) - \frac{1}{\max\{\lambda, 1-\lambda\}}(\Phi(k_1) - \Phi(k_1 + 1))(n_1(k_{12} + 1) + n_2(k_{21} + 1)).$$

The difference between the levels of the welfare is

$$\begin{aligned} & W(g(C_0)) - W(g(\widehat{C}), \widehat{C}) \\ = \quad & (\Phi(k_1) - \Phi(k_1+1))\left[-n^2 + \frac{1}{\max\{\lambda, 1-\lambda\}}(n_1(k_{12}+1) + n_2(k_{21}+1))\right] \leq 0, \end{aligned}$$

because

$$\begin{aligned} & -n^2 + \frac{1}{\max\{\lambda, 1-\lambda\}}(n_1(k_{12}+1) + n_2(k_{21}+1)) \leq -n^2 + 2n_1(k_{12}+1) + 2n_2(k_{21}+1) \\ = \quad & -n^2 + 2n_1(k_1+2-n_1) + 2n_2(k_1+2-n_2) = -n^2 + 2(k_1+2)n - 2n_1^2 - 2n_2^2 \\ \leq \quad & -n^2 + 2(k_1+2)n - n^2 = -2n(n-(k_1+2)) \leq 0, \end{aligned}$$

given that $k_1 < n-1$. For any lower level of the cost $C_1 < \widehat{C}$ cross-group communication is performed through only hard links as well. Since, compared to the case of $\widehat{C}$, no cheap talk links are severed and w.l.o.g. it can be assumed that all hard links are retained and some new hard links are added, Lemma 4 implies that $W(g(C_1), C_1) \geq W(g(\widehat{C}), \widehat{C}) \geq W(g(C_0))$. **QED.**

Proof of Theorem 4

The case of $n = 3$ was already analyzed in the main body of the paper; it remains to show the positive welfare result for $n \geq 4$.

Consider prohibitively costly hard links, $\max\{\lambda, 1-\lambda\}C_0 > \Phi(0) - \Phi(1)$, and construct a pairwise stable equilibrium that generates the greatest welfare in a way described in Lemma 3. In the corresponding truthful network, manager i gets truthful messages from k_i closest managers. This type of cheap talk equilibrium corresponds to the utility-maximizing equilibrium The communication network $g(C_0)$ can be formally described as follows: if $b > \frac{\alpha+\beta+D}{2(\alpha+\beta+2)}$ then $g(C_0)$ is empty, otherwise, let

$$V(b) = \max\{V \in \{1, ..., n\} : b \leq \frac{\alpha+\beta+D}{2V(2V-1+1+\alpha+\beta)}\},$$

then

1. For every $j \in \{V(b)+1, ..., n-V(b)\}$, $g_{ij} = s$ if $|i-j| < V(b)$ and $g_{ij} = 0$ if $|i-j| > V(b)$; if $b > \frac{\alpha+\beta+D}{2V(b)(2V(b)+1+\alpha+\beta)}$, then $g_{ij} = s$ for one and only one manager i such that $|i-j| = V(b)$; if $b \leq \frac{\alpha+\beta+D}{2V(b)(2V(b)+1+\alpha+\beta)}$, then $g_{ij} = s$ for both managers i such that $|i-j| = V(b)$.

2. For all managers $j \in \{1, ..., V(b)\} \cup \{n-V(b)+1, ..., n\}$, $g_{ij} = s$ if and only if $|i-j| \leq M(j,b)$, where $M(j,b) = \max\{M \in \{1, ..., n\} : b \leq \frac{\alpha+\beta+D}{2M(\min\{j-1, n-j\}+M+1+\alpha+\beta)}\}$.

In the pure cheap talk equilibrium, the set of maximal in-degrees $K = \{k_1, ..., k_n\}$ is the following: $k_j = 0$, $j \in N$, if $b > \frac{\alpha+\beta+D}{2(\alpha+\beta+2)}$; otherwise, for every $i \in \{V(b)+1, ..., n-V(b)\}$,

$$k_i = \begin{cases} 2V(b)-1, \text{ if } b > \frac{\alpha+\beta+D}{2V(b)(2V(b)+1+\alpha+\beta)} \\ 2V(b), \text{ if } b \leq \frac{\alpha+\beta+D}{2V(b)(2V(b)+1+\alpha+\beta)}, \end{cases}$$

and for each $j \in \{1, ..., V(b)\} \cup \{n-V(b)+1, ..., n\}$,

$$k_j = \min\{j-1, n-j\} + M(j,b).$$

Given the set of maximal in-degrees K, define

$$\begin{aligned} k_{(n)} &= \max\{k_i \in K\}, \\ k_{(j)} &= \max\{k_i \in K/\{k_{(n)},...,k_{(j-1)}\}\}, \end{aligned}$$

i.e., $k_{(1)} \leq ... \leq k_{(n)}$ is a reordering of K in the increasing order. It can be easily seen, that managers with moderate preferences, $\{V(b)+1,...,n-V(b)\}$, have the highest in-degree $k_{(n)}$. The in-degrees of other managers decrease as their preference biases get closer to the extremes, such that manager 1 (with bias 0) and manager n (with bias $(n-1)b$) have the same in-degree of $k_{(1)}$. Since $M(j,b) \in \{M(j+1,b), M(j+1,b)+1\}$ for $j \in \{1,...,V(b)\}$ (similarly, $M(j+1,b) \in \{M(j,b), M(j,b)+1\}$ for $j \in \{n-V(b)+1,...,n\}$) and $M(V(b),b) = M(n-V(b),b) = V(b)$, the structure of a pairwise stable equilibrium ensures that for every $i = 1,...,n-1$ the difference $k_{(i+1)} - k_{(i)}$ is either 0 or 1.

Now introduce hard links with the cost $C_1 \leq \frac{\Phi(0)-\Phi(1)}{\max\{\lambda,1-\lambda\}}$. If the communication network of the pairwise stable pure cheap talk equilibrium is empty ($b > \frac{\alpha+\beta+D}{2(1+1+\alpha+\beta)}$), then there is no crowding out when hard links become available, and Lemma 4 implies that the welfare increases. Consider now the case where the pure cheap talk communication network is not empty, i.e., $k_{(n)} > 0$, and study three possibilities for C_1 separately

1. $\max\{\lambda,1-\lambda\}C_1 \in (\Phi(k_{(1)}) - \Phi(k_{(1)}+1), \Phi(0) - \Phi(1)]$.

2. $\max\{\lambda,1-\lambda\}C_1 \in (\Phi(k+1) - \Phi(k+2), \Phi(k) - \Phi(k+1)]$, for some $k_{(1)} \leq k < k_{(n)}$.

3. $\max\{\lambda,1-\lambda\}C_1 \in (0, \Phi(k_{(n)}) - \Phi(k_{(n)}+1)]$.

Case 1. For the cost higher than $\frac{\Phi(k_{(1)})-\Phi(k_{(1)}+1)}{\max\{\lambda,1-\lambda\}}$, the maximal in-degrees remain the same and the pure cheap talk equilibrium remains pairwise stable, meaning that w.l.o.g. $g(C_1) = g(C_0)$ and $W(g(C_1),C_1) = W(g(C_0))$.

Case 2. Because the difference $k_{(i+1)} - k_{(i)}$ is either 0 or 1, there must exist i such that $k_{(i)} = k$ and $k_{(i+1)} = k+1$. The structure of the pure cheap talk equilibrium implies that i is an even number less or equal to $2V(b)$. For any C_1 that satisfies

$$\max\{\lambda,1-\lambda\}C_1 \in (\Phi(k+1) - \Phi(k+2), \Phi(k) - \Phi(k+1)],$$

the maximal in-degrees are

$$k'_j = \begin{cases} k+1, & \text{if } k_j \leq k, \\ k_j, & \text{if } k_j > k. \end{cases}$$

Consider a pairwise stable equilibrium that generates the greatest welfare. It must be the case that in the corresponding communication network $g(C_1)$ managers $\{j : k_j > k\}$ receive messages via only soft links, in particular, assume that they receive truthful messages from the same managers as in the pure cheap talk equilibrium. Other managers $\{j : k_j \leq k = k_{(i)}\} = \{1,..., \frac{i}{2}\} \cup \{n - \frac{i}{2},...,n\}$ have the new in-degrees equal to $k+1$ and can receive truthful messages through both, soft and hard links. If $j \in \{1,..., \frac{i}{2}\}$, then the number of soft links directed to manager j in $g(C_1)$ is at least $j-1+V(b)$. This implies that the upper bound on the number of costly hard links directed to j is $k+1-(j-1+V(b))$. Similarly, if $j \in \{n-\frac{i}{2},...,n\}$, then the number of hard links directed to manager j is less or equal than $k+1-(n-j+V(b))$. Thus, the upper bound for the total cost of hard links is

$$2\cdot\frac{\Phi(k)-\Phi(k+1)}{\max\{\lambda,1-\lambda\}}\sum_{j=1}^{\frac{i}{2}}[k+1-(j-1+V(\beta))]$$
$$= i\frac{\Phi(k)-\Phi(k+1)}{\max\{\lambda,1-\lambda\}}\left[k+2-V(\beta)-\frac{i+2}{4}\right].$$

The additional welfare is

$$2n\sum_{j=1}^{\frac{i}{2}}\left[\Phi(k_j)-\Phi(k+1)\right]\geq n\cdot i(\Phi(k)-\Phi(k+1)).$$

The upper bound for the cost is less than the lower bound for the additional benefit, because

$$n\geq 2\left[k+2-V(\beta)-\frac{i+2}{4}\right]\geq\frac{1}{\max\{\lambda,1-\lambda\}}\left[k+2-V(\beta)-\frac{i+2}{4}\right],$$

where the first inequality is satisfied due to $\frac{n}{2}+V(\beta)\geq k_{(n)}\geq k+1$ and $1-\frac{i+2}{4}\leq 0$. As a result, $W(g(C_1),C_1)\geq W(g(C_0))$.

Case 3. If $\max\{\lambda,1-\lambda\}C_1\in(0,\Phi(k_{(n)})-\Phi(k_{(n)}+1)]$, then there is $k\geq k_{(n)}$ such that $\max\{\lambda,1-\lambda\}C_1\in(\Phi(k+1)-\Phi(k+2),\Phi(k)-\Phi(k+1)]$. There are several possibilities to consider:

1. If $k_{(n)}=n-1$, then $g(C_0)$ is either complete, or not. In case it is complete, the pure cheap talk equilibrium remains pairwise stable once hard links become feasible, thus, $W(g(C_1),C_1)=W(g(C_0))$. If $g(C_0)$ is incomplete, then there exists i: $1\leq i<N$ such that

   $$k_{(1)}\leq\ldots\leq k_{(i)}<n-1=k_{(i+1)}=\ldots=k_{(n)}.$$

   Consider a pairwise stable equilibrium that generates the greatest welfare when hard links are available. Condition $\max\{\lambda,1-\lambda\}C_1\leq\Phi(n-1)-\Phi(n)$ insures that in the corresponding communication network $g(C_1)$ each manager $j\in N$ has the in-degree $k'_j=n-1$. Note that for the greater cost $C_2=\frac{\Phi(n-2)-\Phi(n-1)}{\max\{\lambda,1-\lambda\}}$, a pairwise stable equilibrium that generates the greatest welfare has the same communication network, $g(C_2)=g(C_1)$. Since C_2 satisfies the conditions of case 2, then $W(g(C_1),C_1)>W(g(C_2),C_2)\geq W(g(C_0))$.

2. Consider $k_{(n)}\leq n-2$. Note that if the positive welfare result $W(g(C_1),C_1)\geq W(g(C_0))$ holds for $k=n-2$, then by Lemma 4 it also holds for any $k>n-2$. Thus, for the rest of the proof assume that $k\leq n-2$. The in-degrees in the communication network of any equilibrium when hard links are available become $k'_j=k+1$, $j\in N$. The total cost of hard links in the corresponding $g(C_1)$ does not exceed

   $$n(k+1)\frac{\Phi(k)-\Phi(k+1)}{\max\{\lambda,1-\lambda\}},$$

   while the gain in the welfare compared to the cheap talk case is at least

   $$n\cdot n(\Phi(k)-\Phi(k+1)).$$

   The lower bound for the gain is greater than the upper bound for the cost if $k\leq n\max\{\lambda,1-$

$\lambda\} - 1$, meaning that for such C_1 the welfare in *any* equilibrium with hard links exceeds $W(g(C_0))$, in particular, $W(g(C_1), C_1) \geq W(g(C_0))$.

Consider now $n - 2 \geq k > n\max\{\lambda, 1-\lambda\} - 1$ and analyze two possibilities:

(a) If $k > k_{(n)}$, then the lower bound for the additional expected total benefit is

$$n^2(\Phi(k-1) - \Phi(k+1)) = n^2(\Phi(k-1) - \Phi(k) + \Phi(k) - \Phi(k+1)) \\ \geq 2n^2(\Phi(k) - \Phi(k+1)),$$

which exceeds the upper bound for the cost $n(k+1)\frac{\Phi(k)-\Phi(k+1)}{\max\{\lambda,1-\lambda\}}$, because

$$\frac{k+1}{\max\{\lambda, 1-\lambda\}} \leq 2(k+1) < 2n.$$

This means, that *any* equilibrium with hard links outperforms the cheap talk equilibrium in terms of welfare, hence, $W(g(C_1), C_1) \geq W(g(C_0))$.

(b) If $k = k_{(n)}$, then in any equilibrium with hard links each manager j has the in-degree of $k'_j = k+1 > n\max\{\lambda, 1-\lambda\} \geq \frac{n}{2}$, $j \in N$. Take a pairwise stable equilibrium that generates the greatest welfare, and consider how many links in the corresponding communication network $g(C_1)$ can be soft. manager i with the bias b_i such that $|b_i - b_j| = lb$, will report truthfully via cheap talk to manager j if

$$b \leq \frac{\alpha + \beta + D}{2l(k + 2 + \alpha + \beta)}.$$

Note that this inequality is satisfied for $l = V(b) - 1$, because by the definition of $V(b)$ and the fact that $k = k_{(n)} \leq 2V(b)$,

$$\frac{\alpha + \beta + D}{2l(k + 2 + \alpha + \beta)} > \frac{\alpha + \beta + D}{2V(b)(2V(b) + \alpha + \beta)} \geq b.$$

Thus, assuming $l = V(b) - 1$, each manager $j \in \{1, ..., l\}$ gets truthful messages through soft links from $j - 1 + l$ managers $1, ..., j-1, j+1, ..., j+l$. Similarly, each manager $j \in \{n-l, ..., n\}$ gets at least $n - j + l$ truthful messages through soft links. Finally, each manager $j \in \{l+1, ..., n-l-1\}$ gets at least $2l$ truthful messages via cheap talk. Thus, the number of hard links in $g(C_1)$ is bounded from above by

$$n(k+1) - 2\sum_{j=1}^{l}(j - 1 + l) - 2l(n - 2l) = n(k+1) + l^2 + l - 2ln,$$

meaning that the total cost does not exceed

$$\frac{\Phi(k) - \Phi(k+1)}{\max\{\lambda, 1-\lambda\}}(n(k+1) + l^2 + l - 2ln).$$

Since $k = k_{(n)} \leq 2V(b)$ and $l = V(b) - 1$, then $k + 1 \leq 2(l+1)$ and the upper bound for the cost becomes

$$\frac{\Phi(k)-\Phi(k+1)}{\max\{\lambda,1-\lambda\}}(2n(l+1)+l^2+l-2ln)=\frac{\Phi(k)-\Phi(k+1)}{\max\{\lambda,1-\lambda\}}(2n+l^2+l).$$

Note that $\frac{1}{\max\{\lambda,1-\lambda\}}\leq 2$ and $l\leq\frac{n}{2}-1$, because $2l<2V(b)-1\leq k\leq n-2$, which allows to write the upper bound for the cost as:

$$2(\Phi(k)-\Phi(k+1))\left(2n+\left(\frac{n}{2}-1\right)^2+\frac{n}{2}-1\right)=(\Phi(k)-\Phi(k+1))\left(\frac{n^2}{2}+3n\right)$$

The additional welfare is at least

$$n^2(\Phi(k)-\Phi(k+1)).$$

The lower bound for the extra welfare is greater than the upper bound for the cost if $n\geq 6$. Thus, $W(g(C_1),C_1)\geq W(g(C_0))$ for $n\geq 6$.

It remains to show that $W(g(C_1),C_1)\geq W(g(C_0))$ for $n=4$ and $n=5$ as well, assuming that

$$n-2\geq k=k_{(n)}>n\max\{\lambda,1-\lambda\}-1\geq\frac{n}{2}-1$$

and

$$\max\{\lambda,1-\lambda\}C_1\in(\Phi(k+1)-\Phi(k+2),\Phi(k)-\Phi(k+1)].$$

Consider, first, $n=5$ and $k=2,3$. Depending on b, case $k=k_{(n)}=2$ corresponds to pairwise stable pure cheap talk equilibria with the following in-degrees (see Figure 9):

(i) If $\frac{\alpha+\beta+D}{4(3+\alpha+\beta)}<b\leq\frac{\alpha+\beta+D}{2(3+\alpha+\beta)}$, then $k_1=k_5=1$, $k_2=k_3=k_4=2$.

(ii) If $\frac{\alpha+\beta+D}{4(4+\alpha+\beta)}<b\leq\frac{\alpha+\beta+D}{4(3+\alpha+\beta)}$, then $k_1=k_2=k_3=k_4=k_5=2$.

When hard links become available with $\max\{\lambda,1-\lambda\}C_1\in(\Phi(3)-\Phi(4),\Phi(2)-\Phi(3)]$, the maximal in-degrees become $k_1'=...=k_5'=3$. In case (i), the upper bound for the total cost of hard links is

$$5\cdot 3\cdot 2(\Phi(2)-\Phi(3)),$$

which is lower than the gain in the welfare

$$5\cdot[3(\Phi(2)-\Phi(3))+2(\Phi(1)-\Phi(3))],$$

because $\Phi(2)-\Phi(3)<2(\Phi(1)-\Phi(2))$. In case (ii), soft links between the managers with adjacent biases can be a part of the communication network of a pairwise stable equilibrium. Thus, the total cost of hard links in $g(C_1)$ does not exceed $7\cdot 2(\Phi(2)-\Phi(3))$, which, in turn, is lower than the additional welfare, $5\cdot 5(\Phi(2)-\Phi(3))$.

Case $k=k_{(n)}=3$ corresponds to $\frac{\alpha+\beta+D}{4(5+\alpha+\beta)}<b\leq\frac{\alpha+\beta+D}{4(4+\alpha+\beta)}$ and maximal in-degrees $k_1=k_5=2$, $k_2=k_3=k_4=3$ in the pure cheap talk setting (see Figure 10 for an example of $g(C_0)$). When hard links become available with $\max\{\lambda,1-\lambda\}C_1\in(\Phi(4)-\Phi(5),\Phi(3)-\Phi(4)]$, the maximal in-degrees become $k_1'=...=k_5'=4$. Note that soft links between the managers with adjacent biases can be a part of the communication network of a pairwise stable equilibrium, implying that the total cost

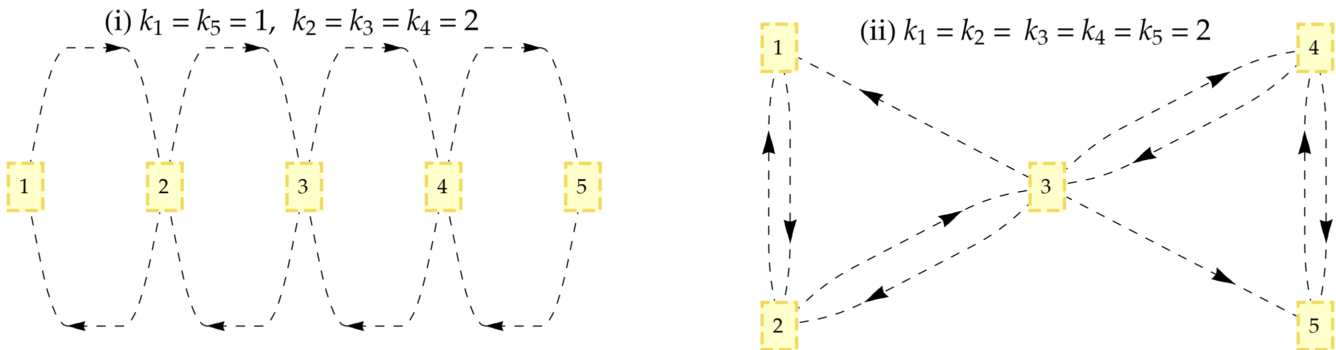


Figure 6: Communication networks of pairwise stable pure cheap talk equilibria when $n = 5$ and $k = 2$.

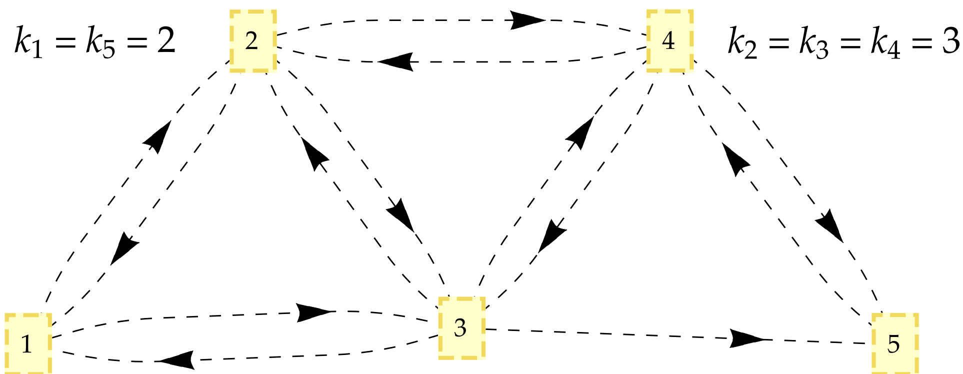


Figure 7: Communication network of pairwise stable pure cheap talk equilibrium when $n = 5$ and $k = 3$.

of hard links in $g(C_1)$ has an upper bound of $12 \cdot 2(\Phi(3) - \Phi(4))$, which is lower than the gain in the welfare, $5 \cdot [3(\Phi(3) - \Phi(4)) + 2(\Phi(2) - \Phi(4))]$. Hence, $W(g(C_1), C_1) \geq W(g(C_0))$, when $n = 5$.

Consider now $n = 4$ and $k = 2$. Depending on b, $g(C_0)$ can have two different structures with the following in-degrees (see Figure 11):

(i) If $\frac{\alpha+\beta+D}{4(3+\alpha+\beta)} < b \leq \frac{\alpha+\beta+D}{2(3+\alpha+\beta)}$, then $k_1 = k_4 = 1$, $k_2 = k_3 = 2$.

(ii) If $\frac{\alpha+\beta+D}{4(4+\alpha+\beta)} < b \leq \frac{\alpha+\beta+D}{4(3+\alpha+\beta)}$, then $k_1 = k_2 = k_3 = k_4 = 2$.

When hard links become available with $\max\{\lambda, 1-\lambda\}C_1 \in (\Phi(3) - \Phi(4), \Phi(2) - \Phi(3)]$, the maximal in-degrees become $k'_1 = ... = k'_4 = 3$. In case (i), the total cost of hard links has an upper bound of $3{\cdot}4{\cdot}2(\Phi(2){-}\Phi(3))$, which is lower than the gain in the welfare $4{\cdot}[2(\Phi(2){-}\Phi(3)){+}2(\Phi(1){-}\Phi(3))]$, because $\Phi(2) - \Phi(3) < \Phi(1) - \Phi(2)$. In case (ii), soft links between the managers with adjacent biases can be a part of the communication network of a pairwise stable equilibrium. Thus, the total cost of hard links in $g(C_1)$ is below $6 \cdot 2(\Phi(2) - \Phi(3))$, which, in turn, is lower than the additional welfare, $4 \cdot 4(\Phi(2) - \Phi(3))$. As a result, $W(g(C_1), C_1) \geq W(g(C_0))$, when $n = 4$. **QED.**

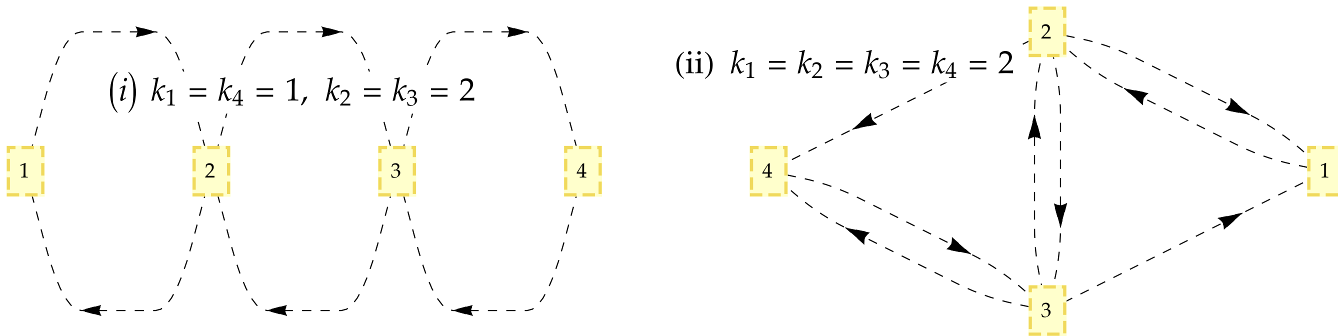


Figure 8: Communication network of pure cheap talk equilibrium when $n = 4$ and $k = 2$.

C Online Appendix: Non-additive signal structure

In this section we show that our results can go beyond additive signals. We consider the following natural non-additive signal structure.

Non-additive signal structure. The state of the world θ is unknown and has a density of Beta distribution with commonly known parameters (α, β):

$$f(\theta) = \frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}.$$

Each player i receives a private signal $s_i \in \{0,1\}$ about θ, where $s_i = 1$ with probability θ and $s_i = 0$ with complementary probability $1-\theta$. Private signals are assumed to be independent and identically distributed. Conditional on the state of the world θ, if the chosen action profile is $\hat{y}^g = \{\hat{y}_1^g, \ldots, \hat{y}_n^g\}$, then the realized payoff (utility) of player i is

$$u_i(\hat{y}^g|\theta) = -\sum_{j=1}^{n}(\hat{y}_j^g - \theta - b_i)^2.$$

Player i's payoff depends on how close his own action and the actions of other players are to player i's ideal action, $\theta + b_i$.

Given that agent i got the private signal s_i and received messages $\widehat{m}^g_{N_i^{-1}(g),i}$, he chooses an action $y_i^g(s_i, \widehat{m}^g_{N_i^{-1}(g),i})$ to maximize his expected payoff,

$$\mathbf{E}\left(-\sum_{j=1}^{N}(y_j^g - \theta - b_i)^2 \,\middle|\, s_i, \widehat{m}^g_{N_i^{-1}(g),i}\right),$$

which means that the agent chooses

$$\begin{aligned} y_i^g(s_i, \widehat{m}_{N_i^{-1}(g),i}) &= \arg\max_{y_i^g}\left\{\mathbf{E}\left(-(y_i^g - \theta - b_i)^2 \,\middle|\, s_i, \widehat{m}^g_{N_i^{-1}(g),i}\right)\right\} \\ &= b_i + \mathbf{E}\left(\theta | s_i, \widehat{m}^g_{N_i^{-1}(g),i}\right). \end{aligned} \tag{C.1}$$

Given the above signal structure, in the following, we, respectively, show that all of our results hold when the signal structure is **non-additive**.

Lemma 5. *Fix a truthful network g and consider player j with the in-degree of $k_j = k_j(g)$. Let s_R be the set of signals that player j gets to know and y_{s_R} be the corresponding action chosen by j. For any $i \in N$ the ex-ante expected input from player j into i's utility is given by*

$$-\int_0^1 \sum_{s_R \in \{0,1\}^{k_j+1}} (y_{s_R} - \theta - b_i)^2 f(\theta, s_R) d\theta = -h(k_j) - (b_j - b_i)^2,$$

where $h(k_j) = \mathbf{E}\left[\mathbf{Var}(\theta|s_R)\right] = \frac{\alpha\beta}{(\alpha+\beta+1+k_j)(\alpha+\beta+1)(\alpha+\beta)}$.

Proof of Lemma 5. Because the chosen action y_{s_R} is given by $y_{s_R} = b_j + \mathbf{E}(\theta|s_R)$ and $f(\theta, s_R) = f(\theta|s_R)P(s_R)$, the expected input from player j into i's payoff becomes

$$\begin{aligned}
h(b_i, b_j, k_j) &= -\int_0^1 \sum_{s_R \in \{0,1\}^{k_j+1}} (b_j + \mathbf{E}(\theta|s_R) - \theta - b_i)^2 f(\theta, s_R) d\theta \\
&= -\sum_{s_R \in \{0,1\}^{k_j+1}} \int_0^1 \Big((\mathbf{E}(\theta|s_R) - \theta)^2 + 2(b_j - b_i)(\mathbf{E}(\theta|s_R) - \theta) + (b_j - b_i)^2\Big) f(\theta|s_R)P(s_R) d\theta \\
&= -\sum_{s_R \in \{0,1\}^{k_j+1}} \left[\int_0^1 (\mathbf{E}(\theta|s_R) - \theta)^2 f(\theta|s_R) d\theta\right] P(s_R) - (b_j - b_i)^2 \\
&= -\mathbf{E}\left[\mathbf{Var}(\theta|s_R)\right] - (b_j - b_i)^2.
\end{aligned}$$

In what follows, we show that $\mathbf{E}\left[\mathbf{Var}(\theta|s_R)\right]$ is exactly $h(k_j)$. Let l denote the number of signals 1 in s_R, then

$$\begin{aligned}
\mathbf{Var}(\theta|s_R) &= \mathbf{E}(\theta^2|s_R) - (\mathbf{E}(\theta|s_R))^2 \\
&= \int_0^1 \theta^2 f(\theta|l,k) d\theta - \left(\frac{\alpha + l}{\alpha + \beta + k_j + 1}\right)^2 \\
&= \frac{B(\alpha + l + 2, \beta + k_j + 1 - l)}{B(\alpha + l, \beta + k_j + 1 - l)} - \left(\frac{\alpha + l}{\alpha + \beta + k_j + 1}\right)^2 \\
&= \frac{(\alpha + l)(\beta + k_j + 1 - l)}{(\alpha + \beta + k_j + 1)^2(\alpha + \beta + k_j + 2)}.
\end{aligned}$$

Using this, $\mathbf{E}\left[\mathbf{Var}(\theta|s_R)\right]$ becomes

$$\begin{aligned}
\mathbf{E}\left[\mathbf{Var}(\theta|s_R)\right] &= \sum_{s_R \in \{0,1\}^{k_j+1}} \frac{(\alpha + l)(\beta + k_j + 1 - l)}{(\alpha + \beta + k_j + 1)^2(\alpha + \beta + k_j + 2)} P(s_R) \\
&= \frac{1}{(\alpha + \beta + k_j + 1)^2(\alpha + \beta + k_j + 2)} A,
\end{aligned}$$

where

$$A = \alpha(\beta + k_j + 1) + (\beta + k_j + 1 - \alpha) \sum_{s_R \in \{0,1\}^{k_j+1}} lP(s_R) - \sum_{s_R \in \{0,1\}^{k_j+1}} l^2 P(s_R).$$

Here $\sum_{s_R \in \{0,1\}^{k_j+1}} lP(s_R) = (k_j + 1)\mathbf{E}(s_1)$, because l is the sum of signals in s_R and all signals are identically distributed. The unconditional expectation of each signal is

$$\mathbf{E}(s_1) = P(s_1 = 1) = \int_0^1 \theta f(\theta) d\theta = \frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} = \frac{\alpha}{\alpha+\beta},$$

hence $\sum_{s_R \in \{0,1\}^{(k_j+1)}} lP(s_R) = (k_j+1)\frac{\alpha}{\alpha+\beta}$

Note that signals s_j are not unconditionally independent: indeed, higher θ will mean higher signals on average. For example, if 9 signals out of 10 are equal to 1, then the probability that the 10-th signal is also 1 is higher compared to the case where the first 9 signals were 0s. However, the signals are *conditionally* independent binary variables with $P(s_j = 1|\theta) = \theta$, given θ. To use this fact, we rewrite $\sum_{s_R \in \{0,1\}^{(k_j+1)}} l^2 P(s_R)$ using the law of iterated expectations in the following way:

$$\sum_{s_R \in \{0,1\}^{(k_j+1)}} l^2 P(s_R) = \int_0^1 \left(\sum_{s_R \in \{0,1\}^{(k_j+1)}} l^2 P(s_R|\theta) \right) f(\theta) d\theta.$$

Since l is equal to the sum of signals in s_R, signals s_j are identically distributed and independent conditionally on θ, the term inside the integral can be rewritten as

$$\begin{aligned}
\sum_{s_R \in \{0,1\}^{(k_j+1)}} l^2 P(s_R|\theta) &= \mathbf{E}(l^2|\theta) = \mathbf{Var}(l|\theta) + (\mathbf{E}(l|\theta))^2 \\
&= (k_j+1)\mathbf{Var}(s_1|\theta) + (k_j+1)^2(\mathbf{E}(s_1|\theta))^2 \\
&= (k_j+1)\theta(1-\theta) + (k_j+1)^2\theta^2.
\end{aligned}$$

Taking the integral:

$$\begin{aligned}
\sum_{s_R \in \{0,1\}^{(k_j+1)}} l^2 P(s_R) &= \int_0^1 \Big((k_j+1)\theta(1-\theta) + (k_j+1)^2\theta^2 \Big) f(\theta) d\theta \\
&= \frac{(k_j+1)}{B(\alpha,\beta)} B(\alpha+1,\beta+1) + \frac{(k_j+1)^2}{B(\alpha,\beta)} B(\alpha+2,\beta) \\
&= \frac{(k_j+1)}{(\alpha+\beta+1)(\alpha+\beta)} [\alpha\beta + (k_j+1)\alpha(\alpha+1)].
\end{aligned}$$

Now $\mathbf{E}[\mathbf{Var}(\theta|s_R)]$ can be written as

$$\begin{aligned}
& \frac{A}{(\alpha+\beta+k_j+1)^2(\alpha+\beta+k_j+2)} \\
= \;& \frac{\alpha(\beta+(k_j+1)) + (\beta+(k_j+1)-\alpha)(k_j+1)\frac{\alpha}{\alpha+\beta} - \frac{(k_j+1)}{(\alpha+\beta+1)(\alpha+\beta)}[\alpha\beta + (k_j+1)\alpha(\alpha+1)]}{(\alpha+\beta+k_j+1)^2(\alpha+\beta+k_j+2)},
\end{aligned}$$

which after several algebraic transformations boils down to

$$\frac{\alpha\beta}{(\alpha+\beta+k_j+1)(\alpha+\beta+1)(\alpha+\beta)} = h(k_j)$$

Finally, the total impact $h(b_i, b_j, k_j)$ becomes

$$h(b_i, b_j, k_j) = -\frac{\alpha\beta}{(\alpha+\beta+k_j+1)(\alpha+\beta+1)(\alpha+\beta)} - (b_j - b_i)^2 = -h(k_j) - (b_j - b_i)^2,$$

which proves the result. **QED.**

Proposition 2. *Consider a triple $\{g, (\mu^g, y^g)\}$ and assume that each element of y^g satisfies the optimality condition (2). Then $\{g, (\mu^g, y^g)\}$ constitutes an equilibrium if and only if the communication network g is truthful and satisfies the following conditions: for any player j with an in-degree $k_j = k_j(g)$ and any player i,*

$$\begin{aligned} g_{ij} = s \quad &\textit{only if } |b_j - b_i| \leq \frac{1}{2(\alpha+\beta+k_j+1)}, \\ g_{ij} = h \quad &\textit{only if } \max\{\lambda, 1-\lambda\}C \leq h(k_j - 1) - h(k_j), \\ g_{ij} = 0 \quad &\textit{only if } \max\{\lambda, 1-\lambda\}C > h(k_j) - h(k_j + 1). \end{aligned}$$

Proof of Proposition 2. The necessary conditions for $g_{ij} = h$ and $g_{ij} = 0$ follow directly from the incentive condition to form/maintain a hard link (5). To derive the necessary condition for $g_{ij} = s$—the incentive compatibility constraint of truthful reporting through a soft link ij—consider player j and let s_R be the set of k_j signals that player j gets to know apart from player i. Specifically, $k_j - 1$ signals from his other communication neighbors $N_j^{-1}(g)/\{i\}$ and his own private signal s_j. Assuming that player j believes that i reports truthfully, let y_{s_R, s_i} be j's action if he has information s_R and i sends him the true signal, $m_{ij} = s_i$; $y_{s_R, 1-s_i}$ be j's action if he has information s_R and i misreports, $m_{ij} = 1 - s_i$. Player i reports truthfully his signal s_i to j if and only if it generates a greater interim expected payoff to i compared to misreporting:

$$-\int_0^1 \sum_{s_R \in \{0,1\}^{k_j}} \left[(y_{s_R, s_i} - \theta - b_i)^2 - (y_{s_R, 1-s_i} - \theta - b_i)^2 \right] f(\theta, s_R | s_i) d\theta \geq 0,$$

which can be rewritten as

$$-\int_0^1 \sum_{s_R \in \{0,1\}^{k_j}} \left[(y_{s_R, s_i} - y_{s_R, 1-s_i})(y_{s_R, s_i} + y_{s_R, 1-s_i} - 2\theta - 2b_i) \right] f(\theta, s_R | s_i) d\theta \geq 0.$$

Recalling that $y_{s_R, s_i} = b_j + \mathbf{E}(\theta | s_R, s_i)$, the condition becomes

$$\begin{aligned} -\int_0^1 \sum_{s_R \in \{0,1\}^{k_j}} &[(\mathbf{E}(\theta|s_R, s_i) - \mathbf{E}(\theta|s_R, 1 - s_i)) \\ &\times (\mathbf{E}(\theta|s_R, s_i) + \mathbf{E}(\theta|s_R, 1 - s_i) + 2b_j - 2\theta - 2b_i) \Big] f(\theta, s_R|s_i) d\theta \geq 0. \end{aligned}$$

Note that

$$f(\theta, s_R|s_i) = \frac{f(\theta, s_R, s_i)}{P(s_i)} = \frac{f(\theta, s_R, s_i)}{P(s_R, s_i)} \frac{P(s_R, s_i)}{P(s_i)} = f(\theta|s_R, s_i) P(s_R|s_i).$$

Let $\Delta = \mathbf{E}(\theta|s_R, s_i) - \mathbf{E}(\theta|s_R, 1-s_i)$ and change the sum and the integral signs

$$-\sum_{s_R \in \{0,1\}^{k_j}} \int_0^1 \left[\Delta(\mathbf{E}(\theta|s_R, s_i) + \mathbf{E}(\theta|s_R, 1-s_i) + 2b_j - 2\theta - 2b_i)\right] f(\theta|s_R, s_i) P(s_R|s_i) d\theta \geq 0.$$

Because $\mathbf{E}(\theta|s_R, s_i)$ and $\mathbf{E}(\theta|s_R, 1-s_i)$ are independent of θ,

$$-\sum_{s_R \in \{0,1\}^{k_j}} \left[\Delta\left(\mathbf{E}(\theta|s_R, s_i) + \mathbf{E}(\theta|s_R, 1-s_i) + 2b_j - 2\int_0^1 \theta f(\theta|s_R, s_i) d\theta - 2b_i\right)\right] P(s_R|s_i) \geq 0;$$

$$-\sum_{s_R \in \{0,1\}^{k_j}} \left[\Delta(\mathbf{E}(\theta|s_R, s_i) + \mathbf{E}(\theta|s_R, 1-s_i) + 2b_j - 2\mathbf{E}(\theta|s_R, s_i) - 2b_i)\right] P(s_R|s_i) \geq 0;$$

$$-\sum_{s_R \in \{0,1\}^{k_j}} \left[\Delta(-\Delta + 2b_j - 2b_i)\right] P(s_R|s_i) \geq 0.$$

If there are l signals 1 in s_R, then

$$\mathbf{E}(\theta|s_R, s_i) = \mathbf{E}(\theta|l + s_i, k_j + 1) = \frac{\alpha + l + s_i}{\alpha + \beta + k_j + 1},$$

$$\mathbf{E}(\theta|s_R, 1-s_i) = \mathbf{E}(\theta|l + 1 - s_i, k_j + 1) = \frac{\alpha + l + 1 - s_i}{\alpha + \beta + k_j + 1}.$$

Thus,

$$\Delta = \frac{\alpha + l + s_i}{\alpha + \beta + k_j + 1} - \frac{\alpha + l + 1 - s_i}{\alpha + \beta + k_j + 1} = \frac{2s_i - 1}{\alpha + \beta + k_j + 1},$$

which is independent on l, and hence on s_R. Using that $\sum_{s_R \in \{0,1\}^{k_j}} P(s_R|s_i) = 1$, the incentive condition becomes

$$-\frac{2s_i - 1}{\alpha + \beta + k_j + 1}\left(-\frac{2s_i - 1}{\alpha + \beta + k_j + 1} + 2(b_j - b_i)\right) \geq 0.$$

If $s_i = 1$, then player i is willing to communicate his signal if and only if

$$-\frac{1}{\alpha + \beta + k_j + 1}\left(-\frac{1}{\alpha + \beta + k_j + 1} + 2(b_j - b_i)\right) \geq 0;$$

$$b_j - b_i \leq \frac{1}{2(\alpha + \beta + k_j + 1)}.$$

If $s = 0$ then truth-telling is incentive compatible if and only if

$$-\frac{-1}{\alpha + \beta + k_j + 1}\left(-\frac{-1}{\alpha + \beta + k_j + 1} + 2(b_j - b_i)\right) \geq 0;$$

$$(b_j - b_i) \geq -\frac{1}{2(\alpha+\beta+k_j+1)}.$$

As a result,

$$|b_j - b_i| \leq \frac{1}{2(\alpha+\beta+k_j+1)},$$

which completes the proof of Proposition 2. **QED.**

Theorem 5. *Take any cost $C_0 > (h(0) - h(1))/\max\{\lambda, 1-\lambda\}$ and consider some pure cheap talk equilibrium $\left\{g(C_0), (\mu^{g(C_0)}, y^{g(C_0)})\right\}$. Let the in-degrees in the equilibrium network $g(C_0)$ be $k_j = k_j(g(C_0))$, $j \in N$. Then for any cost $C_1 \leq (h(0) - h(1))/\max\{\lambda, 1-\lambda\}$ there exists an equilibrium $\left\{g(C_1), (\mu^{g(C_1)}, y^{g(C_1)})\right\}$ in which the players have weakly greater in-degrees:*

$$k_j' = k_j(g(C_1)) \geq k_j \text{ for any } i \in N.$$

Proof of Theorem 5. Proof follows similar steps as in the proof of Theorem 1, replace $\Phi(\cdot)$ with $h(\cdot)$ that is defined in Lemma 5.
QED.

Lemma 6. *There exist a maximal and a pairwise stable equilibrium. Any pairwise stable equilibrium is maximal.*

Proof of Lemma 6. See the proof of Lemma 2.
QED.

Theorem 6. *Take any cost $C_0 > h(0) - h(1)$ and consider some pure cheap talk pairwise stable equilibrium with the communication network $g(C_0)$ and the total welfare $W(g(C_0))$. Then for any cost $C_1 \leq h(0) - h(1)$, there exists a pairwise stable equilibrium with the communication network $g(C_1)$ such that the total welfare $W(g(C_1), C_1) \geq W(g(C_0))$.*

Proof of Theorem 6. Proof follows similar steps as in the proof of Theorem 2, replace $\Phi(\cdot)$ with $h(\cdot)$ that is defined in Lemma 5.
QED.

Lemma 7. *Consider two cost levels, $C_0 \geq C_1$. For each C_i, fix some equilibrium and consider the corresponding communication network $g(C_i) = g^s(C_i) \cup g^h(C_i)$, where $g^s(C_i)$ is the set of soft links of $g(C_i)$ and $g^h(C_i)$ is the set of hard links of $g(C_i)$. If $g^s(C_0) \subseteq g^s(C_1)$ and $g^h(C_0) \subseteq g^h(C_1)$, then $W(g(C_1), C_1) \geq W(g(C_0), C_0)$, where $W(g(C_i), C_i)$ is the total welfare corresponding to the equilibrium network $g(C_i)$ and the cost C_i. Moreover, the same welfare implications hold on a per-individual basis.*

Proof of Lemma 7. See the proof of Lemma 4, replace $\Phi(\cdot)$ with $h(\cdot)$ that is defined in Lemma 5.
QED.

Theorem 7. *Take any cost $C_0 > \frac{h(0)-h(1)}{\max\{\lambda,1-\lambda\}}$ and consider some pure cheap talk pairwise stable equilibrium with the communication network $g(C_0)$ and the total welfare $W(g(C_0))$. Introduce the possibility* *to form hard links with the cost C_1 and consider some pairwise stable equilibrium with the communication network $g(C_1)$ and the total welfare $W(g(C_1),C_1)$. There exists a non-degenerate set $(\underline{C}_1,\overline{C}_1)$ of C_1 such that $W(g(C_1),C_1) < W(g(C_0))$ if and only if the preferential difference b satisfies*

$$b \in \left(\frac{1}{2(\alpha+\beta+n_1+k+1)}, \frac{1}{2(\alpha+\beta+n_1+k)} \right],$$

for some k, where $\max\{\lambda, 1-\lambda\}n-1 < k \le n_2-n_1-1$. *Otherwise, for all C_1: $W(g(C_1),C_1) \ge W(g(C_0))$.*

Proof of Theorem 7. The logic of the proof follows similar steps as in the proof of Theorem 3. However, non-trivial changes in the proof are necessary. The complete proof is as follows. Consider the setting in which only soft links are available. Condition $n_2 > n_1$ and the negative externality effect ensure that cross-group truthful communication in any pairwise stable equilibrium might be one of the following 3 kinds:

1. No cross-group communication, i.e. $k_{21} = k_{12} = 0$.

2. Communication from group N_2 to group N_1, i.e. $k_{12} > 0$, $k_{21} = 0$.

3. Cross-group communication, i.e. $k_{12} > 0$, $k_{21} > 0$.

In what follows, we examine each case separately and show that introducing hard links is welfare increasing in cases 1 and 3, and is welfare decreasing for some parameters in case 2.

Case 1 corresponds to

$$b > \max\left(\frac{1}{2(\alpha+\beta+n_1)}, \frac{1}{2(\alpha+\beta+n_2)} \right) = \frac{1}{2(\alpha+\beta+n_1)}.$$

If costly hard links become feasible, then intra-group soft-link communication remains unchanged. Regarding cross-group communication, if the cost C_1 is such that $\max\{\lambda,1-\lambda\}C_1 > h(n_1-1)-h(n_1)$, then cross-group communication remains empty and $W(g(C_1),C_1) = W(g(C_0))$. If $\max\{\lambda, 1-\lambda\}C_1 \le h(n_1-1)-h(n_1)$, then some cross-group communication via hard links appears. Because the set of links in this equilibrium includes the set of links from the cheap talk case (no crowding out occurs), by Lemma 4, the equilibrium with hard links generates a greater total (and individual) welfare, $W(g(C_1),C_1) \ge W(g(C_0))$.

Case 2 applies when the preference bias b satisfies

$$b \in \left(\frac{1}{2(\alpha+\beta+n_1+k_{12}+1)}, \frac{1}{2(\alpha+\beta+n_1+k_{12})} \right], \tag{C.2}$$

where $n_1 + k_{12} \le n_2$. Consider separately two possibilities: $n_1+k_{12} = n_2$ and $n_1+k_{12} < n_2$.

First, consider $n_1 + k_{12} = n_2$, which means that $k_1 = k_2 = n_2 - 1$. The total welfare in the pure cheap talk equilibrium is

$$W(g(C_0)) = -n(n_1h(k_1) + n_2h(k_2)) - B = -n^2h(n_2-1) - B,$$

where the term B depends only on preference bias b, $B = 2\sum_{i\in N_1}\sum_{j\in N_2} b^2 = 2n_1n_2b^2$.

If the cost C_1 is such that $\max\{\lambda, 1-\lambda\}C_1 > h(n_1-1+k_{12}) - h(n_1+k_{12})$, then the maximal in-degrees remain the same and each pairwise stable equilibrium involves only soft-link communication, meaning that $W(g(C_1),C_1) = W(g(C_0))$. Now let the cost be $\widehat{C}$ such that $\max\{\lambda, 1-\lambda\}\widehat{C} =$

$h(n_1-1+k_{12})-h(n_1+k_{12})=h(n_2-1)-h(n_2)$, which means that in a new pairwise stable equilibrium $k'_{21}=1$, $k'_{12}=k_{12}+1$, and hence, $k'_1=k'_2=n_2$.[31] Given the preference divergence, members of the opposite communities can not report to each other truthfully via soft links, thus, all cross-group soft links are substituted out by costly hard links leading to the following welfare:

$$\begin{aligned} W(g(\widehat{C}),\widehat{C}) &= -n(n_1h(k'_1)+n_2h(k'_2))-B-\frac{1}{\max\{\lambda,1-\lambda\}}(h(n_2-1)-h(n_2))[n_1k'_{12}+n_2] \\ &= -n^2h(n_2)-B-\frac{1}{\max\{\lambda,1-\lambda\}}(h(n_2-1)-h(n_2))[n_1(k_{12}+1)+n_2]. \end{aligned}$$

Thus, the difference between the levels of total welfare is

$$W(g(C_0))-W(g(\widehat{C}),\widehat{C})=(h(n_2-1)-h(n_2))\left[-n^2+\frac{1}{\max\{\lambda,1-\lambda\}}(n_1(k_{12}+1)+n_2)\right]\leq 0,$$

because

$$\begin{aligned} -n^2+\frac{1}{\max\{\alpha,1-\alpha\}}(n_1(k_{12}+1)+n_2) &\leq -n^2+2(n_1(k_{12}+1)+n_2) \\ &= -3n_1^2-n_2^2+2n_1+2n_2\leq 0, \end{aligned}$$

given that $1\leq n_1<n_2$. For any lower cost level $C_1<\frac{h(n_2-1)-h(n_2)}{\max\{\lambda,1-\lambda\}}=\widehat{C}$, cross-group communication is also carried out through only hard links. Compared to the considered case of $\widehat{C}$, no soft links are severed and w.l.o.g. it can be assumed that all hard links are retained and some new hard links are added. Then Lemma 7 implies that the total welfare increases: $W(t(C_1),C_1)\geq W(g(\widehat{C}),\widehat{C})\geq W(g(C_0))$. As a result, any pairwise stable equilibrium with hard links generates a greater total welfare than the pure cheap talk equilibrium.

Second, consider $n_1+k_{12}+1\leq n_2$. In this case $k_1=n_1-1+k_{12}$, $k_2=n_2-1$ and the total welfare in the pure cheap talk case is

$$W(g(C_0))=-n(n_1h(n_1-1+k_{12})+n_2h(n_2-1))-B.$$

If the cost $C_1>\frac{1}{\max\{\lambda,1-\lambda\}}(h(n_1-1+k_{12})-h(n_1+k_{12}))$, then the maximal in-degrees remain the same and the pairwise stable equilibria are pure soft-link, hence, $W(g(C_1),C_1)=W(g(C_0))$. Let the cost be $\widehat{C}=\frac{1}{\max\{\lambda,1-\lambda\}}(h(n_1-1+k_{12})-h(n_1+k_{12}))$. Then cross-group communication becomes $k'_{12}=k_{12}+1$, while k'_{21} remains 0; all cross-group links are hard. The new maximal in-degrees are $k'_1=n_1+k_{12}$ and $k'_2=k_2=n_2-1$, implying the following total welfare

$$\begin{aligned} W(g(\widehat{C}),\widehat{C})= &- n(n_1h(n_1+k_{12})+n_2h(n_2-1))-B \\ &- \frac{1}{\max\{\lambda,1-\lambda\}}(h(n_1-1+k_{12})-h(n_1+k_{12}))[n_1(k_{12}+1)]. \end{aligned}$$

The difference in the levels of the welfare is then

$$W(g(C_0))-W(g(\widehat{C}),\widehat{C})=n_1(h(n_1-1+k_{12})-h(n_1+k_{12}))\left[-n+\frac{1}{\max\{\lambda,1-\lambda\}}(k_{12}+1)\right],$$

[31] Recall the assumption that whenever the player is indifferent, the choice is made in favor of a hard link creation.

which is strictly greater than 0 if and only if $k_{12} > \max\{\lambda, 1-\lambda\}n-1$. Since the inequality is strict, there exists $\underline{C}_1 < \widehat{C}$ such that the welfare difference remains strictly positive for $C_1 \in (\underline{C}_1, \overline{C}_1)$, where $\overline{C}_1 = \widehat{C}$. Thus, given the preference divergence (C.2), condition $\max\{\alpha, 1-\alpha\}n-1 < k_{12} \leq n_2 - n_1 - 1$ is sufficient for existence of the cost that leads to a lower welfare. The necessity follows from the fact that for any $C_1 < \widehat{C}$ Lemma 7 implies that $W(g(C_1), C_1) \geq W(g(\widehat{C}), \widehat{C})$.

Finally, **Case 3** corresponds to a sufficiently small preference divergence:

$$b \leq \frac{1}{2(\alpha+\beta+n_2+1)}.$$

Non-zero cross-group communication in both directions implies that the maximal in-degrees are the same for members of both communities, $k_1 = k_2$. If the truthful cheap talk network $g(C_0)$ is complete ($k_1 = k_2 = n-1$), which corresponds to $b \leq \frac{1}{2(\alpha+\beta+n)}$, then introducing feasible hard links does not alter this pairwise stable equilibrium and $W(g(C_1), C_1) = W(g(C_0))$.

Case where $g(C_0)$ is not complete ($k_1 = k_2 < n-1$) corresponds to the preference divergence

$$b \in \left(\frac{1}{2(\alpha+\beta+n_2+k_{21}+1)}, \frac{1}{2(\alpha+\beta+n_2+k_{21})}\right],$$

where $k_{21} < n_1$. The total welfare of the pure cheap talk pairwise stable equilibrium is

$$W(g(C_0)) = -n^2 h(k_1).$$

If the cost $C_1 > \frac{1}{\max\{\lambda,1-\lambda\}}(h(k_1) - h(k_1+1))$, then the maximal in-degrees are the same as in the cheap talk case and any pairwise stable equilibrium involves only soft communication, meaning that $W(g(C_1), C_1) = W(g(C_0))$. Consider

$$\widehat{C} = \frac{1}{\max\{\lambda, 1-\lambda\}}(h(k_1) - h(k_1+1)).$$

The new maximal in-degrees become $k_1' = k_2' = k_1 + 1$ and all cross-group communication in any pairwise stable equilibrium is performed via hard links, such that each member of community N_1 gets $k_{12}+1$ hard links from members of N_2 and, similarly, each member of N_2 gets $k_{21}+1$ hard links from members of N_1. The total welfare is

$$W(g(\widehat{C}), \widehat{C}) = -n^2 h(k_1+1) - \frac{1}{\max\{\lambda, 1-\lambda\}}(h(k_1) - h(k_1+1))(n_1(k_{12}+1) + n_2(k_{21}+1)).$$

The difference between the levels of the welfare is

$$\begin{aligned} & W(g(C_0)) - W(g(\widehat{C}), \widehat{C}) \\ = \ & (h(k_1) - h(k_1+1))\left[-n^2 + \frac{1}{\max\{\lambda, 1-\lambda\}}(n_1(k_{12}+1) + n_2(k_{21}+1))\right] \leq 0, \end{aligned}$$

because

$$\begin{aligned} & -n^2 + \frac{1}{\max\{\lambda, 1-\lambda\}}(n_1(k_{12}+1) + n_2(k_{21}+1)) \leq -n^2 + 2n_1(k_{12}+1) + 2n_2(k_{21}+1) \\ \leq \ & -n^2 + 2(k_1+2)n - n^2 = -2n(n-(k_1+2)) \leq 0, \end{aligned}$$

given that $k_1 < n-1$. For any lower level of the cost $C_1 < \widehat{C}$ cross-group communication is performed through only hard links as well. Since, compared to the case of $\widehat{C}$, no cheap talk links are severed and w.l.o.g. it can be assumed that all hard links are retained and some new hard links are added, Lemma 7 implies that $W(g(C_1), C_1) \geq W(g(\widehat{C}), \widehat{C}) \geq W(g(C_0))$. **QED.**

Theorem 8. *Let $C_0 > \frac{h(0)-h(1)}{\max\{\lambda, 1-\lambda\}}$ and consider some pure cheap talk pairwise stable equilibrium with the communication network $g(C_0)$ and the total welfare $W(g(C_0))$. Introduce feasible hard links with the cost of $C_1 \leq \frac{h(0)-h(1)}{\max\{\lambda, 1-\lambda\}}$ and consider a pairwise stable equilibrium with the communication network $g(C_1)$ that generates the greatest total welfare $W(g(C_1), C_1)$. There exists a non-degenerate set $(\underline{C}_1, \overline{C}_1)$ of C_1 such that $W(g(C_1), C_1) < W(g(C_0))$ if and only if $n = 3$, $b \in \left(\frac{1}{10}, \frac{1}{8}\right]$ and $\lambda \in (\frac{1}{3}, \frac{2}{3})$. Otherwise, for all C_1: $W(g(C_1), C_1) \geq W(g(C_0))$.*

Proof of Theorem 8. The logic of the proof follows similar steps as in the proof of Theorem 4. However, there are non-trivial changes in the proof that are necessary. The complete proof is as follows. The case of $n = 3$ was already analyzed in the main body of the paper; it remains to show the positive welfare result for $n \geq 4$. Consider prohibitively costly hard links, $\max\{\lambda, 1-\lambda\}C_0 > h(0) - h(1)$, and construct a pairwise stable equilibrium that generates the greatest welfare in a way described in Lemma 3. In the corresponding truthful network, player i gets truthful messages from k_i closest players. Following the construction in the proof of Theorem 4, the communication network $g(C_0)$ can be formally described as follows: if $b > \frac{1}{2(\alpha+\beta+2)}$ then $g(C_0)$ is empty, otherwise, let $V(b) = \max\{V \in \{1, ..., n\} : b \leq \frac{1}{2V(2V-1+1+\alpha+\beta)}\}$, then

1. For every $j \in \{V(b)+1, ..., n-V(b)\}$, $g_{ij} = s$ if $|i-j| < V(b)$ and $g_{ij} = 0$ if $|i-j| > V(b)$; if $b > \frac{1}{2V(b)(2V(b)+1+\alpha+\beta)}$, then $g_{ij} = s$ for one and only one player i such that $|i-j| = V(b)$; if $b \leq \frac{1}{2V(b)(2V(b)+1+\alpha+\beta)}$, then $g_{ij} = s$ for both players i such that $|i-j| = V(b)$.

2. For all players $j \in \{1, ..., V(b)\} \cup \{n-V(b)+1, ..., n\}$, $g_{ij} = s$ if and only if $|i-j| \leq M(j, b)$, where $M(j, b) = \max\{M \in \{1, ..., n\} : b \leq \frac{1}{2M(\min\{j-1, n-j\}+M+1+\alpha+\beta)}\}$.

In the pure cheap talk equilibrium, the set of maximal in-degrees $K = \{k_1, ..., k_n\}$ is the following: $k_j = 0$, $j \in N$, if $b > \frac{1}{2(\alpha+\beta+2)}$; otherwise, for every $i \in \{V(b)+1, ..., n-V(b)\}$,

$$k_i = \begin{cases} 2V(b) - 1, \text{ if } b > \frac{1}{2V(b)(2V(b)+1+\alpha+\beta)} \\ 2V(b), \text{ if } b \leq \frac{1}{2V(b)(2V(b)+1+\alpha+\beta)}, \end{cases}$$

and for each $j \in \{1, ..., V(b)\} \cup \{n-V(b)+1, ..., n\}$, $k_j = \min\{j-1, n-j\} + M(j, b)$. Given the set of maximal in-degrees K, define

$$\begin{aligned} k_{(n)} &= \max\{k_i \in K\}, \\ k_{(j)} &= \max\{k_i \in K / \{k_{(n)}, ..., k_{(j-1)}\}\}, \end{aligned}$$

i.e., $k_{(1)} \leq ... \leq k_{(n)}$ is a reordering of K in the increasing order. It can be easily seen, that players with moderate preferences, $\{V(b)+1, ..., n-V(b)\}$, have the highest in-degree $k_{(n)}$. The in-degrees of other players decrease as their preference biases get closer to the extremes, such that player 1 (with bias 0) and player n (with bias $(n-1)b$) have the same in-degree of $k_{(1)}$. Since $M(j, b) \in \{M(j+1, b), M(j+1, b)+1\}$ for $j \in \{1, ..., V(b)\}$ (similarly, $M(j+1, b) \in \{M(j, b), M(j, b)+1\}$ for

$j \in \{n-V(b)+1,...,n\}$) and $M(V(b),b) = M(n-V(b),b) = V(b)$, the structure of a pairwise stable equilibrium ensures that for every $i = 1,...,n-1$ the difference $k_{(i+1)} - k_{(i)}$ is either 0 or 1.

Now introduce hard links with the cost $C_1 \leq \frac{h(0)-h(1)}{\max\{\lambda,1-\lambda\}}$. If the communication network of the pairwise stable pure cheap talk equilibrium is empty ($b > \frac{1}{2(1+1+\alpha+\beta)}$), then there is no crowding out when hard links become available, and Lemma 4 implies that the welfare increases. Consider now the case where the pure cheap talk communication network is not empty, i.e., $k_{(n)} > 0$, and study three possibilities for C_1 separately

1. $\max\{\lambda, 1-\lambda\}C_1 \in (h(k_{(1)}) - h(k_{(1)}+1), h(0) - h(1)]$.
2. $\max\{\lambda, 1-\lambda\}C_1 \in (h(k+1) - h(k+2), h(k) - h(k+1)]$, for some $k_{(1)} \leq k < k_{(n)}$.
3. $\max\{\lambda, 1-\lambda\}C_1 \in (0, h(k_{(n)}) - h(k_{(n)}+1)]$.

Case 1. For the cost higher than $\frac{h(k_{(1)})-h(k_{(1)}+1)}{\max\{\lambda,1-\lambda\}}$, the maximal in-degrees remain the same and the pure cheap talk equilibrium remains pairwise stable, meaning that w.l.o.g. $g(C_1) = g(C_0)$ and $W(g(C_1), C_1) = W(g(C_0))$.

Case 2. Because the difference $k_{(i+1)} - k_{(i)}$ is either 0 or 1, there must exist i such that $k_{(i)} = k$ and $k_{(i+1)} = k+1$. The structure of the pure cheap talk equilibrium implies that i is an even number less or equal to $2V(b)$. For any C_1 that satisfies $\max\{\alpha, 1-\alpha\}C_1 \in (h(k+1) - h(k+2), h(k) - h(k+1)]$, the maximal in-degrees are

$$k'_j = \begin{cases} k+1, & \text{if } k_j \leq k, \\ k_j, & \text{if } k_j > k. \end{cases}$$

Consider a pairwise stable equilibrium that generates the greatest welfare. It must be the case that in the corresponding communication network $g(C_1)$ players $\{j : k_j > k\}$ receive messages via only soft links, in particular, assume that they receive truthful messages from the same players as in the pure cheap talk equilibrium. Other players $\{j : k_j \leq k = k_{(i)}\} = \{1,..., \frac{i}{2}\} \cup \{n - \frac{i}{2},...,n\}$ have the new in-degrees equal to $k+1$ and can receive truthful messages through both, soft and hard links. If $j \in \{1,..., \frac{i}{2}\}$, then the number of soft links directed to player j in $g(C_1)$ is at least $j-1+V(b)$. This implies that the upper bound on the number of costly hard links directed to j is $k+1-(j-1+V(b))$. Similarly, if $j \in \{n-\frac{i}{2},...,n\}$, then the number of hard links directed to player j is less or equal than $k+1-(n-j+V(b))$. Thus, the upper bound for the total cost of hard links is

$$\begin{aligned}
& 2 \cdot \frac{h(k)-h(k+1)}{\max\{\alpha, 1-\alpha\}} \sum_{j=1}^{\frac{i}{2}} [k+1-(j-1+V(\beta))] \\
= \quad & 2\frac{h(k)-h(k+1)}{\max\{\alpha, 1-\alpha\}} \left[\frac{i}{2}(k+2-V(\beta)) - \sum_{j=1}^{\frac{i}{2}} j \right] \\
= \quad & 2\frac{h(k)-h(k+1)}{\max\{\alpha, 1-\alpha\}} \left[\frac{i}{2}(k+2-V(\beta)) - \frac{\frac{i}{2}+1}{2}\frac{i}{2} \right] \\
= \quad & i\frac{h(k)-h(k+1)}{\max\{\alpha, 1-\alpha\}} \left[k+2-V(\beta) - \frac{i+2}{4} \right].
\end{aligned}$$

The additional welfare is

$$2n\sum_{j=1}^{\frac{i}{2}}\Big[h(k_j)-h(k+1)\Big] \geq n \cdot i(h(k)-h(k+1)).$$

The upper bound for the cost is less than the lower bound for the additional benefit, because

$$n \geq 2\left[k+2-V(\beta)-\frac{i+2}{4}\right] \geq \frac{1}{\max\{\lambda,1-\lambda\}}\left[k+2-V(\beta)-\frac{i+2}{4}\right], \tag{C.3}$$

where the first inequality is satisfied due to $\frac{n}{2}+V(\beta) \geq k_{(n)} \geq k+1$ and $1-\frac{i+2}{4} \leq 0$. As a result, $W(g(C_1),C_1) \geq W(g(C_0))$.

Case 3. If $\max\{\lambda,1-\lambda\}C_1 \in (0,h(k_{(n)})-h(k_{(n)}+1)]$, then there is $k \geq k_{(n)}$ such that $\max\{\lambda,1-\lambda\}C_1 \in (h(k+1)-h(k+2),h(k)-h(k+1)]$. There are several possibilities to consider:

1. If $k_{(n)} = n-1$, then $g(C_0)$ is either complete, or not. In case it is complete, the pure cheap talk equilibrium remains pairwise stable once hard links become feasible, thus, $W(g(C_1),C_1) = W(g(C_0))$. If $g(C_0)$ is incomplete, then there exists i: $1 \leq i < N$ such that

   $$k_{(1)} \leq \ldots \leq k_{(i)} < n-1 = k_{(i+1)} = \ldots = k_{(n)}.$$

   Consider a pairwise stable equilibrium that generates the greatest welfare when hard links are available. Condition $\max\{\lambda,1-\lambda\}C_1 \leq h(n-1)-h(n)$ insures that in the corresponding communication network $g(C_1)$ each player $j \in N$ has the in-degree $k'_j = n-1$. Note that for the greater cost $C_2 = \frac{h(n-2)-h(n-1)}{\max\{\lambda,1-\lambda\}}$, a pairwise stable equilibrium that generates the greatest welfare has the same communication network, $g(C_2) = g(C_1)$. Since C_2 satisfies the conditions of case 2, then $W(g(C_1),C_1) > W(g(C_2),C_2) \geq W(g(C_0))$.

2. Consider $k_{(n)} \leq n-2$. Note that if the positive welfare result $W(g(C_1),C_1) \geq W(g(C_0))$ holds for $k = n-2$, then by Lemma 4 it also holds for any $k > n-2$. Thus, for the rest of the proof assume that $k \leq n-2$. The in-degrees in the communication network of any equilibrium when hard links are available become $k'_j = k+1$, $j \in N$. The total cost of hard links in the corresponding $g(C_1)$ does not exceed $n(k+1)\frac{h(k)-h(k+1)}{\max\{\lambda,1-\lambda\}}$, while the gain in the welfare compared to the cheap talk case is at least $n \cdot n(h(k)-h(k+1))$. The lower bound for the gain is greater than the upper bound for the cost if $k \leq n\max\{\lambda,1-\lambda\}-1$, meaning that for such C_1 the welfare in *any* equilibrium with hard links exceeds $W(g(C_0))$, in particular, $W(g(C_1),C_1) \geq W(g(C_0))$.

   Consider now $n-2 \geq k > n\max\{\lambda,1-\lambda\}-1$ and analyze two possibilities:

   (a) If $k > k_{(n)}$, then the lower bound for the additional expected total benefit is

   $$\begin{aligned} n^2(h(k-1)-h(k+1)) &= n^2(h(k-1)-h(k)+h(k)-h(k+1)) \\ &\geq 2n^2(h(k)-h(k+1)), \end{aligned} \tag{C.4}$$

   which exceeds the upper bound for the cost $n(k+1)\frac{h(k)-h(k+1)}{\max\{\lambda,1-\lambda\}}$, because

   $$\frac{k+1}{\max\{\lambda,1-\lambda\}} \leq 2(k+1) < 2n.$$

This means, that *any* equilibrium with hard links outperforms the cheap talk equilibrium in terms of welfare, hence, $W(g(C_1),C_1) \geq W(g(C_0))$.

(b) If $k = k_{(n)}$, then in any equilibrium with hard links each player j has the in-degree of $k'_j = k+1 > n\max\{\lambda, 1-\lambda\} \geq \frac{n}{2}$, $j \in N$. Take a pairwise stable equilibrium that generates the greatest welfare, and consider how many links in the corresponding communication network $g(C_1)$ can be soft. Player i with the bias b_i such that $|b_i - b_j| = lb$, will report truthfully via cheap talk to player j if

$$b \leq \frac{1}{2l(k+2+\alpha+\beta)}.$$

Note that this inequality is satisfied for $l = V(b)-1$, because by the definition of $V(b)$ and the fact that $k = k_{(n)} \leq 2V(b)$,

$$\frac{1}{2l(k+2+\alpha+\beta)} > \frac{1}{2V(b)(2V(b)+\alpha+\beta)} \geq b.$$

Thus, assuming $l = V(b)-1$, each player $j \in \{1,...,l\}$ gets truthful messages through soft links from $j-1+l$ players $1,...,j-1,j+1,...,j+l$. Similarly, each player $j \in \{n-l,...,n\}$ gets at least $n-j+l$ truthful messages through soft links. Finally, each player $j \in \{l+1,...,n-l-1\}$ gets at least $2l$ truthful messages via cheap talk. Thus, the number of hard links in $g(C_1)$ is bounded from above by

$$\begin{aligned}
&n(k+1) - 2\sum_{j=1}^{l}(j-1+l) - 2l(n-2l) = n(k+1) - 2\frac{l+1}{2}l - 2l(l-1) - 2l(n-2l) \\
&= n(k+1) - l^2 - l - 2l^2 + 2l - 2ln + 4l^2 \\
&= n(k+1) + l^2 + l - 2ln,
\end{aligned}$$

meaning that the total cost does not exceed $\frac{h(k)-h(k+1)}{\max\{\lambda,1-\lambda\}}(n(k+1)+l^2+l-2ln)$. Since $k = k_{(n)} \leq 2V(b)$ and $l = V(b)-1$, then $k+1 \leq 2(l+1)$ and the upper bound for the cost becomes

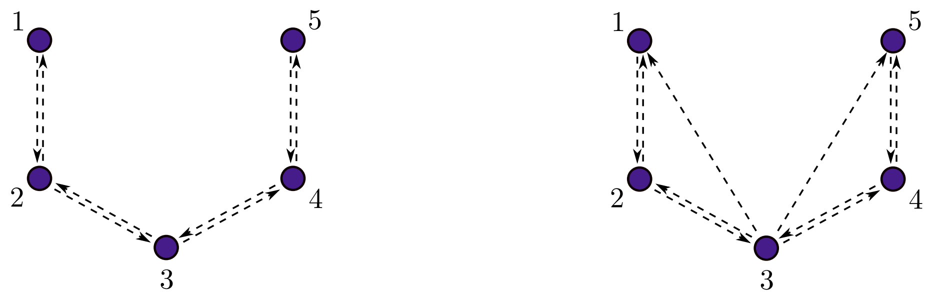
$$\frac{h(k)-h(k+1)}{\max\{\lambda,1-\lambda\}}(2n(l+1)+l^2+l-2ln) = \frac{h(k)-h(k+1)}{\max\{\lambda,1-\lambda\}}(2n+l^2+l).$$

Note that $\frac{1}{\max\{\lambda,1-\lambda\}} \leq 2$ and $l \leq \frac{n}{2}-1$, because $2l < 2V(b)-1 \leq k \leq n-2$, which allows to write the upper bound for the cost as:

$$2(h(k)-h(k+1))\left(2n + \left(\frac{n}{2}-1\right)^2 + \frac{n}{2} - 1\right) = (h(k)-h(k+1))\left(\frac{n^2}{2}+3n\right).$$

The additional welfare is at least $n^2(h(k)-h(k+1))$. The lower bound for the extra welfare is greater than the upper bound for the cost if $n \geq 6$. Thus, $W(g(C_1),C_1) \geq W(g(C_0))$ for $n \geq 6$.

It remains to show that $W(g(C_1),C_1) \geq W(g(C_0))$ for $n=4$ and $n=5$ as well, assuming that $n-2 \geq k = k_{(n)} > n\max\{\lambda,1-\lambda\}-1 \geq \frac{n}{2}-1$ and $\max\{\lambda,1-\lambda\}C_1 \in (h(k+1)-h(k+2), h(k)-h(k+1)]$. Consider, first, $n=5$ and $k=2,3$. Depending on b, case $k = k_{(n)} = 2$ corresponds to pairwise



(i) $k_1 = k_5 = 1, k_2 = k_3 = k_4 = 2$ (ii) $k_1 = k_2 = k_3 = k_4 = k_5 = 2$

Figure 9: Communication networks of pairwise stable pure cheap talk equilibria when $n = 5$ and $k = 2$.

stable pure cheap talk equilibria with the following in-degrees (see Figure 9):

(i) If $\frac{1}{4(3+\alpha+\beta)} < b \leq \frac{1}{2(3+\alpha+\beta)}$, then $k_1 = k_5 = 1$, $k_2 = k_3 = k_4 = 2$.

(ii) If $\frac{1}{4(4+\alpha+\beta)} < b \leq \frac{1}{4(3+\alpha+\beta)}$, then $k_1 = k_2 = k_3 = k_4 = k_5 = 2$.

When hard links become available with

$$\max\{\lambda, 1-\lambda\}C_1 \in (h(3) - h(4), h(2) - h(3)],$$

the maximal in-degrees become $k'_1 = ... = k'_5 = 3$. In case (i), the upper bound for the total cost of hard links is $5 \cdot 3 \cdot 2(h(2) - h(3))$, which is lower than the gain in the total welfare $5 \cdot [3(h(2) - h(3)) + 2(h(1) - h(3))]$, because $h(2) - h(3) < 2(h(1) - h(2))$. In case (ii), soft links between the players with adjacent biases can be a part of the communication network of a pairwise stable equilibrium. Thus, the total cost of hard links in $g(C_1)$ does not exceed $7 \cdot 2(h(2) - h(3))$, which, in turn, is lower than the additional total welfare, $5 \cdot 5(h(2) - h(3))$.

Case $k = k_{(n)} = 3$ corresponds to $\frac{1}{4(5+\alpha+\beta)} < b \leq \frac{1}{4(4+\alpha+\beta)}$ and maximal in-degrees $k_1 = k_5 = 2$, $k_2 = k_3 = k_4 = 3$ in the pure cheap talk setting (see Figure 10 for an example of $g(C_0)$). When hard links become available with

$$\max\{\lambda, 1-\lambda\}C_1 \in (h(4) - h(5), h(3) - h(4)],$$

the maximal in-degrees become $k'_1 = ... = k'_5 = 4$. Note that soft links between the players with adjacent biases can be a part of the communication network of a pairwise stable equilibrium, implying that the total cost of hard links in $g(C_1)$ has an upper bound of $12 \cdot 2(h(3) - h(4))$, which is lower than the gain in the total welfare, $5 \cdot [3(h(3) - h(4)) + 2(h(2) - h(4))]$. Hence, $W(g(C_1), C_1) \geq W(g(C_0))$, when $n = 5$.

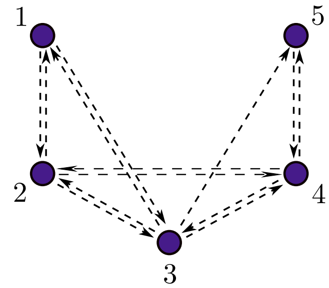
Consider now $n = 4$ and $k = 2$. Depending on b, $g(C_0)$ can have two different structures with the following in-degrees (see Figure 11):

(i) If $\frac{1}{4(3+\alpha+\beta)} < b \leq \frac{1}{2(3+\alpha+\beta)}$, then $k_1 = k_4 = 1$, $k_2 = k_3 = 2$.

(ii) If $\frac{1}{4(4+\alpha+\beta)} < b \leq \frac{1}{4(3+\alpha+\beta)}$, then $k_1 = k_2 = k_3 = k_4 = 2$.

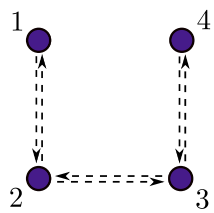
When hard links become available with

$$\max\{\lambda, 1-\lambda\}C_1 \in (h(3) - h(4), h(2) - h(3)],$$

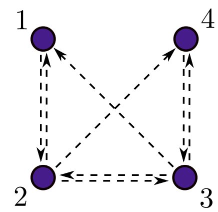


$k_1 = k_5 = 2,\ k_2 = k_3 = k_4 = 3$

Figure 10: Communication network of pairwise stable pure cheap talk equilibrium when $n = 5$ and $k = 3$.



(i) $k_1 = k_4 = 1, k_2 = k_3 = 2$



(ii) $k_1 = k_2 = k_3 = k_4 = 2$

Figure 11: Communication network of pure cheap talk equilibrium when $n = 4$ and $k = 2$.

the maximal in-degrees become $k'_1 = ... = k'_4 = 3$. In case (i), the total cost of hard links has an upper bound of $3 \cdot 4 \cdot 2(h(2) - h(3))$, which is lower than the gain in the total welfare $4 \cdot [2(h(2) - h(3)) + 2(h(1) - h(3))]$, because $h(2) - h(3) < h(1) - h(2)$. In case (ii), soft links between the players with adjacent biases can be a part of the communication network of a pairwise stable equilibrium. Thus, the total cost of hard links in $g(C_1)$ is below $6 \cdot 2(h(2) - h(3))$, which, in turn, is lower than the additional total welfare, $4 \cdot 4(h(2) - h(3))$. As a result, $W(g(C_1), C_1) \geq W(g(C_0))$, when $n = 4$. **QED.**