The Role of Managerial Protections on Bank Capital Requirements*

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Abstract

Can tightening capital requirements lead to greater lending? We introduce a model to illustrate how the effect of capital requirements on bank lending can qualitatively depend on the extent of managerial protections against shareholder actions. Protections encourage managers to pursue unprofitable projects. While managers can still be disciplined by debt, if a financial institution's debt is constrained by capital requirements, then a higher level of investment can serve as a partial substitute. Capital requirements can therefore spur increased investment for banks with managerial protections. Empirically, we study banks facing stress-tests following the great recession and find evidence consistent with our model, that there was an increase in lending for banks with strong protections compared to banks with weak protections. These results provide new insights into the central role that corporate governance and incentives of bank managers have on the effect of bank capital requirements.

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JEL Classification: Managerial protections, Banking, Corporate governance, Stress testing, Capital structure, Ownership structure, Investment.

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Can tightening capital requirements lead to greater lending? Bank regulations that are intended to improve financial stability, such as capital requirements, can have important side effects on lending. Theoretically, there are numerous conflicting channels by which bank capital can affect lending (Bahaj and Malherbe (2020); Bahaj et al. (2016); Admati et al. (2013); Diamond and Rajan (2000)).[1] Empirically, recent studies have analyzed this relationship in the context of the introduction of the bank stress tests after the 2008 financial crisis, which operate like forward-looking capital requirements (Cortés et al. (2020); Acharya et al. (2018); Bassett and Berrospide (2018)).[2] However, little is known about how capital requirements interact with managerial protections against shareholders, which refers to anti-takeover laws, restricted shareholder rights, and other laws and provisions that could enable a manager to obtain private benefits from controlling a firm.

This interaction is important because managerial protections have been shown to affect investment more generally through various channels.[3] As empirical motivation, Figure 1 shows that, among bank holding companies that were subject to the stress tests conducted by the Federal Reserve, those with relatively strong managerial protections exhibited a relative increase in lending compared to those with weak protections. This difference in lending underlines the importance of investigating the channels by which managerial protections affect bank decisions.

This paper shows that the effect of capital requirements on bank lending can qualitatively depend on the degree of managerial protections. We illustrate this result using a model in which a firm chooses its value-maximizing capital structure and level of investment, which can be interpreted as loans in the context of banks, while considering the incentives of managers and regulatory capital requirements. The firm's manager then learns about the quality of the investment project during development and chooses whether to terminate or complete it. Managerial protections provide an incentive for managers to complete unprofitable projects to obtain a private benefit from controlling

[1] On the one hand, the safety from a large capital buffer can lead banks to decrease lending because they have more severe financing frictions (Diamond and Rajan (2000)) or higher charter values that motivate them to lend more cautiously (Keeley (1990)). On the other hand, a greater capital stock can increase the capacity to make loans by improving monitoring (Holmstrom and Tirole (1997)) or by mitigating debt overhang (Admati, DeMarzo, Hellwig, and Pfleiderer (2013)). Others have studied this through lending prospects and the quality of legacy assets (Bahaj, Bridges, Malherbe, and O'Neill (2016)). In a recent paper, Bahaj and Malherbe (2020) studies capital requirements and the forced safety effect.

[2] Some of these studies have shown that the stress tests were negatively associated with lending (Acharya, Berger, and Roman (2018) and Cortés, Demyanyk, Li, Loutskina, and Strahan (2020)), while others have found weaker or contrary effects (Bassett and Berrospide (2018)).

[3] On the one hand, they have been associated with greater levels of investment (Gompers, Ishii, and Metrick (2003)), possibly by providing greater freedom for managers to maintain "empires". On the other hand, Bertrand and Mullainathan (2003) show that managerial protections can lead to reductions in potentially efficient investments, suggesting that they allow managers to enjoy a "quiet life".

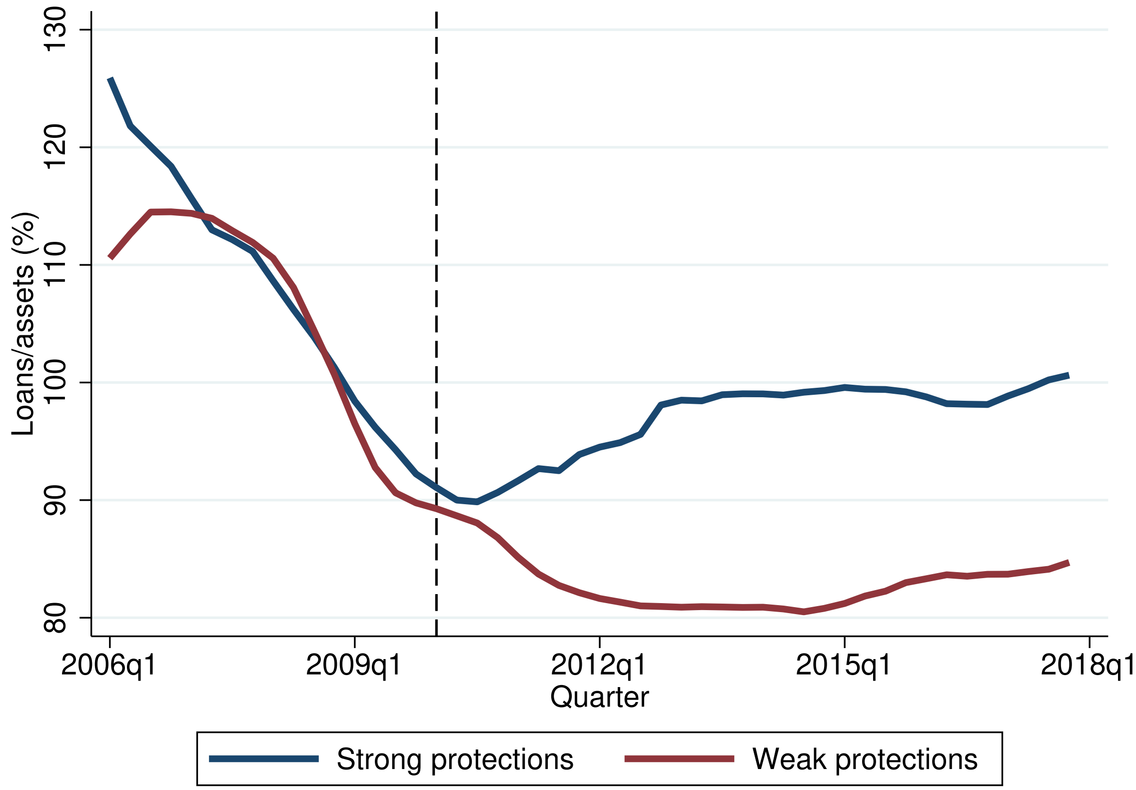

Figure 1: Lending for banks with strong vs. weak managerial protections

This plot shows the mean ratio of loans and unused commitments to assets for a balanced sample of bank holding companies (BHCs) that were subject to the stress tests conducted by the Federal Reserve (the Supervisory Capital Assessment Program in 2009 and annual rounds of the Comprehensive Capital Analysis and Review starting in 2011), split by the strength of managerial protections. The series have been smoothed using a moving average to reduce seasonal fluctuations. Managerial protections are measured using the G-index from Gompers, Ishii, and Metrick (2003). Note that we show that this is robust to alternative governance indices in Section 3.3. BHCs with a G-index greater than the median among this subset are designated as having strong managerial protections, and BHCs with a G-index less than the median are designated as having weak managerial protections. The dashed line indicates the approximate start of the US bank stress tests. See Section 2.2 for more details about the sample construction.

the firm.

To illustrate the mechanism by which managerial protections influence the effect of capital requirements on investment, we first characterize the firm's optimal capital structure in the absence of capital requirements. In particular, if a firm has strong managerial protections, then shareholders have a limited ability to directly prevent excessive project completion by managers. However, protected managers can still be disciplined with debt. The firm's optimal level of debt balances this disciplining effect against the expected losses from liquidation costs if the firm defaults.

Capital requirements that force banks to issue a suboptimally low level of debt affect investment through two channels. On the one hand, distortions to the firm's capital structure reduce the return on investment, which discourages investment. On the other, for a firm with strong managerial protections against shareholder oversight, capital requirements can undermine the role of debt in disciplining the manager. At the same time, under the canonical assumptions of a linear liquidation cost and a concave expected return, the manager's incentive to liquidate unprofitable projects increases with the level of investment. As a result, the value-maximizing level of investment can increase in response to tightening capital requirements, since it partially substitutes for debt in disciplining the manager. The dominant channel determining the firm's investment response to capital requirements depends on other firm characteristics, such as the earnings distribution.

An extended version of the model that allows managers to choose the firm's capital structure and investment has other features consistent with facts from the corporate governance literature. First, the model illustrates channels by which protections can cause managers to issue either too much or too little debt. In particular, if managerial protections and voting power associated with equity share are complementary sources of managerial power, then protections can intensify the incentive for managers to increase their voting power by financing the firm with debt rather than issuing shares to outside equityholders. They can also lead managers to issue less debt to reduce the probability of liquidation.

We bring our model to the data to validate its central predictions. Our empirical analysis uses stress testing implemented by the Federal Reserve in 2009 to study how bank holding companies (BHCs) differentially respond to stress tests. Focusing on BHCs subject to the Supervisory Capital Assessment Program (SCAP), we use quarterly data from FR Y-9C reports to measure bank lending.

We find evidence consistent with our model's central predictions using lending data and a measure of managerial protections. As a first test, we investigate if banks with high managerial protections increase their relative level of lending versus banks with lower managerial protections. Using a difference-in-differences specification, we find a relative increase in lending of 17%. As a second test, we investigate if the treatment effect varies with the earnings distribution of the BHCs. We explore this heterogeneous treatment effect by splitting and estimating our sample based on the empirical distribution of earnings for BHCs. Re-estimating our baseline specification, we find evidence consistent with our model's predictions, that is, that BHCs with fat-tailed earnings distributions are more likely to respond to capital requirements.

We conduct a battery of robustness tests and find evidence consistent with our baseline analysis. Our first set of tests uses alternative governance indexes. Next, we consider alternative measures of lending by BHCs. We then consider the role of outliers and alternative clustering specifications. Finally, we use a sample of BHCs that were not subject to the first wave of stress tests to ensure the results operate through the interaction between managerial protections and capital constraints. In all tests, we find evidence consistent with our model's prediction. These results, taken together, provide additional confidence in our baseline findings.

Overall, this paper provides a tractable theoretical model and supporting empirical evidence suggesting that the effect of capital requirements depends on the corporate governance of banks. Research has provided theoretical evidence that tightening capital requirements can distort banks' provision of liquidity services (Diamond and Rajan, 2000; DeAngelo and Stulz, 2015). At the same time, capital requirements may also introduce governance problems (Dewatripont and Tirole, 2012).[4] This paper offers a new perspective on bank capital requirements, where their efficacy depends on the private benefits of bank managers. Further, this study brings nuance to the understanding of capital structure theory and may have implications for the debate on the effective monitoring of financial institutions.

This paper contributes to the literature on the effect of regulations on lending by presenting a novel channel through which policies that constrain debt, such as bank stress tests and capital requirements, affect investment. The theoretical literature on the role of bank capital offers mixed predictions regarding the effect of tightening capital requirements. One perspective is that the safety from a larger capital buffer could result in more severe financing frictions (Diamond and Rajan, 2000) or higher charter values (Keeley, 1990), leading banks to lend more conservatively.[5] Another perspective is that a greater capital stock can improve monitoring (Holmstrom and Tirole, 1997) and mitigate debt overhang (Admati, DeMarzo, Hellwig, and Pfleiderer, 2013), allowing banks to lend more. It could also increase the value of loans by increasing the probability that a bank remains operational to collect cash flows (Bahaj and Malherbe, 2020). Our model follows the latter view in that we illustrate how policies that constrain debt ratios can increase

[4]However, Hellmann, Murdock, and Stiglitz (2000), Morrison and White (2005), Repullo (2004) and Acharya, Mehran, and Thakor (2015) argue that stringent capital regulation can induce prudence by banks.

[5]Additionally, Thakor (1996) shows that risk-based capital requirements that increase the cost of loans, relative to safe investments, can also decrease lending. Relatedly, for lending channels focused on recent liquidity regulations see, e.g., Brunnermeier and Krishnamurthy (2014), Bosshardt, Kakhbod, and Saidi (2021a) and Bosshardt, Kakhbod, and Saidi (2021b), among others.

lending. However, we propose a novel mechanism based on the interaction between debt and corporate governance.

This paper also contributes to the empirical literature on the effect of tightening capital requirements on lending. Research has suggested a negative effect, such as Acharya, Berger, and Roman (2018), who find that the U.S. stress tests led banks to decrease credit supply. Further, Fraisse, Lé, and Thesmar (2020); Gropp, Mosk, Ongena, and Wix (2018) present additional evidence from other contexts that increasing capital requirements can reduce lending. In contrast, Bassett and Berrospide (2018) find a weak effect or even a positive one in some specifications. Related, Malherbe (2020) studies the effect of capital requirements over the business cycle. This paper provides a further nuanced analysis, suggesting that agency issues, particularly managerial protections, matter when considering the effects of tightening capital requirements.

This paper also contributes to the literature on the effects of managerial protections on firm performance by illustrating how they can distort capital structure and investment. Research has shown that managerial protections are associated with worse firm performance across a variety of indicators, including stock returns, firm valuation, profits, and sales growth (Gompers, Ishii, and Metrick, 2003; Bebchuk, Cohen, and Ferrell, 2009). Further, Bertrand and Mullainathan (2003) present causal evidence suggesting that the introduction of anti-takeover laws led to reduced firm productivity and profitability, possibly due to weaker incentives for managers to minimize costs and enhance value. We contribute to this literature by considering additional mechanisms related to how managerial protections distort capital structure and investment decisions.[6]

1 Model of Bank Investments with Managerial Protections

This section introduces a model in which managerial protections affect how a manager determines which projects to pursue. It then derives the capital structure and level of investment that maximizes the firm's value. Finally, it describes conditions under which policies that constrain the debt ratio can lead to increased investments.[7]

[6]For surveys of the literature on corporate governance, see Shleifer and Vishny (1997) and Tirole (2006).

[7]The model relates to ideas from the classical literature linking capital structure and investment. On the one hand, debt increases the probability of liquidation and damps investment, which is analogous to the debt overhang effect described by Myers (1977). On the other, debt constrains managers from pursuing unprofitable projects, which is analogous to the disciplining effect described by Jensen (1986). The model also relates to subsequent work in which these channels affect the determination of firm debt ratios (Morellec, Nikolov, and Schürhoff (2012)). We consider specifically how the level of investment varies with a firm's capital structure.

1.1 Timeline, Capital, Investment, and Liquidation

1.1.1 Manager and Investors

A manager operates a firm (bank). The firm raises capital to invest in a risky project. A manager can invest in the project and have sufficient funds to finance a fraction α_F of the investment. The manager can acquire external financing for the remaining fraction by issuing claims on the return realized by the project.[8]

There are two types of external investors:

1. **Uninformed Investors (credit)** — receive a contingent claim on the project. Importantly, they do not observe signals of the project's quality until its completion.

2. **Informed Investors (equity)** — are offered equity shares in the project. The fraction of capital provided by informed investors is α_I. They observe a private signal about the project's quality θ prior to its completion.

Informed investors (equity) are passive investors; they invest without getting involved in controlling the firm. The simplest way for them to invest is to buy claims on the residual value of the firm's cash flows in the form of equities. In particular, the informed investors are assumed to receive a fraction of equity shares corresponding to their initial investment.[9]

The external investors have no bargaining power, and their outside option is to forego investment and obtain zero net returns. If the project is liquidated, the creditors have a senior claim on the liquidation value of the firm. Denote the fraction of capital provided by uninformed investors by α_U. Note that the fraction of capital financed by equity can be written as $\alpha_I + \alpha_F = 1 - \alpha_U$.

1.1.2 Timeline

The economy extends over three dates $t = 0,1,2$. Figure 2 outlines the model's timeline. As an overview, at date $t = 0$, the firm raises capital to invest in a risky project. At date $t = 1$, the manager observes a precise signal of the project's profitability and decides whether to liquidate it. If the project is completed rather than liquidated, then at date $t = 2$, its cash flow is realized and distributed to the manager and external investors.

[8] All of our results are robust to supposing that the manager invests a fixed volume of funds rather than a fixed share.

[9] In particular, since the total fraction of capital financed by equity is equal to $\alpha_I + \alpha_F$, informed investors own a fraction $\frac{\alpha_I}{\alpha_I + \alpha_F}$ of the equity value, while the manager owns the remaining fraction of $\frac{\alpha_F}{\alpha_I + \alpha_F}$.

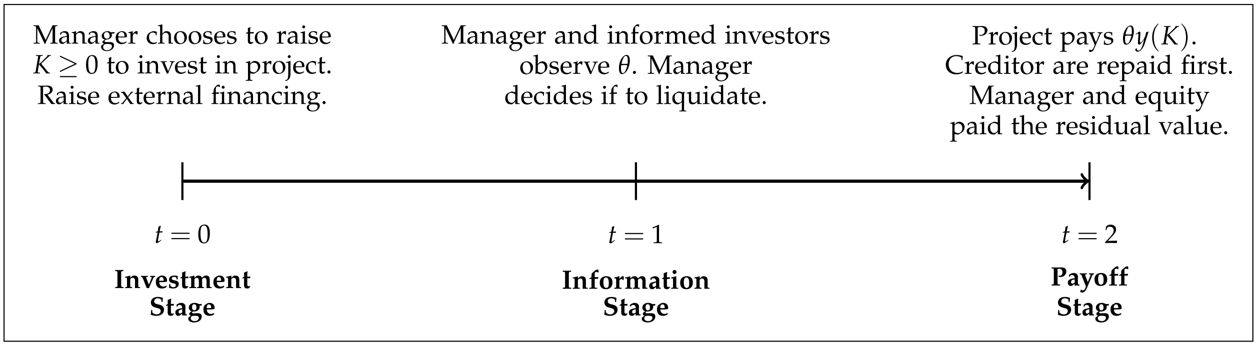

Figure 2: Timeline

This figure summarizes the sequential timeline of the model. In the investment stage ($t = 0$), the manager chooses to raise capital $K \geq 0$, and funds the remainder of the project from credit and equity investors. In the information stage ($t = 1$), θ is revealed about the project to both the manager and equity investor. In the payoff stage ($t = 2$), the project pays $\theta y(K)$. Proceeds are used to pay creditors first, with the residual income paid to managers and equity investors according to their initial fraction of investment.

In more detail, at $t = 0$, the manager chooses to raise capital $K \geq 0$ to invest in a risky project. An investment that continues until $t = 2$ yields a cash flow of $\theta y(K)$, where $\theta \sim H(\cdot)$ is a random variable with log-concave density $h(\theta)$ where $\theta \in [0, \infty)$.[10] We assume that $y(K)$ is increasing and concave. It also has a convex marginal product $y'(K)$ and satisfies the boundary conditions $y(0) = 0$, $y'(0) = \infty$, and $y'(\infty) = 0$.

At $t = 1$, the manager observes θ and decides whether to liquidate the project. Liquidation incurs a fractional cost of $c \in (0,1)$ and therefore yields a liquidation value of $(1 - c)K$.

If the project is continued until $t = 2$, the creditors are repaid and the equityholders, which include the manager and the informed investors, divide, according to their shares, the residual value of the firm.

1.2 Managerial Protections and the Private Benefits of Control

An important way of disciplining a manager is shareholder rights. However, with increased protections, a manager can pursue unprofitable projects to derive private benefits. As a result, managerial protections can distort capital raising within the context of financial firms.

To incorporate this feature, we model the private benefits earned by a manager as

[10] The family of log-concave distributions includes the Uniform, Normal, Exponential, and some classes of the Beta and Gamma distributions. Moreover, it is easy to show that, if the continuously differentiable p.d.f. $h(\cdot)$ is log-concave on $(\underline{\theta}, \bar{\theta})$, then the corresponding c.d.f. $H(\cdot)$ is also log-concave on $(\underline{\theta}, \bar{\theta})$. In addition, the corresponding hazard rate, i.e., $\frac{h(\theta)}{1-H(\theta)}$, is increasing in θ.

a function of managerial protections and their equity investment:

$$R(G, 1-\alpha_U, \alpha_F) \equiv G\,\frac{\alpha_F}{1-\alpha_U}. \tag{1}$$

The private benefit of control increases in the exogenous degree of managerial protections G, which represents the contribution to managerial power of firm provisions or state laws that restrict shareholder rights. Note that the private benefit of control captured in equation (1) specifies a conflict of interest between the manager and the informed investors. This conflict increases in the degree of managerial protections G.

The private benefit of control also increases in the manager's share of the firm's equity. In particular, the manager's equity share represents the manager's voting power, relative to the outside equityholders, which can in turn affect the ability of managers to improve their terms of employment (Morck, Shleifer, and Vishny, 1988) or accrue nonpecuniary benefits from influencing the operations of the firm, according to their personal preferences (Demsetz and Lehn, 1985).[11]

The choice to model managerial protections and the manager's equity share as contributing to the manager's private benefit of control in a complementary way is consistent with evidence showing that ownership concentration is positively associated with factors that increase a manager's potential private benefit of control (Dyck and Zingales, 2004). For simplicity, it is convenient to assume that the private benefit of control depends on managerial protections and the manager's share of the firm's equity in a multiplicative fashion. The results can be extended to a more general functional form $R(G, 1-\alpha_U, \alpha_F, K)$ satisfying $\frac{\partial R}{\partial G} \geq 0$, $\frac{\partial R}{\partial K} \geq 0$, and $R\frac{\partial^2 R}{\partial K \partial(1-\alpha_U)} \leq \frac{\partial R}{\partial(1-\alpha_U)}\frac{\partial R}{\partial K}$.

1.3 Equity Value of Bank

To determine the firm's equity value, we first characterize the payment to the creditors as well as conditions under which the firm is liquidated.

Since creditors are uninformed about the quality of the project, to participate in the investment, they need to write an incentive compatible and individually rational contract with the manager. The following lemma characterizes the terms of the contract and shows that the unique incentive compatible contract is debt.

Lemma 1. *The unique incentive compatible contract between the manager and the uninformed investors is debt. Specifically, the uninformed investors are paid a fixed amount $p(K, \alpha_U)$ whenever the project is not liquidated at $t = 1$. If the project does not generate a large enough return to*

[11]The fraction of equity held by managers is frequently substantial even in large publicly owned firms (La Porta, de Silanes, Shleifer, and Vishny, 1998).

repay the promised amount, then the project is liquidated at $t = 1$, and the uninformed investors are paid the liquidation value up to the value of their investment $\alpha_U K$. Let $\theta^*(K, \alpha_U)$ denote the threshold for θ at which the project is liquidated. Then $p(K, \alpha_U)$ and $\theta^*(K, \alpha_U)$ satisfy the following:

- ***CASE I:*** *If $\alpha_U \leq 1 - c$, then*

$$p(K, \alpha_U) = \alpha_U K \tag{2}$$

$$\theta^*(K, \alpha_U) y(K) = \max\{(1-c)K - G, \alpha_U K\} \tag{3}$$

- ***CASE II:*** *If $\alpha_U \geq 1 - c$, then*

$$p(K, \alpha_U) = \theta^*(K, \alpha_U) y(K) \tag{4}$$

$$\alpha_U K = H\Big(\theta^*(K, \alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big) \underbrace{\theta^*(K, \alpha_U) y(K)}_{= p(K, \alpha_U)} \tag{5}$$

Proof. See Appendix. □

The intuition for this result is as follows. First, consider the "high debt" case where $\alpha_U \geq 1 - c$. If the project is liquidated, then the liquidation value $(1-c)K$ is insufficient to repay the value of the investment by the creditors $\alpha_U K$. Therefore the firm defaults, and the creditors receive all of the liquidation value, since they have a senior claim on it (see Figure 3). Ex ante, they charge a risk premium that is determined by an individual rationality condition that equates the creditors' expected payoff with their cost of investment. The risk premium and the default threshold for the quality of the project θ are related by an incentive compatibility condition for the manager, which requires that, conditional on seeing the project to completion, the creditors receive the return of the firm at the marginal state at which it is continued, which is $\theta^*(K, \alpha_U) y(K)$.

Second, consider the "low debt" case where $\alpha_U \leq 1 - c$. If the project is liquidated, the liquidation value $(1-c)K$ is sufficient to repay the creditors for their investment $\alpha_U K$. Therefore, by the individual rationality condition, the creditors always receive $\alpha_U K$. If the debt ratio is sufficiently small, then the manager is unconstrained by the debt contract and chooses the default threshold $\theta^*(K, \alpha_U)$ by equating his or her own

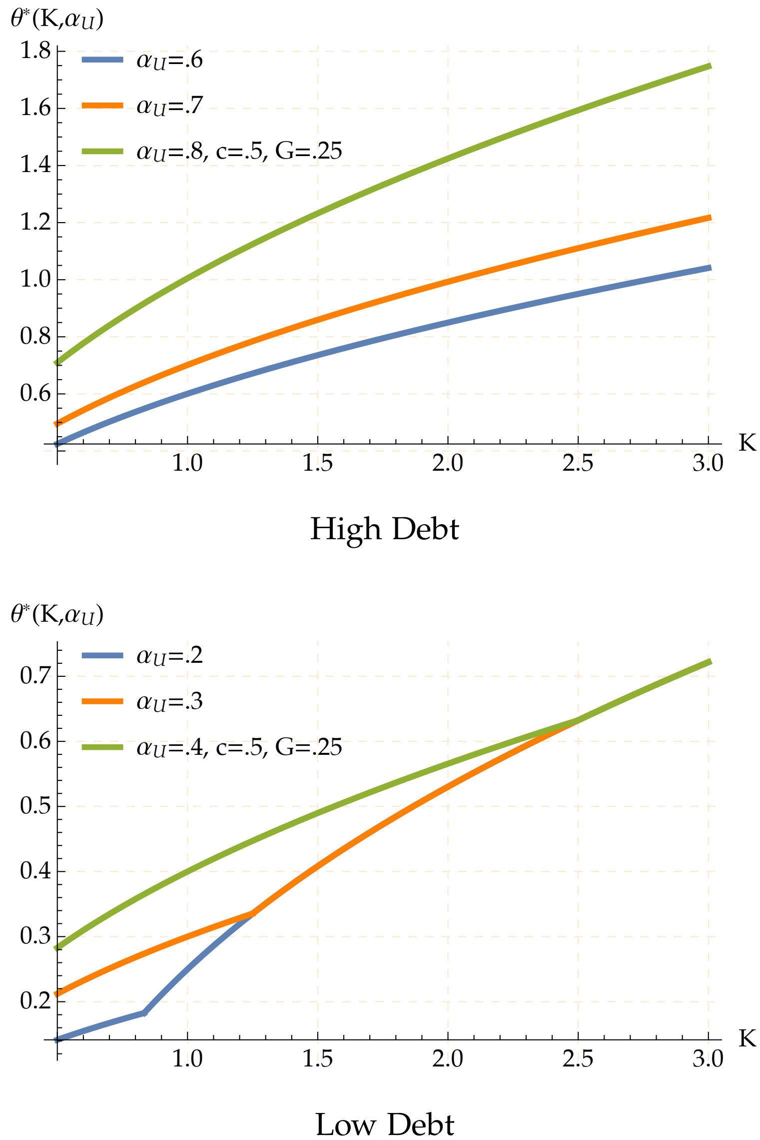

Figure 3: Illustration of Lemma 1

This figure illustrates the $\theta^*(K, \alpha_U)$, as specified in Lemma 1. The horizontal axis corresponds to the capital while the vertical axis corresponds to the liquidation threshold. Panel A illustrates the high debt case ($\alpha_U > 1-c$). Panel B illustrates the low debt case ($\alpha_U < 1-c$). The blue, orange, and green lines corresponds to α_U equalling 0.2, 0.3, and 0.4, respectively. We assume $y(K) = \sqrt{K}$, $c = 0.5$, $G = 0.25$ and the distribution is Uniform$[0, a]$, where a is large enough.

payoffs from either liquidating or continuing the project:

$$\underbrace{\frac{\alpha_F}{1-\alpha_U}\left(\theta^*(K,\alpha_U)y(K) - \overbrace{\alpha_U K}^{=p(K,\alpha_U)} + G\right)}_{\text{continuation payoff}} = \underbrace{\frac{\alpha_F}{1-\alpha_U}((1-c)-\alpha_U)K}_{\text{liquidation payoff}} \tag{6}$$

Increasing the strength of managerial protections increases the manager's incentive to continue projects, which reduces the liquidation threshold. However, feasibility requires the return at the marginal state of continuation to be large enough to pay the creditors the promised amount, which implies $\theta^*(K,\alpha_U)y(K) \geq \alpha_U K$. When the debt ratio is large, relative to the degree of managerial protections, this constraint holds with equality.

After the creditors are repaid, the equity value of the firm is equal to

$$V\Big(K,\alpha_U\Big) = H\Big(\theta^*(K,\alpha_U)\Big)\Big[((1-c)-\alpha_U)K\Big]^+ + \int_{\theta^*(K,\alpha_U)}^{\infty}\Big(\theta y(K) - p(K,\alpha_U)\Big)dH(\theta) - (1-\alpha_U)K$$

where $[A]^+ = \max\{A,0\}$. Note that, because the creditors obtain zero expected profits, the value of the firm is also equal to the ex ante welfare, which is defined as the sum of the net payoffs for the manager, outside equityholders, and creditors.

The manager then chooses to liquidate or continue projects to maximize his or her utility

$$u_m \equiv \frac{\alpha_F}{1-\alpha_U}V\Big(K,\alpha_U\Big) + G\frac{\alpha_F}{1-\alpha_U}\ \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big). \tag{7}$$

1.4 Efficient Capital Structure

To provide a benchmark for evaluating the effect of policies that constrain debt ratios, this subsection derives the efficient capital structure that maximizes the firm's equity value.

To determine the efficient capital structure, we first derive the first-best liquidation rule that maximizes the value of the firm in the absence of external financing frictions and private benefits of control. In particular, eliminating these frictions is equivalent to maximizing the utility of the manager (Equation 7) with $\alpha_F = 1$, $\alpha_U = 1$, and $G = 0$. At $t = 1$, it is straightforward to see that liquidating the project is efficient if and only if the return $\theta y(K)$ is less than the liquidation value $(1-c)K$ or equivalently

$$\theta \leq \theta^{opt}(K) \equiv \frac{(1-c)K}{y(K)}. \tag{8}$$

The efficient capital structure that induces the first-best liquidation rule is given by $\alpha_U = 1 - c$. Note that, for tractability, the model focuses on determinants of capital structure involving managerial discretion and omits other potentially relevant considerations. Accordingly, we interpret this result as a benchmark for a qualitative analysis of how these channels interact with policies that constrain debt ratios, and we do not interpret it quantitatively.

Proposition 1. *If* $\alpha_U = 1 - c$ *then* $\theta^*(K, \alpha_U) = \theta^{opt}(K)$*. If* $G > 0$ *then the converse also holds.*

Proof. See Appendix. □

The intuition is as follows. In the "high debt" case where $\alpha_U > 1 - c$, the creditors accrue losses when the project is liquidated. As a result, they require a greater repayment $\theta^*(K, \alpha_U)y(K)$ in states where the project is not liquidated. This implies an increase in the liquidation threshold $\theta^*(K, \alpha_U)$, which also increases the probability of having to liquidate profitable projects. Alternatively, consider the "low debt" case where $\alpha_U < 1 - c$. If managerial protections enable the manager to obtain a private benefit of control, or $G > 0$, then the manager has an incentive to complete unprofitable projects, which is reflected in a decrease in the liquidation threshold $\theta^*(K, \alpha_U)$; see Figure 4.[12] This trade-off is summarized in the following proposition:

Proposition 2. *The liquidation threshold is increasing in the firm's debt ratio:*

$$\frac{\partial \theta^*(K, \alpha_U)}{\partial \alpha_U} \geq 0$$

Consequently, if $\alpha_U > 1 - c$*, the manager excessively liquidates the project,*

$$\theta^*(K, \alpha_U) > \theta^{opt}(K).$$

If $\alpha_U < 1 - c$ *and* $G > 0$*, the manager excessively continues the project,*

$$\theta^*(K, \alpha_U) < \theta^{opt}(K).$$

Proof. See Appendix. □

1.5 Investment Decision

This subsection first derives the first-best level of investment that maximizes the value of the firm in the absence of external financing frictions and managerial private benefits of control. It then introduces the conditionally efficient level of investment for a given capital structure and illustrates channels by which it can either increase or decrease in response to policies that constrain debt ratios.

[12] In the low debt case, eliminating the private benefit of control by setting $G = 0$ restores the efficient liquidation threshold $\theta^*(K, \alpha_U) = \theta^{opt}(K)$.

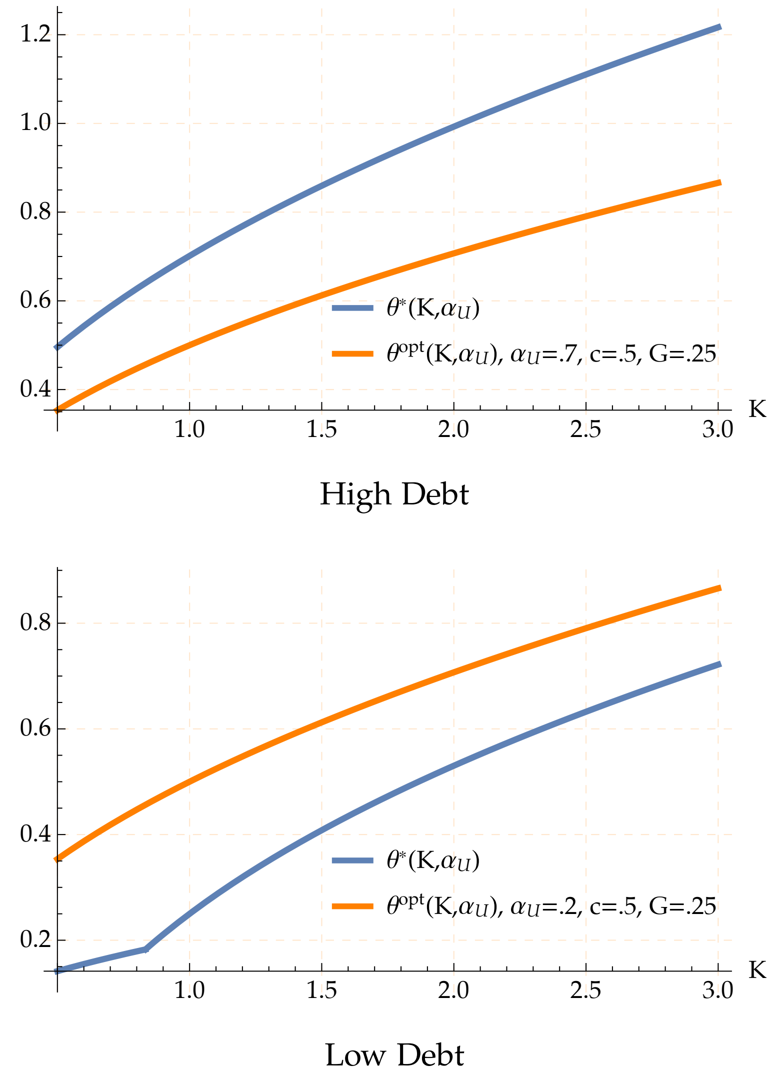

Figure 4: Managerial Decision to Liquidate Projects

This figure illustrates the decision of managers to liquidate projects. The horizontal axis corresponds to the capital while the vertical axis corresponds to the liquidation threshold. Panel A illustrates the high debt case, when $\alpha_U > 1 - c$, and shows that a manager will excessively liquidate the project, $\theta^*(K, \alpha_U) > \theta^{opt}(K)$. Panel B illustrates the low debt case, $\alpha_U < 1 - c$ and $G > 0$, and shows that the manager will excessively continue the project, $\theta^*(K, \alpha_U) < \theta^{opt}(K)$.

1.5.1 Efficient Level of Investment

Given that the first-best liquidation threshold is equal to $\theta^{opt}(K)$, the first-best level of investment solves:

$$\max_K \left\{ \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) + \int_0^{\theta^{opt}(K)} (1-c) K dH(\theta) - K \right\} \tag{9}$$

Proposition 3. *There exists a unique interior capital level* $K^{opt} \in (0, \infty)$ *that solves* (9) *and satisfies the first order condition*

$$1 = (1-c)\, H\Big(\theta^{opt}(K^{opt})\Big) + y'(K^{opt}) \int_{\theta^{opt}(K^{opt})}^{\infty} \theta\, dH(\theta). \tag{10}$$

Proof. See appendix. □

Intuitively, the first-best level of investment is chosen to equate the marginal cost of investment with the marginal expected returns, which consists of the liquidation value from bad realizations plus the returns from good realizations.

1.5.2 The Level of Investment with an Efficient Capital Structure

To determine how capital structure affects the level of investment, consider the level of investment that maximizes the value of the firm for a given capital structure, which is denoted by $K^*(\alpha_U)$. Since the efficient capital structure $\alpha_U = 1-c$ induces the first-best liquidation threshold $\theta^{opt}(K)$ (see Proposition 1), the corresponding conditionally efficient level of investment is equal to the first-best benchmark.

1.6 Investment Decision with an Inefficient Capital Structure

In the prior section, as a benchmark, we characterized the investment decisions based on an efficient capital structure. In this section, we consider cases where the debt ratio is higher or lower than the efficient level and its effect on investments.

1.6.1 Debt Ratio Higher than Efficient Level

If the firm has a greater than the efficient debt ratio, or $\alpha_U > 1-c$, then investment is distorted downward, relative to the first-best benchmark K^{opt}.

Proposition 4. *If* $\alpha_U > 1-c$, *then* $K^*(\alpha_U) < K^{opt}$.

Proof. See appendix. □

To see this, recall that the cost of debt is increasing in the debt ratio to compensate creditors for losses in states where the project is liquidated. Because of the higher cost of debt, the firm may have to liquidate profitable projects (see Proposition 2). This in turn reduces the marginal expected returns to investment.

1.6.2 Debt Ratio Lower than Efficient Level

If the firm has less than the efficient debt ratio, or $\alpha_U < 1 - c$, the conditionally efficient level of investment may be distorted either downward or upward, relative to the first-best benchmark K^{opt}. The direction of the distortion depends on the parameters, such as the concavity of the firm's earnings distribution cdf.

Proposition 5. *Suppose that the debt ratio is less than the efficient level or $\alpha_U < 1 - c$. If there are no managerial protections, or $G = 0$, then the level of investment is equal to the first-best benchmark, and there is no distortion:*

$$K^*(\alpha_U) = K^{opt} = K^*(1 - c)$$

If there are managerial protections, or $G > 0$, then the conditionally efficient level of investment can in general be greater or less than the first-best benchmark. If the debt ratio is low enough, or $\alpha_U \leq 1 - c - \frac{G}{K^{opt}}$, and H is strictly concave then investment is distorted upwards, or $K^*(\alpha_U) > K^{opt}$. If the cdf for the return distribution H is weakly convex, then the conditionally efficient level of investment is distorted downwards, or $K^*(\alpha_U) \leq K^{opt}$.

Proof. See appendix. □

The intuition is that debt disciplines managers by requiring them to liquidate unprofitable projects. In particular, if the manager is not constrained by debt, he or she may continue some unprofitable projects to obtain the private benefit of control (see Proposition (2)). On the one hand, continuing unprofitable projects decreases the marginal expected return to investment, which in turn inhibits investment. On the other, increasing the level of investment increases the rate at which the firm liquidates unprofitable projects.

Lemma 2. *The termination threshold increases in the level of investment: $\frac{\partial \theta^*(K,\alpha_U)}{\partial K} \geq 0$.*

Proof. See appendix. □

Intuitively, the increasing relationship between the level of investment and the probability of liquidation follows from the linear liquidation value and concave expected return. Therefore increasing the level of investment partially corrects the excessive continuation of unprofitable projects by the manager. Accordingly, the level of investment can be interpreted as a partial substitute for debt in disciplining the manager.

Due to these countervailing effects on the incentive to invest, the conditionally efficient level of investment can be either greater or less than the efficient level, depending on the parameters.

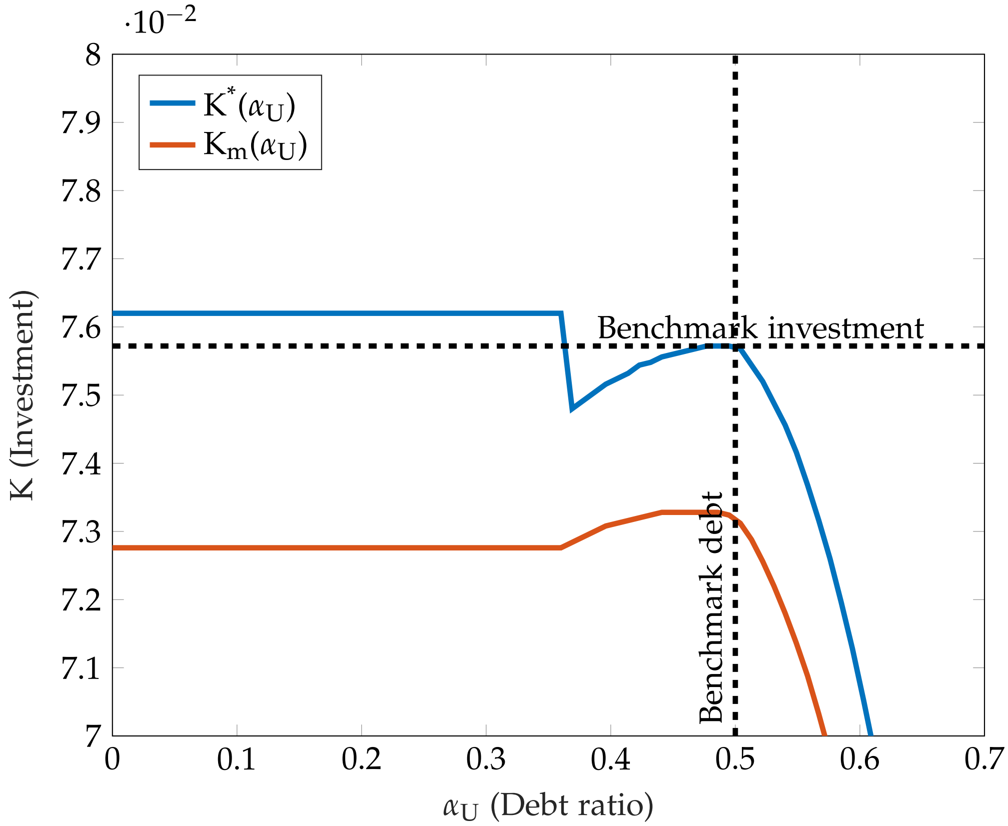

Figure 5: Conditionally Efficient Level of Investment

This plot shows the conditionally efficient level of investment and the manager's preferred level of investment as a function of the debt ratio when the return for the project follows an exponential distribution. The plot illustrates that the manager's preferred level of investment is less than the conditionally efficient level, as shown in Proposition 6.

Figure 5 illustrates a case in which requiring banks to have a lower than the optimal debt ratio can lead to higher investment. The blue line illustrates the level of investment that maximizes the value of the firm for a given capital structure. Moving to the left of the figure, we observe that the level of investment K is greater than the benchmark investment case.

1.7 Incorporating Manager Preferences

This section shows that an extended version of the model allowing managers to choose the capital structure and level of investment of the firm is consistent with other facts in the corporate governance literature.

First, we illustrate channels by which managerial protections can cause managers to issue a suboptimal level of debt, relative to the value-maximizing level. Second, we illustrate a novel channel by which managerial protections can cause managers to underinvest, relative to the value-maximizing level. The model thereby provides new perspectives on evidence linking managerial protections with capital structure and investment.

1.7.1 Manager's preferred capital structure

This section describes channels by which managerial protections can cause a manager to choose an inefficient capital structure. These channels correspond to terms in the derivative of the manager's utility evaluated at the efficient benchmark:

$$\frac{\partial u_m}{\partial \alpha_U}|_{\alpha_U=1-c} = \underbrace{\frac{\alpha_F}{(1-c)^2}V(K,1-c)}_{>0} + \frac{\alpha_F}{1-c}\underbrace{\frac{\partial V}{\partial \alpha_U}|_{\alpha_U=1-c}}_{=0} + \underbrace{G\frac{\alpha_F}{(1-c)^2}\Big(1-H\Big(\theta^*(K,1-c)\Big)\Big)}_{\geq 0} \underbrace{-G\frac{\alpha_F}{1-c}h\Big(\theta^*(K,1-c)\Big)\frac{\partial \theta^*(K,\alpha_U)}{\partial \alpha_U}|_{\alpha_U=1-c}}_{\leq 0} \tag{11}$$

The positive first term, which does not involve managerial protections, shows that managers have an incentive to issue excessive debt to increase their share of the value of the firm. The last two terms illustrate the effect of managerial protections, which can distort the manager's chosen level of debt either upward or downward. On the one hand, the positive term illustrates that protections can intensify the incentive for managers to increase their voting power by issuing debt rather than equity. This term is driven by the complementarity between the degree of managerial protections and the manager's equity share in determining managerial power and is consistent with evidence showing that ownership concentration is positively associated with estimated private benefits of control (Dyck and Zingales, 2004). On the other hand, the negative term illustrates that protections can lead managers to issue less debt to decrease the probability of liquidation. Further, this analysis provides a novel interpretation for the finding of John and Litov (2010) that managerial protections are positively associated with debt ratios.

1.7.2 Manager's preferred level of investment

This section describes channels by which managerial protections can cause a manager to choose an inefficiently low level of investment.[13]

Denote the manager's preferred level of investment that maximizes his or her utility (equation (7)) for a given debt ratio α_U by $K_m(\alpha_U)$.

Proposition 6. *If there are no managerial protections, or $G = 0$, then the manager's preferred level of investment that maximizes his or her utility (equation (7)) for a given debt ratio α_U, $K_m(\alpha_U)$, is equal to the conditionally efficient level $K^*(\alpha_U)$. If there are managerial protections, or $G > 0$, then the manager's preferred level of investment is less than the conditionally efficient level, or $K_m(\alpha_U) < K^*(\alpha_U)$.*

Proof. See appendix. □

The intuition is as follows. Because the manager can obtain the private benefit of control only while the firm is operational, the manager has a stronger incentive to avoid liquidating the project, compared to other equityholders. Since investing less reduces the probability of liquidation, due to the concave expected return (Lemma 2), the manager chooses a lower level of investment to increase the expected private benefit of control. Figure 5 illustrates this result by showing that the conditionally efficient level of investment is greater than the manager's preferred level of investment for different levels of the debt ratio. Appendix Section A.2 shows that this result is robust to alternatively assuming a competitive equity market in which the informed investors make zero rents.

1.8 Recap of Model

The model illuminates the unique distortionary effects of managerial protections on financial institutions and regulatory capital controls. Our model shows that capital requirements that force banks to issue a suboptimally low level of debt affect investment through two channels. On the one hand, distortions to the firm's capital structure reduce the return on investment, which discourages investment. On the other, for a firm with strong managerial protections, capital requirements can undermine the role of debt in disciplining the manager. At the same time, under the canonical assumptions of a linear liquidation cost and a concave expected return, the manager's incentive to liquidate unprofitable projects is increasing in the level of investment. As a result, the

[13]This result differs from the pro-investment effect of the stress tests in Proposition 5 in that it describes the effect allowing the manager to decide the investment level, conditional on a given capital structure, whereas Proposition 5 analyzes the effect of varying the capital structure on the value-maximizing level of investment.

value-maximizing level of investment can increase in response to tightening capital requirements since it serves as a partial substitute for debt in disciplining the manager.

The dominant channel determining the firm's investment response to capital requirements depends on other firm characteristics, such as the earnings distribution. For example, the pro-investment channel can dominate if the firm has an earnings distribution with a strictly concave cdf, such as an exponential distribution. In contrast, the anti-investment channel can dominate if the firm has an earnings distribution with a convex or linear cdf, such as a uniform distribution.

2 Bringing the Model to the Data

This section brings the model to the data by studying BHCs following the first wave of stress-testing in 2009. We first lay out testable implications of the model. Next, we discuss the data and the paper's empirical approach. Finally, we present our empirical estimates and robustness tests.

2.1 Testable Implication, Shock to Capital Requirement

A central result of the model comes from Proposition 5. As a reminder, this proposition suggests that, if a firm starts at its efficient debt ratio, $\alpha_U = 1 - c$, it can increase investment in response to a policy that requires a reduction of the ratio.

A testable implication for banks is that they can increase investment in response to a policy that requires a reduction of their debt ratios, such as capital requirements or stress-testing. As illustrated in the model, this response is conditional on the private benefits accruing to bank managers.

Given this central prediction, we set out to empirically validate our model by studying BHCs and their lending around stress-testing by the Federal Reserve in 2009.[14] We use the comprehensive capital analysis and review (CCAR) initiated by the Federal Reserve to test our model empirically. The Federal Reserve used forward-looking evaluation of the internal capital planning processes of large, complex bank holding companies (Board of Governors of the Federal Reserve System, 2011). Commonly referred to as

[14]Following the crisis, the Supervisory Capital Assessment Program (SCAP) in 2009 and Comprehensive Capital Analysis, Review (CCAR) conducted annually since 2011, have required a subset of large BHCs to maintain sufficient capital to lend under potential adverse scenarios (Goldstein and Sapra, 2014). The SCAP was conducted for 19 BHCs with assets exceeding $100 billion as of 2009. The first three rounds of the CCAR from 2011 to 2013 were conducted for the subset of these firms that remained registered as BHCs, and subsequent rounds of the CCAR were conducted for a wider set of BHCs with assets exceeding $50 billion until 2018.

Table 1: Summary statistics for stress-tested bank holding companies.

The table presents the descriptive statistics for the sample of the bank holding companies across all years in our sample. Observations are at the company-quarter observation level. The sample covers the first quarter of 2006 through the fourth quarter of 2017.

	N	Mean	SD	P25	P75
G-index (2-18)	480	9.10	2.51	7.00	12.00
Loans/assets (%)	480	95.78	37.46	81.72	103.99
Tier 1 capital/assets (%)	480	8.17	1.76	6.75	9.53
NPLs/loans (%)	480	1.41	1.21	0.53	2.11
Non-interest expenses/assets (%)	480	1.16	0.57	0.84	1.22
Net income/assets (%)	480	0.81	1.19	0.68	1.23
Liquid assets/assets (%)	480	11.65	9.71	3.24	19.75
Sensitivity to market risk (%)	480	6.83	7.35	1.69	10.16
Log assets	480	19.85	1.22	18.79	21.26

stress tests, the CCAR implemented a cross-institution study of the capital plans of the 19 largest U.S. bank holding companies (BHC).

2.2 Summary of Datasets

Our empirical analysis studies the 19 bank holding companies subject to the SCAP in the first quarter of 2009. From these BHCs, we draw on their publicly available quarterly filed FR Y-9C reports to measure bank-level variables. Next, we merge regulatory filings with numerous governance indexes that proxy for the level of managerial protections against shareholder actions and takeover threats. Our baseline analysis relies on the Governance Index or G-index, constructed by Gompers, Ishii, and Metrick (2003).[15] However, as more recent papers have suggested revisions to the original G-index, we also include the E-index (Bebchuk, Cohen, and Ferrell, 2009), as well as the D-index, (Karpoff, Schonlau, and Wehrly, 2017) in our analysis.

Summary statistics for our merged dataset are presented in Table 1. Here we construct a balanced sample covering the period from the first quarter of 2006 to the fourth quarter of 2017. All nominal variables are normalized to fourth-quarter 2010 dollars using the GDP deflator.

[15] Similar to Gompers, Ishii, and Metrick (2003), we omit firms with dual-class stock.

2.3 Measuring Private Benefits of Control

We measure the private benefits of control using the level of managerial protection from shareholder actions. This is motivated by Equation 1, where the level of private benefit of control increases in the exogenous degree of managerial protections G, which represents the contribution to managerial power of firm provisions or state laws that restrict shareholder rights. This factor captures the lack of ability of shareholders to discipline managers of the firm.

We measure the level of managerial protection using a variety of governance indexes, a proxy for managerial protection from shareholder discipline.[16] Research has explored the role of shareholder rights for firms and has proposed indexes to capture the degree of rights. Our baseline approach relies on the Governance Index or G-index constructed in Gompers, Ishii, and Metrick (2003), which represents the number of firm provisions and state laws that provide defenses against shareholder actions and takeover threats. This measure, calculated annually, accounts for 24 different governance rules and their role in shareholder value. In further tests, discussed below, we show that our empirical results are robust to alternative governance indexes, including those proposed by Bebchuk, Cohen, and Ferrell (2009) and Karpoff, Schonlau, and Wehrly (2017).

2.4 Identifying BHCs with Strong Managerial Protections

We first set out to identify which banks had a relatively high level of managerial protections. Our baseline approach designates a BHC as having such protections if its G-index is greater than the median in this sample, and we similarly designate a BHC as having weak managerial protections if its G-index is less than the median. BHCs whose G-index is equal to the median are excluded, leaving 10 BHCs for the analysis.

Our sample is restricted to the subset of these BHCs that were listed as BHCs during the entire sample period and that could be merged to the G-index dataset in 2006, the last year in which the G-index data is available. The set of BHCs designated as having strong managerial protections has a median G-index of 11, which corresponds to the 83rd percentile among all firms in 2006, while the set of BHCs designated as having weak managerial protections has a median G-index of 7, which corresponds to the 25th percentile among all firms.

Because we are comparing two groups of BHCs, we need to consider the comparability of BHCs with strong and weak managerial protections. As a first approach, we

[16]Throughout the paper, managerial protection refers to anti-takeover laws, restricted shareholder rights, and other laws and provisions that could help manager obtain private benefits from controlling a firm.

Table 2: Comparison of observables for bank holding companies

This table presents the means of bank characteristics within the subsample of bank holding companies (BHCs) with a high managerial protections (i.e. greater than the median G-index among stress-tested BHCs) in 2009Q4. It also presents the t-statistic for the coefficient η from estimating the regression $Y_{it} = \eta I\left(Protection\right)_i + \epsilon_{it}$ and computed using robust standard errors.

	Low Protections	High Protections	T-statistic
Tier 1 capital/assets	7.244	9.406	2.887
NPLs/loans	3.589	2.521	-1.332
Non-interest expenses/assets	1.085	1.565	1.882
Net income/assets	0.0677	0.260	.246
Liquid assets/assets	13.04	7.549	-1.063
Sensitivity to market risk	5.613	4.558	-.379
Log assets	20.85	18.89	-3.802

estimate

$$W_{it} = \eta I\left(Protection_i\right) + \epsilon_{it}, \tag{12}$$

where i indexes BHC at quarter-time t and W_{it} represents one of the control variables. BHCs that have high managerial protections are measured by $I\left(Protection_i\right)$.

Comparing the two groups of BHCs, we present the estimates of Equation 12 in Table 2. Here we compare the treatment and control groups with respect to the control variables for our sample of BHCs in the last quarter of the pre-policy period. It presents the mean for each variable and group in the period preceding the stress tests as well as the t-statistic on the coefficient η from estimating the regression.

We find that BHCs with high and low managerial protections are similar with respect to most of the characteristics. The only two characteristics for which the difference between the two groups is statistically significant at 5% are log assets and the Tier 1 capital to assets ratio. The observed similarity between the treatment and control groups mitigates concerns that characteristics correlated with the extent of managerial protections could confound the results. Further, this study of BHC characteristics motivates which controls we use in our main empirical specification.

2.5 Empirical Strategy

We use a difference-in-differences analysis comparing BHCs subject to stress testing in 2009. We compare BHCs that had relatively high managerial protections and a control set of BHCs with relatively low managerial protections and that were similar along other important dimensions. Specifically, we estimate:

$$Y_{it} = \beta_1 \ I\left(Protection_i\right) \times I\left(Post_t\right) + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it}, \tag{13}$$

where i indexes BHC at quarter-time t. We include BHC fixed effects (ψ_i) to control for time-invariant heterogeneity at the BHC level. We include quarter fixed effects (ϕ_t) to control for time-varying trends in loan provisions. $I\left(Post_t\right)$ is an indicator and takes the value of one when a quarter is on or after 2010Q1, a year after implementation of the first stress test. $I\left(Protection_i\right)$ is an indicator for whether BHC i's 2006 G-index is above the median among the set of stress-tested BHCs. We use alternative governance indexes as a proxy for managerial protections in further robustness tests. Our primary coefficient of interest is β_1, which captures the within-BHC change in loan provisions following the stress tests. We cluster standard errors at the BHC level.

We include a vector of control variables ($\mathbf{X}_{it-1}$), which correspond to the CAMELS rating system. Controlling for bank fundamentals, we include capital adequacy (C), as measured by the fraction of tier 1 equity capital to assets; asset quality (A), as measured by the fraction of nonperforming loans; manager quality (M), as measured by non-interest expenses to total assets; earnings (E), as measured by return on assets; liquidity (L), as measured by the fraction of liquid assets; sensitivity to market risk (S), as measured by the absolute value of the difference between short-term assets and short-term liabilities divided by total assets. We also include lag measures of the logarithm of total assets to control for size.

We are careful when specifying our dependant variable, Y_{it}. Our baseline specification uses the ratio of loans and unused commitments to assets at the BHC level ($Loans/Assets_{it}$). This is because commitments are partly chosen by customers; we consider the sum of loans and unused commitments to better reflect the lending choices of banks. In an additional robustness test, we also consider $\left(Adjusted\ Loans/Assets_{it}\right)$ and total amount loans without scaling.

3 Managerial Protections, Capital Requirements, and Bank Lending

3.1 Capital Requirements on Bank Lending

We first assess our empirical specification by plotting aggregate bank lending for those facing capital requirements. As previously shown in Figure 1, we plot the level of lending of banks with strong and weak managerial protections. This plot shows the mean ratio of loans and unused commitments to assets for a balanced sample of bank holding companies (BHCs) that were subject to the stress tests conducted by the Federal Reserve (the SCAP in 2009 and annual rounds of the CCAR starting in 2011), split by the strength of managerial protections.[17] Interpreting this figure, we find an important difference in the lending activity of BHCs with strong and weak managerial protections, providing the first set of evidence in support of our model.

As our second approach, we study the difference between banks by estimating the relative trend between the two groups using,

$$Loans/assets_{it} = \sum_{t \neq 2010Q1} \beta_t \, I\left(Protection\right)_i \times I\left(Post\right)_t + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it} \tag{14}$$

Estimating Equation 14, we can graphically evaluate whether banks with strong managerial protections are indeed responding to stress tests. Plotting the coefficients, relative to the quarter of the stress-tests, Figure 6 presents three pieces of evidence. First, it shows that there are no differential pre-trends in the lending between banks with strong and weak managerial protections. This absence of pre-trends suggests that the stress tests did not, on average, coincide with the differential increase in lending in the pre-period. Second, after the stress-tests, there is a gradual and significant increase in lending by banks with strong managerial protections, relative to those with weak ones. Third, the relative increase in lending is persistent, suggesting that the treatment effect remains important for banks, consistent with our proposed economic channel. These pieces of evidence together support the appropriateness of our empirical specification for studying our model.

Next we move to our main regression framework to estimate the difference in bank lending. Table 3 presents estimates from Equation 13 and provides evidence in support

[17] BHCs with a G-index greater than the median in this subset are designated as having strong managerial protections, and BHCs with a G-index less than the median are designated as having weak managerial protections.

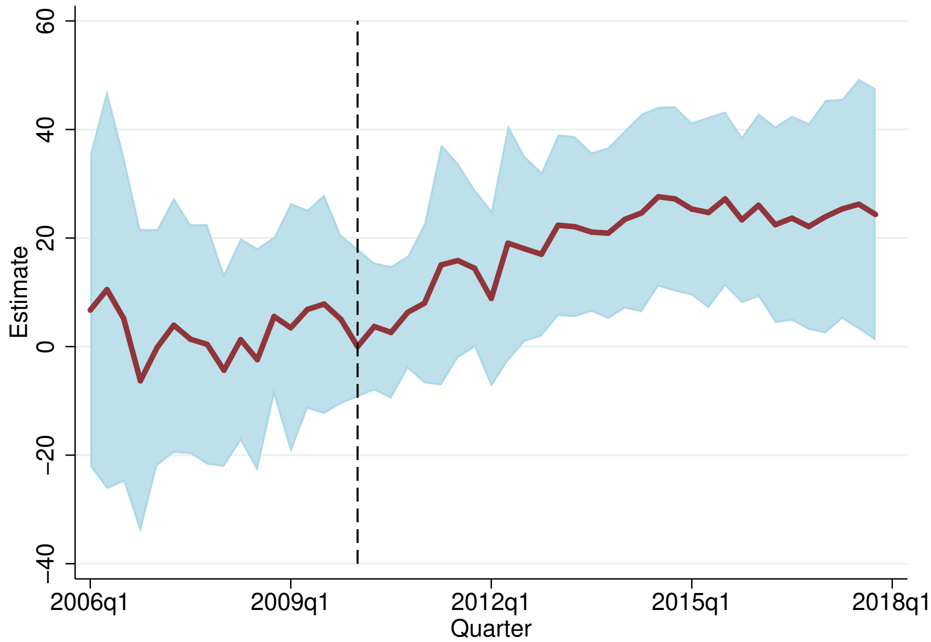

Figure 6: Bank lending by BHCs around stress-testing

This figure presents the estimates β_t from estimating the regression $Loans/assets_{it} = \sum_{t \neq 2010Q1} \beta_t I\left(Protection_i\right) \times I\left(Post_t\right) + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it}$, where $Loans/assets_{it}$ is the ratio of loans and unused commitments to assets for bank holding company (BHC) i at quarter t, $I\left(Protection_i\right)$ is an indicator for whether a BHC's 2006 G-index is above the median among the set of stress-tested BHCs, $I\left(Post_t\right)$ is an indicator for whether a quarter occurs on or after 2010Q1, ψ_i represents BHC fixed effects, ϕ_t represents year fixed effects, and $\mathbf{X}_{it-1}$ is a set of control variables that includes the fraction of tier 1 equity capital to assets, the fraction of non-performing loans, non-interest expenses to total assets, return on assets, the fraction of liquid assets, the difference between short-term assets and short-term liabilities divided by total assets, and the logarithm of total assets. Nominal variables are normalized to 2010Q4 dollars using the GDP deflator. The figure also presents 95% confidence intervals computed using BHC-clustered standard errors.

of Proposition 5. The point estimates in column (1) from our baseline specification indicate that the stress-tested BHCs with managerial protections increased the asset share of loans, relative to other stress-tested BHCs, by approximately 16%, which is about 17% of the mean ratio of loans to assets and 30% of the standard deviation. The estimated coefficient is significant at the 5% level. These results provide the first set of empirical evidence on the importance of managerial protections to bank lending, through capital requirements, in support of our theoretical model.

Table 3: Bank Lending and Managerial Protections

This table presents results from estimating the regression $Loans/assets_{it} = \beta_1\ I(Protection_i) \times I(Post_t) + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it}$, where $Loans/assets_{it}$ is the ratio of loans and unused commitments to total assets for bank holding company (BHC) i at quarter t, $I(Protection_i)$ is an indicator for whether a BHC's 2006 G-index is greater than the median among the set of stress-tested BHCs, $I(Post_t)$ is an indicator for whether the quarter occurs on or after 2010Q1, ψ_i represents BHC fixed effects, ϕ_t represents quarter fixed effects, and X_{it-1} is a set of control variables as described in subsection 2.2. Column (1) runs the estimation on the full sample. Column (2) runs the estimation on the subsample of BHCs with a dominantly concave return distribution. Column (3) runs the estimation on the subsample of BHCs with a dominantly convex return distribution. Standard errors are clustered at the bank level and are robust to heteroscedasticity. ***, **, * denote significance at the 1%, 5%, and 10% level, respectively.

	$(Loans/assets)_{it}$		
	(1) Full sample	(2) Concave	(3) Convex
$I(Protection_i) \times I(Post_t)$	16.002**	19.649***	-2.991
	(2.52)	(7.96)	(-0.69)
Tier 1 capital/assets	3.754**	0.242	1.562
	(2.58)	(0.15)	(1.23)
NPLs/loans	1.377	-3.166	3.354*
	(0.46)	(-2.01)	(2.23)
Non-interest expenses/assets	9.342*	-0.686	9.461
	(2.20)	(-0.39)	(1.90)
Net income/assets	-0.147	-0.373	1.231
	(-0.38)	(-0.95)	(1.21)
Liquid assets/assets	-1.157	-0.117	-0.849
	(-1.44)	(-0.47)	(-1.30)
Sensitivity to market risk	0.396	-0.349	0.211
	(0.72)	(-1.29)	(0.73)
Log assets	9.973	-6.242	-12.232**
	(1.17)	(-1.11)	(-2.89)
Observations	480	192	288
R^2	0.952	0.969	0.975
BHC FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes

3.2 Heterogeneous Treatment: Return Distribution of BHCs

We further consider the empirical implications of Proposition 5 by studying the heterogeneous treatment effect of stress tests based on the earnings distribution of the BHCs. As discussed in Section 1.6, BHCs with fat-tailed earnings distributions are more likely to respond to capital requirements. That is, under the canonical assumptions of a linear liquidation cost and a concave expected return, the manager's incentive to liquidate unprofitable projects is increasing in the level of investment. As a result, the value-maximizing level of investment can increase in response to tightening capital requirements since it serves as a partial substitute for debt in disciplining the manager.

To study the differential treatment of BHCs, we first compute the empirical cdf of each BHC's return distribution using observations of return on assets from the start of the sample, in the first quarter of 2006, until the approximate start of the financial crisis in the third quarter of 2007 (Bernanke, 2009).[18] Second, we take a numerical approximation of the earnings distribution and estimate the curvature.[19] Armed with the empirical distribution of earnings, we split our data based on the relative skewness of earnings.

Splitting the sample based on the earnings distribution, we re-estimate Equation 13 and find further evidence consistent with Proposition 5. From Table 3, we see that the subset of BHCs with skewed earnings distributions experienced a relative increase in lending after the introduction of stress testing. Estimates in column (2) indicate that the positive effect of stress testing on the lending for BHCs with a high degree of managerial protections is robust to restricting to BHCs with a dominantly concave return distribution. In contrast, the estimate from column (3) indicates that this result is not present among BHCs with a dominantly convex return distribution.

Establishing this heterogenous treatment effect has important implications for interpreting the results. Specifically, we see that the treatment effect of capital requirements depends on the characteristics of the BHC. Further, this has implications for how policymakers design capital requirements for banks in the future.

[18]See Appendix B, which describes our methodology for determining the concavity of a BHC's earnings distribution cdf. Note, we think of the manager as responding to the stress tests based on the typical return distribution of the firm, so we omit the crisis because it may have caused uncharacteristic returns.

[19]BHCs for which the median second derivative is negative are identified as having a dominantly concave cdf, while those for which the median second derivative is positive are identified as having a dominantly convex cdf.

3.3 Robustness

3.3.1 BHCs Not Subject to Stress Tests

A natural concern with the prior analysis is that loan activity increases only through the managerial protection channel and not through its interaction with BHCs subject to stress tests. This would imply that BHCs with high managerial protections increase their loans/assets relative to their counterparts, regardless of the stress tests. If this were so, it would cast significant doubt on the main channel proposed in the paper.

We rule out this possibility by re-estimating our baseline analysis of BHCs that were not subject to the first stress test in 2009. Specifically, we restrict our analysis to BHCs present during the entire sample period, consistent with a balanced sample, and remove the BHCs included in the first stress tests. Next, we manually link to the 2006 G-index for the largest remaining banks. For comparability, we use the same cutoff for managerial protection as our baseline and then remove the BHCs whose managerial protection score is equal to the median.

Re-estimating Equation 13 with BHCs that were not subject to stress-testing, we do not find evidence that loan activity is operating purely through the managerial protection channel. As suggested by column (1) of Table 4, the coefficient of the interaction term is statistically indistinguishable from zero. Similarly, when splitting the sample based on the earnings distribution of the BHC, the coefficients of the interaction terms of columns (2) and (3) are also statistically indistinguishable from zero, providing further confidence in our main results.

3.3.2 Additional Robustness Tests

While establishing the central predictions of our model, we conduct a battery of additional robustness tests to provide further confidence in our results linking managerial entrenchment to bank lending. We report the results of each robustness test in Table 5, with each row presenting the estimates from a separate regression. For comparability, we first repeat row (1) of Table 3 in row (1) of Table 5.

3.3.3 Alternative Measures of Managerial Protection

Our first set of robustness tests considers alternative governance index specifications. While the G-index proposed by Gompers, Ishii, and Metrick (2003) is a popular measure of managerial entrenchment, it is not without limitations.

Table 4: BHCs Not Subject to Stress Testing

Difference-in-differences regression of loans on G-index for bank holding companies not subject to stress-tests. This table presents results from estimating the regression $Loans/assets_{it} = \beta_1 \, I(Protection_i) \times I(Post_t) + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it}$, where $Loans/assets_{it}$ is the ratio of loans and unused commitments to total assets for bank holding company (BHC) i at quarter t, $I(Protection_i)$ is an indicator for whether a BHC's 2006 G-index is greater than the median among the set of stress-tested BHCs, $I(Post_t)$ is an indicator for whether the quarter occurs on or after 2010Q1, ψ_i represents BHC fixed effects, ϕ_t represents quarter fixed effects, and X_{it-1} is a set of control variables as described in Section 2.2. T-statistics computed using BHC-clustered standard errors are reported in parentheses. Column (1) runs the estimation on the full sample. Column (2) runs the estimation on the subsample of BHCs with a dominantly concave return distribution. Column (3) runs the estimation on the subsample of BHCs with a dominantly convex return distribution. ***, **, * denote significance at the 1%, 5%, and 10% level, respectively.

	(1) Full sample	(2) Concave	(3) Convex
$I(Protection_i) \times I(Post_t)$	0.498	0.015	0.195
	(0.89)	(0.03)	(0.53)
Tier 1 capital/assets	0.659	-1.173	0.543
	(0.72)	(-1.18)	(0.47)
NPLs/loans	-0.497	-2.164	0.082
	(-0.55)	(-1.05)	(0.14)
Non-interest expenses/assets	0.516	-1.677*	0.758
	(0.41)	(-2.24)	(1.50)
Net income/assets	0.151	0.054	-0.003
	(0.65)	(0.10)	(-0.02)
Liquid assets/assets	0.220	1.119***	-0.124
	(0.77)	(4.81)	(-0.39)
Sensitivity to market risk	-0.695***	-1.059*	-0.974***
	(-3.14)	(-2.68)	(-3.44)
Log assets	-1.398	0.655	-6.468
	(-0.32)	(0.05)	(-0.86)
Observations	898	240	610
R^2	0.868	0.931	0.869
BHC FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes

Table 5: Robustness

This table provides further robustness around our main specification of Table 3. Each row represents a separate regression. The test are organised around five categories: the baseline results (Panel A), alternative index measures (Panel B), alternative loan metrics (Panel C), and alternative empirical specifications (Panel D). Row 1 repeats the main coefficient of interest from Table 3, and is repeated for comparison. Row 2 replaces the G-index with the E-index proposed by Bebchuk, Cohen, and Ferrell (2009). Row 3 replaces the G-index with the D-index proposed by Karpoff, Schonlau, and Wehrly (2017). Row 4 reports replaces indicator of High G-index with the level of G-index. Row 5 replaces the indicator of High G-index with the level of the E-index. Row 6 replaces the indicator of High G-index with the level of the D-index. Row 7 reports results that do not normalize the dependent variable. Row 8 reports results when scaling the dependent variable with total risk weighted assets, after deductions. Row 9 reports results using a jackknifing re-sampling technique to rule out the disproportionate influence of outlier data points. Row 10 reports results standard errors double-clustered at the BHC and quarter level. All regressions control for variables as described in subsection 2.2. All regressions includes BHC and quarter fixed effects. Excluding row (10), all standard errors are clustered at the BHC level and are robust to heteroscedasticity. A dash indicates an insufficient number of observations for estimate. ***, **, * denote significance at the 1%, 5%, and 10% level, respectively.

	$(Loans/assets)_{it}$		
Panel A: Baseline	(1) Full sample	(2) Concave	(3) Convex
Baseline			
(1) $I(Protection_i) \times I(Post_t)$	16.002**	19.649***	-2.991
	(2.52)	(7.96)	(-0.69)
Panel B: Alternative Indexes			
High E-Index			
(2) $I(Protection_i) \times I(Post_t)$	23.243***	25.683***	13.730
	(5.49)	(6.77)	(1.94)
High D-Index			
(3) $I(Protection_i) \times I(Post_t)$	25.925***	24.919	-
	(4.06)	(2.55)	(-)
Level of G-Index			
(4) $Index_i \times I(Post_t)$	2.936***	1.862***	-0.253
	(3.31)	(10.99)	(-0.67)
Level of E-Index			
(5) $Index_i \times I(Post_t)$	5.970***	7.348***	1.582
	(4.40)	(7.40)	(1.50)
Level of D-Index			
(6) $Index_i \times I(Post_t)$	8.886***	6.230	-
	(4.43)	(2.35)	(-)
BHC Level Controls	Yes	Yes	Yes
BHC FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes

continued on next page...

Robustness (Continued)

Panel B: Alternative Loans Metrics	$(Loans/assets)_{it}$ (1) Full sample	(2) Concave	(3) Convex
log(Loans)			
(7) $I(Protection_i) \times I(Post_t)$	0.102	0.219***	-0.014
	(1.40)	(11.55)	(-0.14)
Loans/Total Risk Weighted Assets After Deductions			
(8) $I(Protection_i) \times I(Post_t)$	16.150**	19.221***	-2.204
	(2.56)	(7.98)	(-0.51)
Panel C: Alternative Empirical Specifications			
Jackknife Estimator			
(9) $I(Protection_i) \times I(Post_t)$	16.002*	19.649**	-2.991
	(1.88)	(3.63)	(-0.41)
Double Clustering			
(10) $I(Protection_i) \times I(Post_t)$	16.002**	19.649***	-2.991
	(2.62)	(6.90)	(-0.72)
BHC Level Controls	Yes	Yes	Yes
BHC FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes

Replacing our governance index with alternative measures, we find qualitative and quantitatively similar estimates to our baseline specification. As a reminder, each row represents a separate regression. We first replace High G-index with High E-index, put forward by Bebchuk, Cohen, and Ferrell (2009), in row (2). Next, we replace the G-index with the D-index, put forward by Karpoff, Schonlau, and Wehrly (2017) in row (3). In both estimates, we find similar results to our baseline specification. We also consider the levels of the G-, E-, and D-indexes in rows (4), (5), and (6), respectively. Again, we find qualitatively similar estimates to the baseline results. These estimates help mitigate concerns that our results are sensitive to the choice of index.

3.3.4 Alternative Measures of Lending

Our baseline specification uses total loans and unused commitments normalized by total risk-weighted assets before deductions. A concern is that a specific loan measure or normalizing metric may drive our baseline results. As a robustness test, we also consider alternative measures of loans as the dependent variable.

Using two alternative measures of loans, we again find similar results to our baseline estimates. In row (7), we replace our dependent variable with log(total loans). In row (8), we also consider banks' choice when making deductions by normalizing total loans by total risk-weighted assets after deductions.[20] Again, both of these estimates provide similar estimates to our baseline specification.

3.3.5 Alternative Specifications

Our third set of estimates addresses concerns about estimation errors. One potential concern with our analysis is that outliers drive it. If this were so, we would be overestimating the effects of managerial protections for loan provisions. We address this concern using a jackknife re-sampling technique in row (9), to mitigate the possibility that outliers drive our results. Lastly, we re-estimate our baseline specification when double-clustered at the BHC and quarter level, in row (10). In all tests, we find results consistent with our baseline estimates.

4 Conclusion

This paper theoretically and empirically demonstrates how managerial protections can affect the investment response of policies that constrain firm debt ratios, such as bank capital requirements and stress tests. We introduce a model in which managerial protections against shareholder actions can distort a manager's decisions about which projects to pursue. The model demonstrates that policies intended to constrain debt ratios depend on managerial protections. Specifically, such policies reduce the return to investment, but they also magnify the role of investment as a partial substitute for the effect of debt in disciplining managers from continuing unprofitable projects.

Taking the model to the data, we document the interaction between managerial protections and the U.S. bank stress tests in determining bank lending. Using a sample of BHCs that were included in the initial stress tests conducted by the Federal Reserve, we show that the introduction of the tests was associated with relatively greater lending for BHCs with strong managerial protections.

[20] These deductions are the allowances for loan and lease losses and the allocated transfer risk reserve, a reserve for certain foreign loans. This approach follows Berger and Bouwman (2017) and Acharya, Berger, and Roman (2018) by adding these reserves back to measure the full value of the assets financed.

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INTERNET APPENDIX
FOR ONLINE PUBLICATION

A Model

A.1 Omitted Proofs

Lemma 1. *The unique incentive compatible contract between the manager and the uninformed investors is debt. Specifically, the uninformed investors are paid a fixed amount $p(K,\alpha_U)$ whenever the project is not liquidated at $t=1$. If the project does not generate a large enough return to repay the promised amount, then the project is liquidated at $t=1$, and the uninformed investors are paid the liquidation value up to the value of their investment $\alpha_U K$. Let $\theta^*(K,\alpha_U)$ denote the threshold for θ at which the project is liquidated. Then $p(K,\alpha_U)$ and $\theta^*(K,\alpha_U)$ satisfy the following:*

- ***CASE I:*** *If $\alpha_U \leq 1-c$, then*

$$p(K,\alpha_U) = \alpha_U K \tag{2}$$
$$\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\} \tag{3}$$

- ***CASE II:*** *If $\alpha_U \geq 1-c$, then*

$$p(K,\alpha_U) = \theta^*(K,\alpha_U)y(K) \tag{4}$$
$$\alpha_U K = H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\underbrace{\theta^*(K,\alpha_U)y(K)}_{=p(K,\alpha_U)} \tag{5}$$

Proof of Lemma 1. Fix K and α_F. Let $\xi(\theta;\alpha_U)$ denote the payment to the uninformed investors. Then, incentive compatibility requires that

$$\theta \in \arg\max_{\theta'}\left[\frac{\alpha_F}{1-\alpha_U}\max\Big\{\theta y(K) - \xi(\theta';\alpha_U) + G, ((1-c)-\alpha_U)K, 0\Big\}\right] \tag{15}$$

where the first term is the manager's payoff if she does not terminate the project at $t=1$ and the second term is her payoff when she terminates/liquidates the project at $t=1$. Let $[0,\theta^*(K,\alpha_U))$ (i.e., the termination region) denote the region where in the manager terminates the project. And, when $\theta \in [\theta^*(K,\alpha_U),\infty)$ the manager continues the project at $t=1$.

We need to consider several cases.

Case 1 (High debt). Suppose $1-c-\alpha_U \leq 0$, equivalently, $(1-c-\alpha_U)K \leq 0$. Then, due to Eq. (15), we only need to consider $\theta y(K) - \xi(\theta;\alpha_U) + G$. Since $\theta^*(K,\alpha_U)$ is the termination threshold, thus, following the above specifications, $\theta < \theta^*(K,\alpha_U)$ implies termination and $\theta > \theta^*(K,\alpha_U)$ leads to continuation of the project.

Clearly, when $\theta > \theta^*(K,\alpha_U)$, i.e., θ is in the continuation region, the manager has no incentive to instead misreport it by θ' where $\theta' < \theta^*(K,\alpha_U)$. Simply because if she does so, she misses out $G > 0$. So, for any $\theta', \theta'' \geq \theta^*(K, \alpha_U)$ we must have

$$\xi(\theta'; \alpha_U) = \xi(\theta''; \alpha_U) = p(K, \alpha_U) \leq \inf_{\theta > \theta^*(K, \alpha_U)} \theta y(K) = \theta^*(K, \alpha_U) y(K).$$

Moreover, when $\theta < \theta^*(K, \alpha_U)$, the manager, in order to accrue her private benefit $G > 0$, may want to misreport her true type θ to some higher type θ', for which $\theta' > \theta^*(K, \alpha_U)$. Therefore, for $\theta \leq \theta^*(K, \alpha_U)$, the manager should not have incentive to misreport it to $\theta' > \theta^*(K, \alpha_U)$. For this, we must have

$$p(K, \alpha_U) \geq \sup_{\theta \leq \theta^*(K, \alpha_U)} \theta y(K) = \theta^*(K, \alpha_U) y(K).$$

Therefore,

$$p(K, \alpha_U) = \theta^*(K, \alpha_U) y(K).$$

We further note that, when the manager does not liquidate project, she must be able to pay $p(K, \alpha_U)$ to the uninformed investors, which is enforced by the terms of the contract. Otherwise, she will be penalized by more than her private control rent G. This enforceability can be easily implemented by regulators observing firm's fundamental θ at $t = 2$.

Finally, the uninformed investors individual rationality (IR) must be satisfied. As a result, the termination threshold $\theta^*(K, \alpha_U)$ satisfies the following uninformed investor's IR constraint:

$$H\Big(\theta^*(K, \alpha_U)\Big)(1 - c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y(K) = \alpha_U K.$$

Case 2 (low debt). Suppose $1 - c - \alpha_U > 0$. Then, we need to consider how large the private reward of the manager is. In this case we need to consider when the following two payoffs (termination vs. continuation payoffs) meet

$$\underbrace{\theta^*(K, \alpha_U) y(K) - p(K, \alpha_U) + G}_{\text{continuation payoff}} = \underbrace{(1 - c - \alpha_U)K}_{\text{termination payoff}} \tag{16}$$

Suppose manager's protection G is sufficiently *large* so that

$$(1 - c - \alpha_U)K - G \leq 0.$$

In this case, then we are back to Case 1, as the above equality never holds.

Next, suppose that G is sufficiently *small* so that

$$(1 - c - \alpha_U)K - G = ((1 - c)K - G) - \alpha_U K > 0.$$

Thus, due to (16), we have $\theta^*(K, \alpha_U) y(K) - p(K, \alpha_U) > 0$. Moreover, the uninformed investors' individually rational (IR) constraint is also satisfied because

$$\underbrace{H\Big(\theta^*(K,\alpha_U)\Big)\alpha_U K}_{\substack{\text{ex-ante termination payoff}\\ \text{of uninformed investors}}} + \underbrace{\Big(1-H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y(K)}_{\substack{\text{ex-ante continuation payoff}\\ \text{of uninformed investors}}} \geq \alpha_U K.$$

and the above IR constraint binds when $p(K,\alpha_U) = \alpha_U K$. Hence,

$$\theta^*(K,\alpha_U)y(K) = (1-c)K - G.$$

Putting together when debt is small, ie. $\alpha_U < 1-c$, the termination rule $\theta^*(K,\alpha_U)$ is given by

$$\theta^*(\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\}.$$

Payment rule $p(K,\alpha_U)$: Finally, we note that the payment rule $p(K,\alpha_U)$ (both in high and low debt cases) is characterized by the uninformed investors' IR constraint. That is, in the high debt case (i.e., $\alpha_U \geq 1-c$) $p(K,\alpha_U)$ satisfies

$$H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1-H\Big(\theta^*(K,\alpha_U)\Big)\Big)p(K,\alpha_U) = \alpha_U K,$$

and in the in the low debt case (i.e., $\alpha_U \leq 1-c$) $p(K,\alpha_U)$ satisfies

$$H\Big(\theta^*(K,\alpha_U)\Big)\alpha_U K + \Big(1-H\Big(\theta^*(K,\alpha_U)\Big)\Big)p(K,\alpha_U) = \alpha_U K.$$

□

Proposition 1. *If $\alpha_U = 1-c$ then $\theta^*(K,\alpha_U) = \theta^{opt}(K)$. If $G > 0$ then the converse also holds.*

Proof of Proposition 1. Fix K. Given that $\theta^{opt}(K) = \frac{(1-c)K}{y(K)}$, the proof immediately follows by Lemma 1. □

Proposition 2. *The liquidation threshold is increasing in the firm's debt ratio:*

$$\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha_U} \geq 0$$

Consequently, if $\alpha_U > 1-c$, the manager excessively liquidates the project,

$$\theta^*(K,\alpha_U) > \theta^{opt}(K).$$

If $\alpha_U < 1-c$ and $G > 0$, the manager excessively continues the project,

$$\theta^*(K,\alpha_U) < \theta^{opt}(K).$$

Proof of Proposition 2. Let $\alpha \equiv 1-\alpha_U$. Thus, to prove the claim it is sufficient to show $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} \leq 0$. To do this, we need to consider two cases (high debt and low debt). Throughout fix K.

Case1. Low debt $(1-c>\alpha_U)$. In this case depending on the size of the manager's private rent (managerial protection), G, the termination threshold satisfies $\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K-G, \alpha_U K\} = \max\{(1-c)K-G,(1-\alpha)K\}$. Therefore, rearranging gives

$$\theta^*(K,\alpha_U) = \max\{\frac{(1-c)K-G}{y(K)}, \frac{(1-\alpha)K}{y(K)}\}.$$

Suppose $(1-c)K-G < (1-\alpha)K$. Then, using the implicit function theorem, we have $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} = \frac{-K}{y(K)} < 0$. And, when $(1-c)K-G > (1-\alpha)K$, then $\theta^*(K,\alpha_U) = \frac{(1-c)K-G}{y(K)}$ thus $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} = 0$. Putting together, $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} \leq 0$.

Case2. High debt $(1-c<\alpha_U)$. For ease of notation let us denote the termination threshold by ℓ. In this case, the termination threshold satisfies the uninformed investor's IR constraint:

$$H(\ell)(1-c)K + (1-H(\ell))\ell y(K) = \alpha_U K = (1-\alpha)K.$$

We need to first show such threshold exists. Let us define for $\ell \geq \theta^{opt}(K) = \frac{(1-c)K}{y(K)}$,

$$\gamma(\ell) \equiv H(\ell)(1-c)K + \Big(1-H(\ell)\Big)\ell y(K) - \alpha_U K.$$

Thus, $\gamma(\frac{(1-c)K}{y(K)}) < 0$. As a result, the termination threshold is bigger that $\frac{(1-c)K}{y(K)}$. Taking derivative w.r.t. ℓ and substituting $(1-c)K$ by $\theta^{opt}(K)y(K)$ imply

$$\gamma'(\ell) = \left[y(K) + \Big(\theta^{opt}(K)-\ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\right]\Big(1-H(\ell)\Big).$$

Clearly, $\lim_{\ell\uparrow\theta^{opt}(K)}\gamma'(\ell) > 0$. However, since (by the monotone hazard rate assumption) $\frac{h(\ell)}{1-H(\ell)}$ is increasing ℓ, thus

$$\lim_{\ell\to\infty}\left[y(K) + \Big(\theta^{opt}(K)-\ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\right] = -\infty.$$

Moreover, $y(K) + \Big(\theta^{opt}(K)-\ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}$ is decreasing in ℓ, thus, there exists a unique $\tilde{\ell}$ so that

$$\gamma'(\tilde{\ell}) = 0.$$

Hence, $\tilde{\ell}$ is the unique maximizer of $\gamma(\ell)$. Clearly, if $\gamma(\tilde{\ell}) \leq 0$ then the IR constraint is always violated (i.e., issuing debt is impossible). So, $\gamma(\tilde{\ell}) > 0$ and there exists $\theta^*(K,\alpha_U)$ (where $\theta^{opt}(K) < \theta^*(K,\alpha_U) < \tilde{\ell}$) so that $\gamma(\theta^*(K,\alpha_U)) = 0$, that is

$$\begin{aligned}\gamma(\theta^*(K,\alpha_U)) &= H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1-H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y(K) - (1-\alpha)K \\ &= 0\end{aligned}$$

Next, using the implicit function theorem, we have $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha}\gamma'\Big(\theta^*(K,\alpha_U)\Big)+K=0$, thus

$$\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha}=\frac{-K}{\gamma'(\theta^*(K,\alpha_U))}<0,$$

where the last inequality follows because $\gamma'(\ell)>0$ for $\theta^{opt}(K)<\ell<\tilde{\ell}$, finishing the proof.

□

Proposition 3. *There exists a unique interior capital level $K^{opt}\in(0,\infty)$ that solves (9) and satisfies the first order condition*

$$1=(1-c)\,H\Big(\theta^{opt}(K^{opt})\Big)+y'(K^{opt})\int_{\theta^{opt}(K^{opt})}^{\infty}\theta\,dH(\theta). \tag{10}$$

Proof of Proposition 3. We prove it in two steps. We first show that K^{opt} exists. Then, we show that it is unique.

Step 1. (Existence) Taking a derivate w.r.t. K from (9) and dividing by $1-H\Big(\theta^{opt}(K)\Big)$ imply

$$y'(K)\mathbf{E}[\theta|\theta>\theta^{opt}(K)]-1-\frac{H\Big(\theta^{opt}(K)\Big)}{1-H\Big(\theta^{opt}(K)\Big)}(1-c) \tag{17}$$

To show that the above equation has a solution we use continuity and the Rolle's theorem. We note that since $y(\cdot)$ (by assumption) is concave and increasing, thus $y'(K)\leq\frac{y(K)}{K}$. Next, recall that $\theta^{opt}(K)=\frac{(1-c)K}{y(K)}$. Therefore, since $y'(K)\leq\frac{y(K)}{K}$, thus $\theta^{opt}(K)$ is increasing in K. Moreover, since $y'(0)=\infty$, and $y'(\infty)=0$, thus $\lim_{K\to0}\theta^{opt}(K)=0$ and $\lim_{K\to\infty}\theta^{opt}(K)=\infty$. Therefore,

$$\lim_{K\to0}y'(K)\mathbf{E}\Big[\theta|\theta>\theta^{opt}(K)\Big]-1=\infty$$

and

$$\lim_{K\to0}\frac{H\Big(\theta^{opt}(K)\Big)}{1-H\Big(\theta^{opt}(K)\Big)}(1-c)=0.$$

Thus,

$$\lim_{K\to0}\left[y'(K)\mathbf{E}[\theta|\theta>\theta^{opt}(K)]-1-\frac{H\Big(\theta^{opt}(K)\Big)}{1-H\Big(\theta^{opt}(K)\Big)}(1-c)\right]=\infty. \tag{18}$$

Next, we consider when $K \to \infty$. Recall that $y'(K) \leq \frac{y(K)}{K}$, thus $\theta^{opt}(K)y'(K) < 1 - c < 1$. Moreover, one can show that (using the assumption that $\ln h(\theta)$ is concave):

$$\frac{H\left(\theta^{opt}(K)\right)}{1 - H\left(\theta^{opt}(K)\right)}\left(\mathbf{E}\left[\theta|\theta > \theta^{opt}(K)\right] - \theta^{opt}(K)\right) \leq 1.$$

Thus, given that $\theta^{opt}(K) = \frac{(1-c)K}{y(K)}$, we must have

$$y'(K)\mathbf{E}\left[\theta|\theta > \theta^{opt}(K)\right] - 1 < \frac{y'(K)}{\frac{H\left(\theta^{opt}(K)\right)}{1-H\left(\theta^{opt}(K)\right)}}$$

Note that

$$\lim_{K\to\infty}\{\frac{y'(K)}{\frac{H\left(\theta^{opt}(K)\right)}{1-H\left(\theta^{opt}(K)\right)}}\} = 0,$$

because $y'(\infty) = 0$. Therefore, since $\lim_{K\to\infty} H\left(\theta^{opt}(K)\right) = 1$, thus

$$\lim_{K\to\infty}\left[y'(K)\mathbf{E}\left[\theta|\theta > \theta^{opt}(K)\right] - 1 - \frac{H\left(\theta^{opt}(K)\right)}{1 - H\left(\theta^{opt}(K)\right)}(1-c)\right] = -\infty. \tag{19}$$

Putting (18) and (19) together along with continuity of (17) imply that the solution K^{opt} exists.

Step 2. (Uniqueness) To prove the uniqueness we establish that

$$\frac{\partial}{\partial K}\left(y'(K)\mathbf{E}\left[\theta|\theta > \theta^{opt}(K)\right] - 1 - \frac{H\left(\theta^{opt}(K)\right)}{1 - H\left(\theta^{opt}(K)\right)}(1-c)\right) < 0.$$

Recall that $\frac{\partial\theta^{opt}(K)}{\partial K} > 0$. Thus,

$$\frac{\partial}{\partial K}\left[\frac{H\left(\theta^{opt}(K)\right)}{1 - H\left(\theta^{opt}(K)\right)}(1-c)\right] > 0.$$

Therefore, to prove that above claim, we only need to show that

$$\frac{\partial \theta^{opt}(K)}{\partial K} y'(K) \frac{\partial \mathbf{E}[\theta|\theta > \theta^{opt}(K)]}{\partial \theta^{opt}(K)} + y''(K)\mathbf{E}\Big[\theta|\theta > \theta^{opt}(K)\Big] < 0.$$

It is clear that

$$\frac{\partial \theta^{opt}(K)}{\partial K} y'(K) \frac{\partial \mathbf{E}[\theta|\theta > \theta^{opt}(K)]}{\partial \theta^{opt}(K)} > 0$$

(because $\frac{\partial \theta^{opt}(K)}{\partial K} > 0$, $y'(K) > 0$, $\frac{\partial \mathbf{E}[\theta|\theta > \theta^{opt}(K)]}{\partial \theta^{opt}(K)} > 0$) and

$$y''(K)\mathbf{E}\Big[\theta|\theta > \theta^{opt}(K)\Big] < 0,$$

(because $y(\cdot)$ is concave).

Next, note that $\frac{\partial \theta^{opt}(K)}{\partial K} = \frac{1-c-\theta^{opt}(K)y'(K)}{y(K)} > 0$ and $0 \leq \frac{\partial \mathbf{E}[\theta|\theta > \theta^{opt}(K)]}{\partial \theta^{opt}(K)} \leq 1$, thus

$$\begin{aligned}
&\frac{\partial \theta^{opt}(K)}{\partial K} y'(K) \frac{\partial \mathbf{E}[\theta|\theta > \theta^{opt}(K)]}{\partial \theta^{opt}(K)} + y''(K)\mathbf{E}[\theta|\theta > \theta^{opt}(K)] \\
&< y''(K)\mathbf{E}[\theta|\theta > \theta^{opt}(K)] + (1-c)\frac{y'(K)}{y(K)} - \theta^{opt}(K)\frac{y'(K)^2}{y(K)} \\
&< (1-c)\frac{y'(K)}{y(K)} + \theta^{opt}(K)\left(y''(K) - \frac{y'(K)^2}{y(K)}\right) \\
&= (1-c)\left(\frac{y'(K)}{y(K)} + K[\frac{d}{dK}\left(\frac{y'(K)}{y(K)}\right)]\right)
\end{aligned}$$

Now, recall that $y'(\cdot)$ and $\frac{1}{y(K)}$ are convex and decreasing, and thus $\frac{y'(K)}{y(K)}$ is convex and decreasing.[21] As a result,

$$\frac{y'(K)}{y(K)} + K[\frac{d}{dK}\left(\frac{y'(K)}{y(K)}\right)] < 0$$

so the upper bound is negative, finishing the proof. □

Proposition 4. *If $\alpha_U > 1-c$, then $K^*(\alpha_U) < K^{opt}$.*

Proof of Proposition 4. When debt is high, due to Proposition 2, we have $\theta^*(K,\alpha_U) > \theta^{opt}(K)$. Thus, $K^*(\alpha_U)$ solves

$$K^*(\alpha_U) \in \arg\max_K \left\{ H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K - C_{h.\ debt} \right\} \tag{20}$$

where the high debt **cost term** is

$$C_{h.\ debt} \equiv \int_{\theta^{opt}(K)}^{\theta^*(K,\alpha_U)} K(\frac{\theta y(K)}{K} - (1-c))dH(\theta).$$

[21]Note that for any convex and decreasing function $\phi(x)$ we have $|\phi'(x)| > \frac{\phi(x)}{x}$.

Clearly, the derivative of

$$H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K$$

evaluated at $K = K^{opt}$ is zero (by (10)). Hence, to prove the claim we only need to show that the derivative of the high debt **cost term** $C_{h.\ debt}$ evaluated at $K = K^{opt}$ is positive.[22]

We prove this statement in several steps.

It observes that

$$\begin{aligned}\frac{dC_{h.\ debt}}{dK} = &\int_{\theta^{opt}(K)}^{\theta^*(K,\alpha_U)} \Big(\theta y'(K) - (1-c)\Big) dH(\theta) \\ &+ \frac{\partial \theta^*(K,\alpha_U)}{\partial K} h\Big(\theta^*(K,\alpha_U)\Big)\Big(\theta^*(K,\alpha_U)y(K) - (1-c)K\Big).\end{aligned}$$

Note that

$$(1-c)K = \theta^{opt}(K)y(K)$$

and

$$(1-c) = \frac{\partial \theta^{opt}(K)}{\partial K} y(K) + \theta^{opt}(K)y'(K).$$

Therefore,

$$\begin{aligned}\frac{dC_{h.\ debt}}{dK} &\geq \frac{\partial \theta^{opt}(K)}{\partial K} y(K)\Big(H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))\Big) + \frac{\partial \theta^*(K,\alpha_U)}{\partial K}\Big(\theta^*(K,\alpha_U) - \theta^{opt}(K)\Big) \\ &= y(K)\left[-\frac{\partial \theta^{opt}(K)}{\partial K}\Big(\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))\Big) + \frac{\partial \theta^*(K,\alpha_U)}{\partial K}\Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\right],\end{aligned}$$

where the last equality follows from differentiating (4) with respect to K.

Next to finish the proof, we show that

$$-\frac{\partial \theta^{opt}(K)}{\partial K}\Big(\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))\Big) + \frac{\partial \theta^*(K,\alpha_U)}{\partial K}\Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big) > 0. \qquad (22)$$

[22] In more details, consider the F.O.C. of (20) w.r.t. K implying

$$0 = \Big(H(\theta^{opt}(K))(1-c) - 1 + \int_{\theta^{opt}(K)}^{\infty} \theta y'(K) dH(\theta)\Big) - \frac{dC_{h.\ debt}}{dK}. \qquad (21)$$

When K goes to zero, then $H(\theta^{opt}(K))(1-c) - 1 + \int_{\theta^{opt}(K)}^{\infty} \theta y'(K) dH(\theta)$ goes to infinity and $\frac{dC_{h.\ debt}}{dK}$ goes to zero. Hence, the R.H.S. in (21) goes to infinity. Moreover, when $K = K^{opt}$ (using Proposition 3), then $H(\theta^{opt}(K))(1-c) - 1 + \int_{\theta^{opt}(K)}^{\infty} \theta y'(K) dH(\theta) = 0$. Hence, to prove the main result, we only need to show that $\frac{dC_{h.\ debt}}{dK}|_{K=K^{opt}} > 0$, implying $K^*(\alpha_U) < K^{opt}$.

Note that differentiating (4) with respect to K implies that

$$\frac{\partial \theta^*(K,\alpha_U)}{\partial K}\left[1 - H\Big(\theta^*(K,\alpha_U)\Big) - \Big(\theta^*(K,\alpha_U) - \theta^{opt}(K)\Big)h\Big(\theta^*(K,\alpha_U)\Big)\right]$$
$$= \frac{\partial \theta^{opt}(K)}{\partial K}\left[\frac{\alpha_U}{1-c} - H(\theta^{opt}(K)) - \Big(H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))\Big)\right]$$

therefore, showing (22) is equivalent to show

$$\left(\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))\right)\Big(\theta^*(K,\alpha_U) - \theta^{opt}(K)\Big)h\Big(\theta^*(K,\alpha_U)\Big)$$
$$> \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\Big(H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))\Big) \tag{23}$$

To show (23) we first note that since $\alpha_U > 1-c$ and $\theta^{opt}(K) < \theta^*(K,\alpha_U)$ thus

$$\frac{\alpha_U}{1-c} - H(\theta^{opt}(K)) > 1 - H(\theta^{opt}(K)) > 1 - H\Big(\theta^*(K,\alpha_U)\Big) \tag{24}$$

In addition, when $H(\cdot)$ is convex in the set $[\theta^{opt}(K), \theta^*(K,\alpha_U)]$ then

$$h\Big(\theta^*(K,\alpha_U)\Big) > \frac{H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))}{\theta^*(K,\alpha_U) - \theta^{opt}(K)} \tag{25}$$

and (24) and (25) together show (23).

Similarly, when $G(\cdot)$ is concave in the set $[\theta^{opt}(K), \theta^*(K,\alpha_U)]$ then since $\frac{h(z)}{1-H(z)}$ is increasing in z (the monotone hazard rate assumption) and $\theta^{opt}(K) < \theta^*(K,\alpha_U)$ thus

$$\frac{1 - H\Big(\theta^*(K,\alpha_U)\Big)}{1 - H(\theta^{opt}(K))} \leq \frac{h\Big(\theta^*(K,\alpha_U)\Big)}{h(\theta^*)} \leq \left(\frac{\theta^*(K,\alpha_U) - \theta^{opt}(K)}{H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))}\right)h\Big(\theta^*(K,\alpha_U)\Big)$$

and as a consequence,

$$\Big(1 - H(\theta^{opt}(K))\Big)\Big(\theta^*(K,\alpha_U) - \theta^{opt}(K)\Big)h\Big(\theta^*(K,\alpha_U)\Big)$$
$$\geq \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\Big(H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))\Big) \tag{26}$$

Replacing $1 - H(\theta^{opt}(K))$ with $\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))$ in (26) and noting $\frac{\alpha_U}{1-c} - H(\theta^{opt}(K)) > 1 - H(\theta^{opt}(K))$ finish the proof.[23]

□

[23]Notice that in the case where $H(\cdot)$ is convex in $\theta^{opt}(K)$ and concave in $\theta^*(K,\alpha_U)$ similar analysis hold. In this case (due to the Rolle's theorem) there exists $\tilde{\theta} \in [\theta^{opt}(K), \theta^*(K,\alpha_U)]$ so that $H'(\tilde{\theta}) = \frac{H(\theta^*(K,\alpha_U)) - H(\theta^{opt}(K))}{\theta^*(K,\alpha_U) - \theta^{opt}(K)}$.

Lemma 2. *The termination threshold increases in the level of investment:* $\frac{\partial \theta^*(K,\alpha_U)}{\partial K} \geq 0$.

Proof of Lemma 2. To prove the claim, we need to consider two cases (high debt and low debt).

Case1. Low debt $(1-c > \alpha_U)$. In this case depending on the size of the manager's private rent (managerial protection), G, the termination threshold satisfies $\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\} = \max\{(1-c)K - G, \alpha_U K\}$. Therefore, rearranging gives

$$\theta^*(K,\alpha_U) = \max\{\frac{(1-c)K - G}{y(K)}, \frac{\alpha_U K}{y(K)}\}.$$

Suppose $(1-c)K - G > \alpha_U K$. Then, $\theta^*(K,\alpha_U) = \frac{(1-c)K-G}{y(K)}$. Since $y(K)$ is increasing and concave thus $\frac{-1}{y(K)}$ and $\frac{K}{y(K)}$ are both increasing, thus $\theta^*(K,\alpha_U)$ is increasing in K. A similar argument holds when $\theta^*(K,\alpha_U) = \frac{(1-\alpha)K}{y(K)}$. Therefore, $\theta^*(K,\alpha_U)$ is increasing in K, i.e., $\frac{\partial \theta^*(K,\alpha_U)}{\partial K} \geq 0$.

Case2. High debt $(1-c < \alpha_U)$. For ease of notation let us denote the termination threshold by ℓ. In this case, the termination threshold satisfies the uninformed investor's IR constraint:

$$H(\ell)(1-c)K + (1-H(\ell))\ell y(K) = (1-\alpha)K = \alpha_U K.$$

We need to first show such threshold exists. Let us define for $\ell \geq \theta^{opt}(K) = \frac{(1-c)K}{y(K)}$,

$$\gamma(\ell) \equiv H(\ell)(1-c)K + \Big(1 - H(\ell)\Big)\ell y(K) - \alpha_U K.$$

Thus, $\gamma(\frac{(1-c)K}{y(K)}) < 0$. As a result, the termination threshold is bigger that $\frac{(1-c)K}{y(K)}$. Taking derivative w.r.t. ℓ and substituting $(1-c)K$ by $\theta^{opt}(K)y(K)$ imply

$$\gamma'(\ell) = \left[y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\right]\Big(1 - H(\ell)\Big).$$

Clearly, $\lim_{\ell \uparrow \theta^{opt}(K)} \gamma'(\ell) > 0$. However, since (by the monotone hazard rate assumption) $\frac{h(\ell)}{1-H(\ell)}$ is increasing ℓ, thus

$$\lim_{\ell \to \infty}\left[y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\right] = -\infty.$$

Moreover,

$$y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}$$

is decreasing in ℓ, thus, there exists a unique $\tilde{\ell}$ so that

$$\gamma'(\tilde{\ell}) = 0.$$

Hence, $\tilde{\ell}$ is the unique maximizer of $\gamma(\ell)$. Clearly, if $\gamma(\tilde{\ell}) \leq 0$ then the IR constraint is always violated (i.e., issuing debt is impossible). So, $\gamma(\tilde{\ell}) > 0$ and there exists $\theta^*(K,\alpha_U)$ (where $\theta^{opt}(K) < \theta^*(K,\alpha_U) < \tilde{\ell}$) so that $\gamma(\theta^*(K,\alpha_U)) = 0$, that is

$$\begin{aligned}\gamma(\theta^*(K,\alpha_U)) &= H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y(K) - \alpha_U K \\ &= 0,\end{aligned}$$

and $\gamma'(\ell) > 0$ for $\theta^{opt}(K) < \ell < \tilde{\ell}$.

Next, by the implicit function theorem, we have

$$\frac{\partial\theta^*(K,\alpha_U)}{\partial K}\gamma'(\theta^*(K,\alpha_U)) + H\Big(\theta^*(K,\alpha_U)\Big)(1-c) + \Big(1-H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y'(K) - \alpha_U = 0.$$

Therefore,

$$\begin{aligned}\frac{\partial\theta^*(K,\alpha_U)}{\partial K} &= \frac{-1}{\gamma'\Big(\theta^*(K,\alpha_U)\Big)}\left[H\Big(\theta^*(K,\alpha_U)\Big)(1-c) + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y'(K) - \alpha_U\right] \\ &> 0.\end{aligned}$$

The last equality follows, because $\gamma'\Big(\theta^*(K,\alpha_U)\Big) > 0$ and

$$H\Big(\theta^*(K,\alpha_U)\Big)(1-c) + \Big(1 - H(\theta^*(K,\alpha_U))\Big)\theta^*(K,\alpha_U)y'(K) - \alpha_U < 0,$$

because $y'(K) < \frac{y(K)}{K}$ and

$$\gamma\Big(\theta^*(K,\alpha_U)\Big) = H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y(K) - \alpha_U K = 0,$$

finishing the proof. □

Proposition 5. *Suppose that the debt ratio is less than the efficient level or $\alpha_U < 1 - c$. If there are no managerial protections, or $G = 0$, then the level of investment is equal to the first-best benchmark, and there is no distortion:*

$$K^*(\alpha_U) = K^{opt} = K^*(1-c)$$

If there are managerial protections, or $G > 0$, then the conditionally efficient level of investment can in general be greater or less than the first-best benchmark. If the debt ratio is low enough, or $\alpha_U \leq 1 - c - \frac{G}{K^{opt}}$, and H is strictly concave then investment is distorted upwards, or $K^*(\alpha_U) > K^{opt}$. If the cdf for the return distribution H is weakly convex, then the conditionally efficient level of investment is distorted downwards, or $K^*(\alpha_U) \leq K^{opt}$.

Proof of Proposition 5. We prove the proposition in the following two parts.

Concave $H(\cdot)$ with low debt ratio capital structure. When $\alpha_U < 1-c$ (i.e., low debt), due to Proposition 2, we have $\theta^*(K,\alpha_U) < \theta^{opt}(K)$. Thus, $K^*(\alpha_U)$ solves

$$K^*(\alpha_U) \in \arg\max_K \left\{ H\left(\theta^{opt}(K)\right)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K - C_{l.\ debt} \right\}$$

where the low debt **cost term** is

$$C_{l.\ debt} \equiv \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}} K\left(1-c-\frac{\theta y(K)}{K}\right) dH(\theta).$$

Clearly, the derivative of

$$H\left(\theta^{opt}(K)\right)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K$$

evaluated at $K = K^{opt}$ is zero (by (10)). Hence, to prove the claim we only need to show that the derivative of the low debt **cost term** $C_{l.\ debt}$ evaluated at $K = K^{opt}$ is negative.

Moreover, when $\alpha_U < 1-c$ then (due to (2))

$$\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\}.$$

Suppose that at $K = K^{opt}$ we have

$$\max\{(1-c)K^{opt} - G, \alpha_U K^{opt}\} = (1-c)K^{opt} - G$$

(which is ensured when $K^{opt} > \frac{G}{1-c-\alpha_U}$). Therefore,

$$\theta^*(K^{opt},\alpha_U)y(K^{opt}) = (1-c)K^{opt} - G$$

and taking a derivative w.r.t. K implies

$$\frac{\partial\theta^*(K^{opt},\alpha_U)}{\partial K} y(K^{opt}) + \theta^*(K^{opt},\alpha_U)y'(K^{opt}) = 1-c. \tag{27}$$

Next, to finish the proof, using (27), we show that the margin of the low debt **cost term** $C_{l.\ debt}$ evaluated at $K = K^{opt}$ is negative.

$$
\begin{aligned}
\frac{dC_{l.\ debt}}{dK} &= -\frac{\partial \theta^*(K,\alpha_U)}{\partial K} h(\theta^*(K,\alpha_U))\Big((1-c)K - \theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}(K)} (1-c-\theta y'(K)) dH(\theta) \\
&\leq -\frac{\partial \theta^*(K,\alpha_U)}{\partial K} h(\theta^*(K,\alpha_U))\Big((1-c)K - \theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \Big(H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)\Big)\Big(1-c-\theta^*(K,\alpha_U)y'(K)\Big) \\
&= -\frac{\partial \theta^*(K,\alpha_U)}{\partial K} h(\theta^*(K,\alpha_U))\Big((1-c)K - \theta^*(K,\alpha_U)y(K)\Big) + \\
&\qquad \Big(H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)\Big)\frac{\partial \theta^*(K,\alpha_U)}{\partial K} y(K) \\
&= \frac{\partial \theta^*(K,\alpha_U)}{\partial K} y(K)\left(H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)) - \Big(\theta^{opt}(K) - \theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U))\right) \\
&< 0,
\end{aligned}
$$

where the last inequality follows because $\frac{\partial \theta^*(K,\alpha_U)}{\partial K} \geq 0$ and

$$
H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)) - \Big(\theta^{opt}(K) - \theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U)) < 0,
$$

because $H(\cdot)$ is concave (using the Taylor expansions). As a consequence, the margin of the low debt cost term is negative at K^{opt}, implying that $K^*(\alpha_U) > K^{opt}$.

Convex and weakly convex (e.g., Uniform distribution) $H(\cdot)$ **with low debt ratio capital structure.** It is also possible to show that managerial protections can be associated with less investment for firms with low debt ratios, or $\alpha_U < 1-c$. In this section we show if the cdf for the return distribution H is weakly convex, then the conditionally efficient level of investment is less than the first-best benchmark, or $K^*(\alpha_U) \leq K^{opt}$.

Here we show when debt ratio is low and the distribution of θ is uniform (i.e., $\alpha_U < 1-c$ and $H(\cdot) \sim \text{Uniform}[a,b]$) then (similar to the high debt capital structure environment) we obtain $K^*(\alpha_U) \leq K^{opt}$. A similar result identically holds when the cdf $H(\cdot)$ is weakly convex on its compact support.

Recall that when $\alpha_U < 1-c$ (i.e., low debt), due to Proposition 2, we have $\theta^*(K,\alpha_U) < \theta^{opt}(K)$. Thus, then $K^*(\alpha_U)$ solves

$$
K^*(\alpha_U) \in \arg\max_K \left\{ H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K - C_{l.\ debt} \right\}
$$

where the low debt **cost term** is

$$
C_{l.\ debt} \equiv \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}} K\left(1-c-\frac{\theta y(K)}{K}\right) dH(\theta).
$$

Clearly, the derivative of

$$H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K$$

evaluated at $K = K^{opt}$ is zero (by (10)). Hence, to prove the claim that $K^*(\alpha_U) \le K^{opt}$ we only need to show that the derivative of the low debt **cost term** $C_{l.\ debt}$ evaluated at $K = K^{opt}$ is positive.

This claim follows because:

$$\begin{aligned}
\frac{dC_{l.\ debt}}{dK} &= -\frac{\partial\theta^*(K,\alpha_U)}{\partial K} h(\theta^*(K,\alpha_U))\Big((1-c)K - \theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}(K)} (1-c-\theta y'(K)) dH(\theta) \\
&\ge -\frac{\partial\theta^*(K,\alpha_U)}{\partial K} h(\theta^*(K,\alpha_U))\Big((1-c)K - \theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \Big(H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U))\Big)\Big(1-c-\theta^{opt}(K)y'(K)\Big) \\
&= -\frac{\partial\theta^*(K,\alpha_U)}{\partial K} h(\theta^*(K,\alpha_U))\Big((1-c)K - \theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \Big(H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U))\Big)\frac{\partial\theta^{opt}(K)}{\partial K} y(K) \\
&\ge \frac{\partial\theta^*(K,\alpha_U)}{\partial K} y(K)\Bigg(H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)) - \Big(\theta^{opt}(K) - \theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U))\Bigg) \\
&= 0,
\end{aligned} \tag{28}$$

where the third relation follows because $(1-c)K = y(K)\theta^{opt}(K)$ and thus

$$1 - c = y'(K)\theta^{opt}(K) + \frac{\partial\theta^{opt}(K))}{\partial K} y(K);$$

the fourth relation follows because (due to supermodularity of the threshold $\theta^*(K,\alpha_U)$ in α_U and K) when $\theta^*(K,\alpha_U) < \theta^{opt}(K)$ we have

$$\frac{\partial\theta^*(K,\alpha_U)}{\partial K} < \frac{\partial\theta^*(K,1-c)}{\partial K} = \frac{\partial\theta^{opt}(K)}{\partial K}.$$

Finally, since the derivative of the low debt **cost term** $C_{l.\ debt}$ evaluated at $K = K^{opt}$ is positive, thus $K^*(\alpha_U) \le K^{opt}$. Note that when $H(\cdot)$ is weakly convex on its compact support, then the last equality in (28) will be replaced by " $\ge$ " because for the convex C.D.F. $H(\cdot)$ we have

$$H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)) - \Big(\theta^{opt}(K) - \theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U)) \ge 0.$$

□

Proposition 6. *If there are no managerial protections, or $G = 0$, then the manager's preferred level of investment that maximizes his or her utility (equation (7)) for a given debt ratio α_U, $K_m(\alpha_U)$, is equal to the conditionally efficient level $K^*(\alpha_U)$. If there are managerial protections, or $G > 0$, then the manager's preferred level of investment is less than the conditionally efficient level, or $K_m(\alpha_U) < K^*(\alpha_U)$.*

Proof of Proposition 6. Suppose

$$K_m(\alpha_U) = \arg\max_K u_m = \arg\max_K \frac{\alpha_F}{1-\alpha_U} V\Big(K, \alpha_U\Big) + G\frac{\alpha_F}{1-\alpha_U}\Big(1 - H(\theta^*(K, \alpha_U))\Big).$$

Then, by the corresponding F.O.C. condition,

$$\frac{\partial u_m}{\partial K}|_{K=K_m(\alpha_U)} = 0. \tag{29}$$

Moreover, as shown in section 1.5.2, we have

$$\frac{\partial V(K, \alpha_U)}{\partial K}|_{K=K^*(\alpha_U)} = 0. \tag{30}$$

Then, evaluating $\frac{\partial u_m}{\partial K}$ at $K = K^*(\alpha_U)$ implies

$$\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} = \frac{\alpha_F}{1-\alpha_U}\left(\underbrace{\frac{\partial V(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)}}_{=0,\text{ by (30)}} - G\, h\Big(\theta^*(K,\alpha_U)\Big)\underbrace{\frac{\partial \theta^*(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)}}_{>0,\text{ by Lemma 2}}\right) < 0$$

the above inequality then shows that $\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} < 0$, as a consequence, $K_m(\alpha_U) < K^*(\alpha_U)$ for all level of debt α_U. □

A.2 Competitive Equity Market

The baseline model introduced in Section 1.1 assumes that informed investors obtain a fraction $\frac{\alpha_I}{1-\alpha_U}$ of the realized equity value. This section shows that Proposition 6, which broadly says that managerial protections can reduce a manager's preferred level of investment, is robust to alternatively assuming a competitive equity market in which the informed investors make zero rents, similar to the uninformed investors.

In that case, the individual rationality constraint for informed investors holds with equality, which implies

$$\lambda_I(K,\alpha_U) = \frac{\alpha_I K}{\int_{\theta^*(K,\alpha_U)}^{\infty} \theta y(K) dH(\theta) + H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K - \alpha_U K} \tag{31}$$

where $\lambda_I(K,\alpha_U)$ is the *endogenous* shares from the realized equity value that goes to informed investors, contributing $\alpha_I K$ in investment. Note that $\lambda_I(K,\alpha_U)$ implies that the ex-ante individual rationality constraint for informed investors bind. Hence, informed investors and debt holders are ex-ante identical. Importantly, this complex object $\lambda_I(K,\alpha_U)$ does depend on K.

Given the competitive market for informed investors, the manager's problem, fixing α_F, is to solve

$$\max_{K_m,\alpha_U} u_m = \max_{K_m,\alpha_U} \underbrace{V(K,\alpha_U)}_{\text{ex-ante realized equity value}} + \underbrace{\Big(1-\lambda_I(K,\alpha_U)\Big)\ G\ \Big(1-H(\theta^*(K,\alpha_U))\Big)}_{\text{private benefit of control}} \tag{32}$$

where

$$V\Big(K,\alpha_U\Big) = H\Big(\theta^*(K,\alpha_U)\Big)\Big[((1-c)-\alpha_U)K\Big]^+ + \int_{\theta^*(K,\alpha_U)}^{\infty} \Big(\theta y(K) - p(K,\alpha_U)\Big) dH(\theta) - (1-\alpha_U)K$$

(see subsection 1.1) and $\Big(1-\lambda_I(K,\alpha_U)\Big)\ G\ \Big(1-H(\theta^*(K,\alpha_U))\Big)$ is the manager's private benefit of control given that the project is not terminated at $t=1$, which is proportional to multiplication of the manager's protection G and the manager's equity share $\Big(1-\lambda_I(K,\alpha_U)\Big)$, consistent with the logic in (1).

While $\lambda_I(K,\alpha_U)$ is endogenous and complex, the following proposition shows that we still have $K_m(\alpha_U) < K^*(\alpha_U)$. That is, even when informed investors, like debt-holders, are ex-ante in zero gain, i.e., their individual rationality constraint binds, the underinvestment result in Proposition 6 still holds.

Proposition A.1. *Fix $\alpha_F < c$. Suppose the market for informed investors is competitive, i.e., informed investors' share from the realized equity value is*

$$\lambda_I(K,\alpha_U) = \frac{\alpha_I K}{\int_{\theta^*(K,\alpha_U)}^{\infty} \theta y(K) dH(\theta) + H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K - \alpha_U K}.$$

Then, for all level of debt α_U, we have $K_m(\alpha_U) < K^*(\alpha_U)$.

Proof of Proposition A.1. The proof follows similar steps as in the proof of Proposition 6. To show that for all α_U, $K_m(\alpha_U) < K^*(\alpha_U)$, it is enough to prove that $\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} < 0$. Note that since $\lambda_I(K, \alpha_U)$ depends on K, we first note that

$$\begin{aligned} \frac{\partial \lambda_I(K, \alpha_U)}{\partial K}|_{K=K^*(\alpha_U)} &= \frac{\partial}{\partial K}\left\{\frac{\alpha_I K}{V(K, \alpha_U) + (1-\alpha_U)K}\right\}|_{K=K^*(\alpha_U)} \\ &= \left(\frac{V(K, \alpha_U) + (1-\alpha_U)K - K(\frac{\partial V(K,\alpha_U)}{\partial K} + 1 - \alpha_U)}{V(K, \alpha_U) + (1-\alpha_U)K}\right)\left(\frac{\alpha_I}{V(K, \alpha_U) + (1-\alpha_U)K}\right)|_{K=} \\ &> 0 \end{aligned} \tag{33}$$

where that inequality follows by $\frac{\partial V(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)} = 0$, (as shown in section 1.5.2, see also (30). Using this result, next we have

$$\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} = \underbrace{\frac{\partial V(K, \alpha_U)}{\partial K}|_{K=K^*(\alpha_U)}}_{\text{=0, by (30)}} + \underbrace{\frac{\partial}{\partial K}\left(\Big(1-\lambda_I(K, \alpha_U)\Big)\, G\, \Big(1 - H(\theta^*(K, \alpha_U))\Big)\right)|_{K=K^*(\alpha_U)}}_{\text{<0, by (33) and Lemma 2}} < 0,$$

(note that $\Big(1-\lambda_I(K, \alpha_U)\Big)$ is decreasing in K, by (33), and $\Big(1 - H(\theta^*(K, \alpha_U))\Big)$ is decreasing in K by Lemma 2) finishing the proof. □

B Numerical Approximation of Earnings Distribution

This section describes a methodology for distinguishing bank holding companies (BHCs) with a relatively concave return distribution cdf. This methodology is motivated by Proposition 5, which shows that managerial protections cause a firm's conditionally efficient level of investment to be greater than the first-best benchmark if the debt ratio is low enough, or $\alpha_U \leq 1 - c - \frac{G}{K^{opt}}$, and the return distribution cdf H is strictly concave

First, we compute the empirical cdf of each BHC's return distribution using observations of return on assets during from the start of the sample 2006Q1 until the approximate start of the financial crisis in 2007Q3, (Bernanke, 2009). We think of the manager as responding to the stress tests based on the typical return distribution of the firm, so we omit the crisis because it may have caused uncharacteristic returns. Order the observed return on assets by θ_k, and let $H(\theta_k)$ denote the empirical cdf.

Second, we compute a numerical approximation of the second derivative of the cdf at each point by the formula

$$\frac{\frac{H(\theta_{k+1})-H(\theta_k)}{\theta_{k+1}-\theta_k} - \frac{H(\theta_k)-H(\theta_{k-1})}{\theta_k-\theta_{k-1}}}{\frac{(\theta_{k+1}-\theta_k)+(\theta_k-\theta_{k-1})}{2}}$$

BHCs for which the median second derivative is negative are identified as having a dominantly concave cdf, while BHCs for which the median second derivative is positive are identified as having a dominantly convex cdf.

Consistent with Proposition 5, Table 3 shows that, among the subset of BHCs with a dominantly concave return distribution, those with a higher degree of managerial protections experienced a relative increase in lending after the introduction of stress-testing.