# Cash Policies and Firm Size

Ali Kakhbod, A. Max Reppen, Tarik Umar, and Hao Xing

**Abstract**

Understanding firms' demand for cash is critical for answering many economic questions. We develop a model of firm dynamics allowing for *heterogeneous size*. The firm faces costly financing, fixed costs, and decreasing returns to scale. Surprisingly, the firm's demand for cash is U-shaped in firm size. When the firm is small, growth lowers cash demand because the relative size of the fixed costs declines sharply. Eventually, growth increases cash demand as the scale of the cash flow shocks increases. Consequently, cash holdings and issuance amounts (payout rates) are U-shaped (hump-shaped) in firm size. We find empirical support for these predictions.

*JEL classification: E22, G30, G31, G35*

*Keywords*: Cash holdings, liquidity management, financial flexibility, costly financing

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# 1. Introduction

U.S. corporations hold significant amounts of cash. In 2021, cash represented 21% of a firm's market capitalization for the average Compustat firm. Understanding how a firm's demand for cash varies with its size has long captured the attention of economists (e.g., Keynes, 1936; Baumol, 1952; Friedman, 1959; Meltzer, 1963*b*,*a*). This relation is important for reasoning about the cross-section of financial constraints, the cross-section of responses to aggregate shocks and regulations, and how monetary policy transmission affects economic activity, interest rates, and inflation (Johnson, 1962). Accordingly, a large literature on corporate cash balances has examined shifts in cash holdings across firms and over time (Graham and Leary, 2018). However, characterizing a firm's demand for cash is a challenging task as it is affected by many considerations. And, empirical work presents some puzzling patterns in the data to rationalize. For example, why are cash holdings increasing at both small firms (Bates, Kahle and Stulz, 2009) and large firms (Faulkender, Hankins and Petersen, 2019) over time?

A surprising new insight in our paper is that firms' demand for cash can be U-shaped in firm size. To determine a firm's demand for cash, we develop and solve a new model of firm dynamics that allows for *heterogeneous size*. We relax the common assumption of homogeneity in firm size by solving a model with two state variables: cash holdings and a stock of productive capital. The model firm optimizes firm value by managing the size of its cash reserve and productive capital stock. Because we relax the homogeneity assumption, a small firm is not simply a scaled-down version of a large firm. Instead, as our model firm invests and grows, the marginal returns to investing decline (Basu and Fernald, 1997). Also, because we model a fixed component of issuance costs, in addition to a proportional component of issuance costs, external financing costs relative to firm size decrease as a firm grows (Lee et al., 1996; Altınkılıç and Hansen, 2000). Also, the firm pays a fixed cost when operating (e.g., rent and lease costs, salaries, utility bills, insurance, and coupon payments on long-term debt) that consumes more of a firm's cash flows when the firm is small (Lev, 1974).[1]

[1]While assuming homogeneity in size makes models more tractable, this modeling choice requires assuming that a firm's capital stock is static or that a firm's environment scales linearly with a firm's capital stock so that small firms and large firms have identical policies. Thus, models assuming homogeneity in size cannot model fixed costs or diminishing returns to scale.

The U-shaped demand for cash emerges due to the interaction between fixed costs and firm size. The demand for cash depends on the probability of requiring external financing and the cost of external financing. When the model firm is small, the cost of external financing is higher relative to the firm's size because of the fixed component of issuance costs. Also, the fixed operating costs consume more of the cash flows, which increases the probability of requiring external financing. Additionally, the firm's investment opportunities are more attractive when a firm is small because of diminishing returns to scale, but investing is a use of cash holdings, increasing the need for costly financing. As the small firm grows, the demand for cash initially decreases steeply because the fixed costs are "fixed" so their effect on the value of cash is largest when a firm is small. Also, because of diminishing returns to scale, the marginal returns to investing decline more steeply when the firm is small. Eventually, as the firm continues to grow, its demand for cash increases again because the scale of the cash flow shocks becomes larger, which increases the likelihood of running out of cash.[2]

As a result of the U-shaped demand for cash in firm size, the model predicts that a firm's cash holdings are U-shaped in firm size. Additionally, the model predicts that when the firm raises cash externally, the firm's optimal issuance amount is U-shaped in firm size. And, the U-shaped demand for cash in firm size leads to payout rates being hump-shaped in firm size. The hump-shaped relation emerges because when the demand for cash is high, the firm is more reluctant to make payouts to equity holders. When fixed operating costs are higher, the model predicts that these non-monotonicities in firm size are more convex for cash holdings and issuances and more concave for payout rates.

We take these predictions to the data. Examining variation in cash holdings within a firm, we show that cash holdings are visibly U-shaped in firm size and that the convexity is statistically significant. We also find in the data that issuance amounts are U-shaped in firm size and that payout rates are hump-shaped in firm size. Consistent with the mechanism in our model, the U-shaped patterns for cash holdings and issuance amounts are stronger for firms with higher fixed operating costs according to the measure in Novy-Marx (2010) and in industries with higher fixed costs, notably the mining, construction,

[2]The scale of a firm's cash flow shocks increases with size because a firm's cash flow shocks are the product of a size-invariant productivity shock with volatility $\sigma$ and the size of the capital stock $k^{\alpha}$, where $\alpha < 1$ captures the degree of diminishing returns to scale.

manufacturing, and wholesale industries. Also, consistent with fixed costs being relatively larger when a firm is small, these non-monotonicities emerge primarily when a firm is smaller than its average size in the sample. In Internet Appendix D, we show that these patterns are robust. For example, we show that these non-monotonicities in firm size hold for different proxies of firm size and hold before and after the median sample year of 1996.

Our findings contribute to the broad literature on a firm's cash holdings. A puzzle has been the large increase in firms' cash holdings over the past several decades. Bates, Kahle and Stulz (2009) and Begenau and Palazzo (2021) suggest that the increase is because public firms are becoming smaller with riskier cash flows. However, Faulkender, Hankins and Petersen (2019) show that the increase in cash holdings is also concentrated in large firms. The U-shaped demand for cash in firm size may partially explain why cash holdings increased more so at both small and large firms since U.S. industries have become more concentrated with a few larger firms and many smaller firms (Autor et al., 2017; Gutiérrez and Philippon, 2017; Grullon, Larkin and Michaely, 2019; Covarrubias, Gutiérrez and Philippon, 2019).[3] Cash balances are also important for the large literature on financial constraints. While the propensity to save cash flows is used to measure external financing constraints (Almeida, Campello and Weisbach, 2004), our model supports the critique in Riddick and Whited (2009) that savings propensities reflect too many forces to measure financial constraints. In our model, cash hoarding behavior varies with many factors, including uncertainty, fixed operating costs, external financing costs, and investment opportunities. Our findings also extend the discussion of cash in monetary economics. Several papers argue that cash demand increases proportionally with size (Keynes, 1936; Samuelson, 1941), while others predict economies of scale in cash demand (Baumol, 1952). Related empirical analyses at different levels of aggregation (e.g., firm-level, industry-level, and money-supply) have since been performed and have found mixed results (Friedman, 1959; Meltzer, 1963*b*,*a*; Mulligan, 1997). Unlike these studies, we predict and find empirical evidence of non-monotonicities in cash holdings and firm size within a firm.

[3]Other papers suggest the increase in cash holdings is due to the opportunity cost of funds (Azar, Kagy and Schmalz, 2016); the increased importance of intangible assets (Falato et al., 2022); and low managerial ownership (Nikolov and Whited, 2014).

Our prediction that issuance amounts are U-shaped in firm size contributes to the limited literature on issuance amounts. Our model helps rationalize the empirical finding in McLean (2011) that over time firms are saving more of their issuances as cash. Again, if industry concentration is increasing, such that the dispersion in firm size increases, then the U-shape demand for cash in our model predicts an increase in the demand for cash holdings and issuance amounts. Prior papers on issuance amounts generally do not characterize how a firm's issuance amounts vary with a firm's size. To the best of our knowledge, the only paper in the trade-off theory literature that considers issuance amounts, Hovakimian, Opler and Titman (2001), finds no relation. In addition, only one paper in the market timing literature, Chang, Dasgupta and Hilary (2006), takes issuance amounts into account, though several papers consider them in the context of discussing issuance costs (Smith, 1977; Lee et al., 1996; Chen and Ritter, 2000; Altınkılıç and Hansen, 2000; Butler, Grullon and Weston, 2005; Gomes, 2001; Hennessy and Whited, 2007). In the Internet Appendix, we examine the results in Hovakimian, Opler and Titman (2001) and Chang, Dasgupta and Hilary (2006) and show that the U-shape we document holds significantly even when accounting for their covariates and also improves their explanatory power (adjusted $R^2$) by 50%–80%.[4]

Our results showing a hump shape in payout rates and firm size contribute to the large literature on payouts. Because the hump-shaped relation means payout rates are lower when a firm is smaller and when a firm is larger, our model helps rationalize why Fama and French (2001) find that payout rates are declining over time at both smaller and larger firms. The hump-shaped relation of payouts in firm size coupled with higher industry concentration may again help rationalize this finding. Empirical studies examining payouts do not account for this hump-shaped relation (Becker, Ivković and Weisbenner, 2011; Becker, Jacob and Jacob, 2013; Desai and Jin, 2011; Hail, Tahoun and Wang, 2014; Hanlon and Hoopes, 2014; Bliss, Cheng and Denis, 2015; Kumar and Vergara-Alert, 2020). In Internet Appendix Table D.5, we show for the main specification in Bliss, Cheng and Denis (2015) that our predicted hump-shaped relation holds and is statistically significant even when accounting for their covariates and that it increases the specification's adjusted-$R^2$ by 15% and the magnitude of interest by 46%.

[4]Internet Appendix Tables D.2, D.3, and D.4 include a discussion of the results in Chang, Dasgupta and Hilary (2006) and Hovakimian, Opler and Titman (2001).

This paper also extends the dynamic liquidity management literature in several ways. We uniquely characterize and solve for the firm's optimal value via a dynamic programming equation with productive capital stock $k$ and cash holdings $c$ as two state variables.[5] Most other models examining a firm's cash policy do not consider firm size because they do not model investment and assume that cash-flow shocks are independent and identically distributed through time, which implies that a firm's economics is constant through time (e.g., Décamps et al., 2011; Anderson and Carverhill, 2012). Bolton, Chen and Wang (2011) model cash accumulation and investment, but assume constant returns to scale and linear issuance costs to satisfy their homogeneity assumption. The authors do not model heterogeneous size. Models with investment and assuming homogeneity cannot model a fixed component of issuance costs, a fixed operating cost, and diminishing returns to scale. These features of our model are necessary to produce the new mechanism: that a firm's cash demand, and consequently a firm's cash policies, are non-monotonic in firm size.

Lastly, before concluding the paper, we feature several additional predictions of our new model with heterogeneous size to guide future research. Specifically, we show that increasing the proportional component of issuance costs increases the demand for cash more when a firm is large. The rationale is that when a firm is large, the primary incentive to hoard cash is to manage the proportional issuance cost because the fixed component of issuance costs is relatively smaller. We also show that an increase in uncertainty increases cash hoarding behavior more when a firm is large because changes in uncertainty have larger-scale effects on cash flow volatility and thereby the probability of issuance for a given cash level. And, the model predicts that increases in fixed operating costs affect firm behavior more when the firm is small because they are relatively larger.

## 2. Model

To characterize how a firm's demand for cash varies with firm size, we develop a model of a firm's liquidity management that allows for *heterogeneous* size.

[5] We prove that the value function of the firm is the unique solution to this equation and show that the numerical algorithm converges to the unique solution of this dynamic programming equation (the numeric solution converges to the firm's value function) by proving a comparison theorem for (discontinuous) viscosity solutions to the Hamilton–Jacobi–Bellman (HJB) equation.

The firm's cash flows are stochastic and are a function of the size of a firm's productive capital stock and the firm's productivity. We assume that the firm's productivity $Z$ evolves according to

$$dZ_t = \mu dt + \sigma dW_t, \tag{1}$$

where $W_t$ is a one-dimensional Brownian motion under the risk-neutral measure and $\mu$ and $\sigma$ are positive constants. Thus, shocks $dZ_t$ are assumed to be i.i.d. with mean $\mu dt$ and variance $\sigma^2 dt$. The firm's cumulative cash flows $Y$ follow the dynamics

$$dY_t = k_t^{\alpha} dZ_t, \tag{2}$$

where $k$ is the size of the firm's productive capital stock and $\alpha \in (0,1)$ is a scale parameter following Bertola and Caballero (1994). Therefore, production exhibits decreasing returns to scale.[6] Also, as a firm grows, the scale of the cash flow shocks increases as the size-invariant productivity shock with volatility $\sigma$ is multiplied by the size of the capital stock $k$ according to $k^{\alpha}\sigma$.

The firm can invest in the productive capital stock. As is standard in capital accumulation models, for an investment process $i$, the dynamics of the capital stock follow

$$dk_t = \Big(i_t - \delta k_t\Big) dt, \tag{3}$$

where $\delta \geq 0$ is the depreciation rate. We assume that investment is irreversible, i.e., $i_t \geq 0$. As is common in capital accumulation models, investment is subject to a convex adjustment cost

$$g(k_t, i_t) = \frac{\theta}{2}\left(\frac{i_t}{k_t}\right)^2 k_t, \tag{4}$$

for a positive constant $\theta$ that measures the degree of the adjustment cost.

The firm determines its investment and cash management strategies, which include when to pay a dividend and when to raise equity. The value of the cash reserve follows

[6]Our model can accommodate $\alpha = 1$ or $\alpha > 1$; however, we find strong support in the data for the model's predictions with regard to investment when assuming $\alpha < 1$. Diminishing returns to scale is quite common in the literature. See Caballero (1991), Basu and Fernald (1997), Gomes (2001), and Grullon and Ikenberry (2021). Also, in Internet Appendix D.5, we show empirically that the investment-to-depreciation ratio is declining and convex in firm size and that firms' return on assets and return on equity decline in firm size, which is consistent with diminishing returns to scale.

the dynamics

$$dc_t = (r - \lambda_c)c_t dt + dY_t - bdt - i_t dt - g(k_t, i_t)dt - dD_t + dI_t. \tag{5}$$

Cash earns a return equal to the risk free rate ($r$) net of a carrying cost of holding cash ($\lambda_c$).[7] Even though cash earns a lower rate of return in the firm than out of the firm, the firm trades off the delay in dividend payouts against the risk management benefits of maintaining a cash reserve. $D_t$ is the cumulative dividend payout, and $I_t$ is the cumulative equity issuance. Both $D_t$ and $I_t$ are nondecreasing processes.

The firm pays a fixed cost when operating, $b$, which reduces a firm's cash reserve, especially when the firm has lower capital $k_t$ and thus lower cash flows. Effectively, the fixed cost when operating $b$ creates operating leverage. Because operating leverage and financial leverage are substitutes (Lev, 1974), a more general choice is to model the fixed cost as a fixed coupon payment on a firm's long-term debt.[8] In other words, the firm is partially financed by a fixed consol bond, which is a common modeling choice (e.g., Duffee, 1995; Hennessy, 2004; Manso, 2008).

Equity issuance is costly. For a lump-sum issuance of size $I$, the cost is

$$\lambda(I) = \lambda_f + \lambda_p I, \tag{6}$$

where $\lambda_f$ and $\lambda_p$ are constants representing the constant component and the proportional component of issuance costs, respectively. The fixed component of issuance costs results in issuance costs scaled by size decreasing in size. Prior work finds declining costs relative to size (Lee et al., 1996; Altınkılıç and Hansen, 2000).

[7] This assumption is standard in models with cash. For example, see Bolton, Chen and Wang (2011), Kim, Mauer and Sherman (1998) and Riddick and Whited (2009). If $\lambda_c = 0$, then the firm finds it optimal to hold as much cash as it can (indefinitely postponing the dividend) to prevent costly equity issuance. The equity is still valuable because equity holders could always choose to extract the cash via a dividend. The more realistic case is when $\lambda_c > 0$. Cash may earn low returns because interest earned on a firm's cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate (Graham, 2000; Faulkender and Wang, 2006). Agency problems may lower cash returns (Jensen, 1986; Harford, 1999; Dittmar and Shivdasani, 2003; Pinkowitz, Stulz and Williamson, 2006; Dittmar and Mahrt-Smith, 2007; Harford, Mansi and Maxwell, 2008; Demarzo et al., 2012; Caprio, Faccio and McConnell, 2011; Gao, Harford and Li, 2013; Kakhbod, 2021; Ward and Ying, 2022).

[8] Modeling the fixed cost $b$ as a fixed coupon payment on long-term debt is a more general choice that encapsulates the fact that in default certain fixed costs (e.g., lease expense, and salaries) have priority over payouts to equity holders much like payouts to debt holders. After discussing default in more detail, we provide additional clarification in Footnote 10.

Even if a firm neither pays out cash nor invests, its cash reserve can run out due to negative productivity shocks. When this happens, the firm compares the benefits of two options: (1) issuing equity and continuing (continuation value) and (2) the residual value for equity holders after liquidation (liquidation value). If the latter outweighs the former, the firm defaults. Therefore, the default time of the firm is

$$\tau = \inf\{t \geq 0 : c_t < 0\}.$$

When the firm defaults, its capital stock $k_\tau$ is fire sold. The recovery rate $\ell$ is assumed to be constant. The liquidation value $\ell k_\tau$ is used to pay off the long-term debt with the face value $b/r_{\text{debt}}$, where $r_{\text{debt}}$ is the cost of financing for long-term debt. If there is any value after paying the long-term debtholder, the remaining value, $\left(\ell k_\tau - b/r_{\text{debt}}\right)_+$, is distributed to the equity holders.[9,10]

*2.1. The firm's problem*

The firm chooses investment, dividend payout, equity issuance, and default timing to maximize the present value of dividend payouts net of equity issuance costs:

$$\sup_{i\geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}\left[\int_0^\tau e^{-rs} dD_s - \sum_j e^{-r\sigma_j}\left(I_j + \lambda(I_j)\right) + 1_{\{\tau<\infty\}} e^{-r\tau}\left(\ell k_\tau - b/r_{\text{debt}}\right)_+\right], \tag{7}$$

where $\{\sigma_j\}$ is a sequence of stopping times when the lump sum of equity of size $I_j$ is issued at each $\sigma_j$.[11]

The capital stock size and the cash reserve value, $k_t$ and $c_t$, are the two state variables for the firm's problem. The firm's value function is

$$\begin{aligned} V(k_t, c_t) = \sup_{i\geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}_t \Bigg[ \int_t^\tau e^{-r(s-t)} dD_s - \sum_{\sigma_j \geq t} e^{-r(\sigma_j - t)} \left(I_j + \lambda(I_j)\right) \\ + 1_{\{\tau<\infty\}} e^{-r(\tau-t)} (\ell k_\tau - b/r_{\text{debt}})_+ \Bigg]. \end{aligned} \tag{8}$$

It follows from the dynamic programming that the value function $V$ satisfies the HJB

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[9] $a_+ = \max\{a, 0\}$.

[10] We could assume that $\eta$ fraction of $b$ is the fixed coupon payment and the balance is a non-financial fixed operating cost. Then, the face value of the debt to be paid off in liquidation would be $\eta b/r_{debt}$. Without loss of generality, we currently assume $\eta = 1$.

[11] Note that because there is no information asymmetry between existing and new investors, one can simply think of the problem through the lens of one representative investor.

equation:

$$0 = \min \Bigg\{ rV - \sup_{i \geq 0} \Bigg\{ \Big[ i - \delta k \Big] \partial_k V + \Big[ (r - \lambda_c) c + k^\alpha \mu - b - i - g(k,i) \Big] \partial_c V + \frac{1}{2} k^{2\alpha} \sigma^2 \partial_{cc}^2 V \Bigg\},$$
$$\partial_c V - 1, V(k,c) - \sup_{I \geq 0} \Big[ V(k, c+I) - I - \lambda(I) \Big] \Bigg\}. \tag{9}$$

In this equation, the firm chooses among three alternatives: investment (the group of terms on the first line of the right-hand side of the equation), dividend payout (the first group of terms on the second line), and equity issuance (the second group on the second line).

Regarding the investment alternative, $rV$ represents the required rate of return on equity, which equals the risk free rate demanded by risk-neutral investors. The term $\partial_k V$ is a firm's marginal benefit of capital; hence, $[i - \delta k]\partial_k V$ captures the marginal effect of net investment on equity value. The term $\partial_c V$ is the firm's marginal cost of cash; hence, $[(r - \lambda_c)c + k^\alpha \mu - b - i - g(k,i)]\partial_c V$ is the effect of a firm's expected savings on equity value. The term $\frac{1}{2} k^{2\alpha} \sigma^2 \partial_{cc}^2 V$ captures the effect of the volatility of cash holdings due to volatility in production on equity value.

To determine the optimal investment strategy, note that the first-order condition of $i$ in Equation (9) and the constraint $i \geq 0$ yields

$$\partial_k V - \Big[ 1 + \theta \frac{i^*}{k} \Big] \partial_c V \leq 0 \tag{10}$$

for the optimal investment $i^*$. This inequality is an equality when $i^* > 0$. Equation (10) shows that $i^* > 0$ if and only if $\partial_c V < \partial_k V$, i.e., when the marginal benefit of capital is larger than the marginal cost of cash. Moreover, $i^* > \delta k$ if and only if $1 + \theta \delta < \frac{\partial_k V}{\partial_c V}$. That is, when the ratio between the marginal benefit of capital and the marginal cost of cash is sufficiently high, the firm invests more than depreciation.

Regarding the dividend payout alternative, the firm postpones the dividend payout until the marginal cost of reducing the cash reserve matches the marginal benefit of the dividend payout, i.e., $\partial_c V = 1$.

Regarding the issuance alternative, at each point $(k,c)$ in the state space, the equity holders compare the value of the firm without issuance $V(k,c)$ to the best value for issuance $\sup_{I \geq 0} \Big[ V(k, c+I) - I - \lambda(I) \Big]$, where $V(k, c+I) - I - \lambda(I)$ is the firm value post-issuance, net of issuance costs. The firm only issues equity when the latter value is weakly larger.

The dynamic programming principle implies that all three alternatives in the HJB Equation (9) are nonnegative, that only one alternative equals zero at each point $(k, c)$ in the state space, and that the corresponding action is optimal for the firm.

The HJB Equation (9) is coupled with several boundary conditions. The boundary condition at $c = 0$ is determined by comparing the default and issuance values:

$$0 = \min\left\{V(k,0) - \Big(\ell k - b/r_{\text{debt}}\Big)_+, V(k,0) - \sup_{I\geq 0}\Big[V(k,I) - I - \lambda(I)\Big]\right\}. \tag{11}$$

In this equation, the boundary value $V(k, 0)$ dominates the default value $(\ell k - b/r_{\text{debt}})_+$ and the best issuance value $\sup_{I\geq 0}[V(k, I) - I - \lambda(I)]$, and is only equal to one of the terms for each value of $k$. If $V(k, 0)$ equals the former value, it is optimal for the firm to default; otherwise, issuance is optimal with an optimal size of $I$. The HJB equation (9) is numerically solved in a sufficiently large domain, with the payout and issuance boundaries determined endogenously in this domain.[12]

## 3. The Model Solution

In this section, we present and discuss the results of our model for a baseline set of parameters. The baseline values for the parameters are presented in Table 1, along with the rationale. In Section 5, we examine the comparative statics (varying the baseline parameters).

[Insert Table 1 Here]

### 3.1. Numeric results

Figure 1 depicts several interesting properties of the model firm's choices as its productive capital stock $k$ (horizontal axis) and cash holdings $c$ (vertical axis) vary. Because our model is two-dimensional with the two state variables $k$ and $c$, several *regions*

[12]When $k = 0$, the adjustment cost $g$ is infinitely large for $i > 0$. When $k = k_{max}$ with a sufficiently high $k_{max}$, the benefit of investment vanishes in comparison to the adjustment cost due to diminishing returns to scale. At both ends, we impose the value of no investment as boundary conditions. After $k_{max}$ is fixed, when the cash reserve is sufficiently high, we expect that the firm optimally pays out excess cash. Therefore, we impose the boundary condition $\partial_c V = 1$ at $c = c_{max}$ and $k \in [0, k_{max}]$ with a sufficiently high $c_{max}$. We check that the first group of terms on the right-hand side of (9) is negative to ensure that $c_{max}$ is sufficiently large. See the appendix for more technical discussions on the boundary conditions and convergence of the numeric scheme.

are labeled in the legend. (By contrast, models assuming homogeneity in size generate one-dimensional predictions.) We briefly name the different regions and then discuss them in more detail below. Lastly, we rationalize the novel non-monotonicities predicted in Figure 1.

Region $A$ in Figure 1 is the dividend payout region. Region $B$ ($C$) is the positive (negative) net investment region, where net investment is $(i^* - \delta k)$. Region $D$ is the no-investment region. The black line is the issuance target, representing a firm's optimal cash balance immediately after issuing equity.

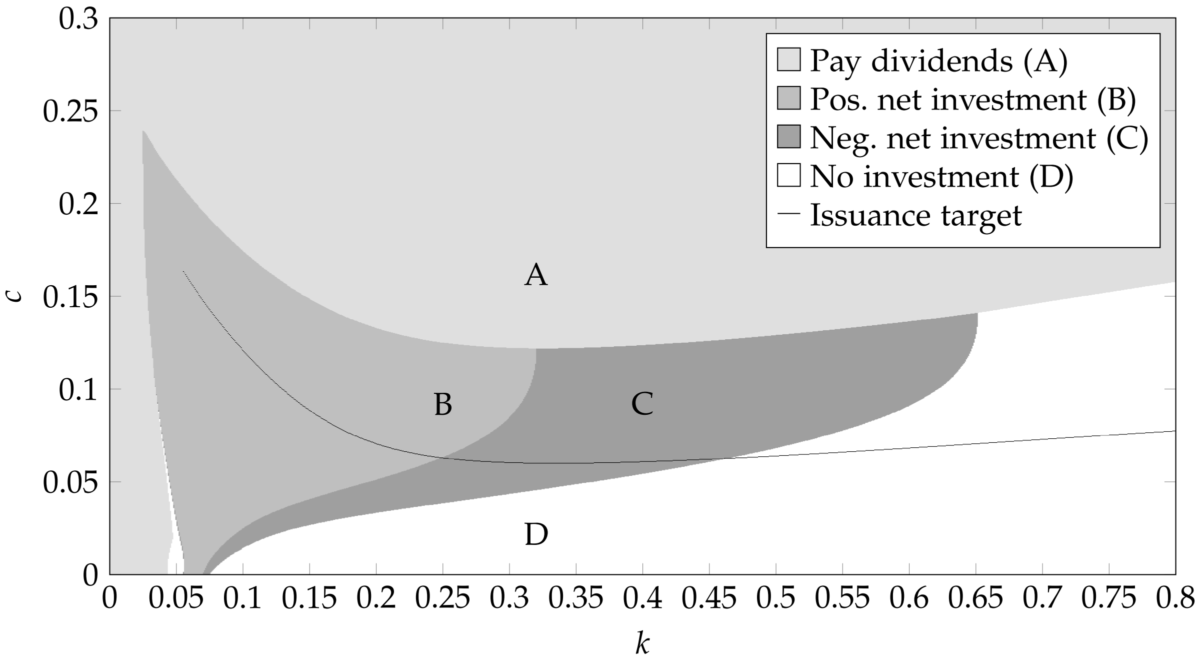



Figure 1: **Optimal regions**

*Parameters used are summarized in Table 1.*

In the payout region (A), the firm pays a dividend. The model firm only pays out a dividend when the marginal cost of reducing the cash reserve matches the marginal benefit of the dividend payout. If cash holdings $c$ is initially higher than the boundary of the payout region ($A$) and the investment regions ($B$ and C), a lump-sum dividend is paid out so that the state process $(k, c)$ lands exactly on the boundary after dividend payout. When the state process $(k, c)$ reaches the payout boundary from below, a minimal dividend is paid out to reflect the state process below so that the state process remains lower than the payout boundary. The payout boundary exists because of the liquidity premium $\lambda_c$, which implies that cash in the firm earns a lower return than investors can earn on the cash outside of the firm. The firm retains cash to economize over the issuance costs, but the value of these precautionary savings declines as the cash balance increases, and eventually the marginal benefit equals the liquidity premium.

The two investment regions ($B$ and $C$ in Figure 1) exist because of diminishing returns to scale. In the positive net investment region, labeled $B$, investment $i$ is larger than the depreciation $\delta k$; thus, capital builds up and the firm grows. In the negative net investment region, labeled $C$, the firm still invests, i.e., $i > 0$, but the investment is less than the depreciation $\delta k$. Because of diminishing returns to scale, the firm is generally in the positive net investment region ($B$) when smaller and in the negative net investment region ($C$) when larger. The interface between the positive ($B$) and negative ($C$) net investment regions is where investment exactly offsets depreciation. Internet Appendix Figure C.1 provides a heat map of net investment, $i - \delta k$.

The no-investment region ($D$) exists because of costly financing. This region predominately emerges when cash is low because the probability of costly financing is high. Thus, the firm may find it optimal to reduce investment to preserve cash and avoid costly financing. Interestingly, when the firm is small, even when cash is very low, the firm may desire to invest even though investing immediately results in costly issuance. Specifically, Figure 1 shows that investment regions $B$ and $C$ touch the horizontal axis when capital is low. The rationale is that when capital is low, the cash flows available to build the cash reserve are small. Consequently, it would take a long time to build up the cash reserve. Additionally, when capital is low, investment opportunities are strong. Thus, the firm may find it optimal to take advantage of these strong investment opportunities immediately by raising costly equity instead of waiting to fund the investments internally. For a more in-depth discussion of the determinants of investment, please see Internet Appendix C.1.

The issuance target (the black line in Figure 1) denotes a firm's optimal cash holdings after raising external financing. In other words, the model firm issues enough equity to land on the issuance target after issuance. The issuance boundary is the interval of $k$ when $c = 0$ for which there exists an issuance target line above ($k > 0.05$ in Figure 1). For this interval of $k$, when $c$ reaches zero, the firm issues a lump sum amount of equity, the size of which is specified by the black line or issuance target. Even though Equation (9) indicates that for each point $(k, c)$ in the state space, the firm compares the value of the firm without issuance to the best value for issuance, we show that the firm optimally issues equity only when the cash reserve runs out. (See the proof of Proposition 1 in Appendix A.)[13] Lumpy issuances help the firm economize over the fixed component of issuance costs. After issuance, the state process jumps upward to the issuance target and continues its dynamics.

Figure 1 also shows that the firm may choose to default or liquidate the firm. The default boundary is the interval of $k$ for which there does not exist an issuance target line ($k < 0.05$ in Figure 1). When $c$ reaches zero, if $k$ is in the default region, then the firm chooses not to issue and instead defaults. The default region emerges because of the adjustment costs and the fixed costs. If the firm is small enough, it would take equity holders too long or be too expensive to increase the size of the capital enough relative to the fixed operating costs and the fixed component of issuance costs.

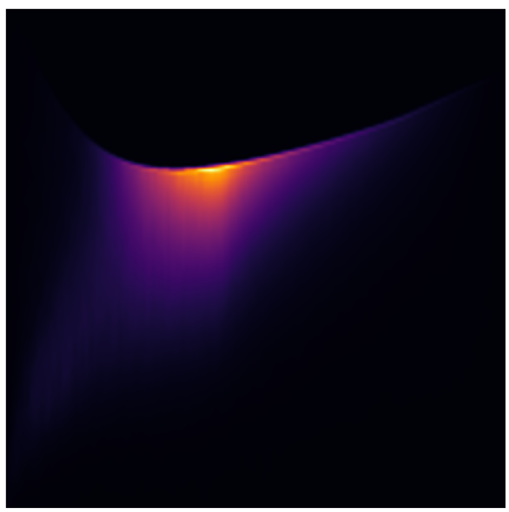


Figure 2: **A firm's time-series location in the capital-cash $(k, c)$ space**

*The heat map is generated by simulating paths until $t = 10$ starting from each grid point in a $1000 \times 1000$ grid in the figure domain. Brighter (more yellow) regions indicate the firm's more common location. The firm spends less time in the darker (black and dark purple) regions. Parameters used are summarized in Table 1. The scale is slightly different from that of Figure 1 because of cropping.*

[13] It will optimally raise cash only when it has to. First, cash within the firm earns a below-market return $r - \lambda_c$. Second, there is a time value for external financing costs. Therefore, without any benefit for early issuance, it is always better to defer external financing when $c > 0$. This argument highlights the pecking order between cash and external financing in the model.

Figure 2 presents a heat map of the firm's position in the $(k, c)$ space over time. We simulate paths until $t = 10$ starting in a $1000 \times 1000$ grid in the figure domain. The figure shows that the firm spends more time around the payout region for an intermediate level of $k$. The firm is less frequently in a position of large or small $k$ and less often has low cash $c$.

*3.2. The demand for cash is U-shaped in firm size.*

We can use our model to determine how a firm's demand for cash varies with the firm's size. We compute the marginal value of cash in the model ($\partial_c V$) as the difference in the value of the firm for a small increment in cash holdings scaled by the size of the small increment in cash holdings. Figure 3 plots the marginal value of cash, $\partial_c V$, against a firm's capital $k$ for the fixed cash level $c = 0.05$.

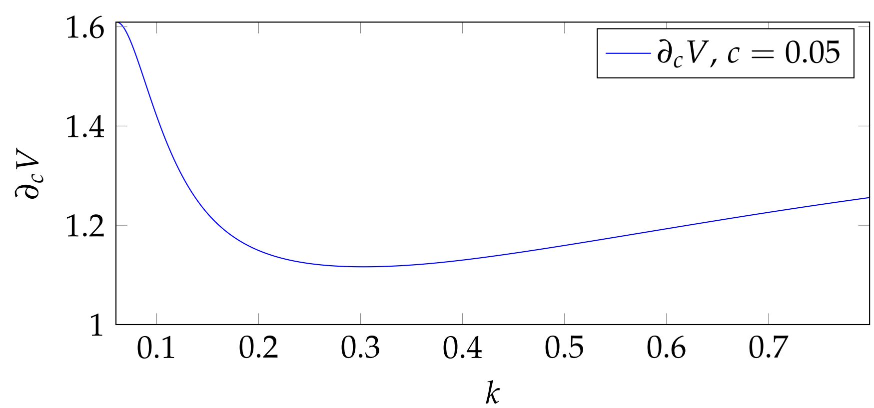



Figure 3: **Marginal value of cash and firm size**

*Parameters used are summarized in Table 1.*

Figure 3 highlights a surprising and novel prediction of our model: that a firm's demand for cash is non-monotonic, or U-shaped, in firm size. This pattern emerges because of the interaction between fixed costs and firm size. Cash helps the firm economize over issuance costs. Accordingly, the value of cash depends on the probability of needing external financing and the costs of that external financing. When a firm is small, the costs of financing are higher because of the fixed component of issuance costs. A small firm also has a higher chance of requiring external financing because of the fixed operating cost, which consumes more of the firm's expected cash flows, lowering profitability and increasing the likelihood of requiring external financing. Also, a small firm has a stronger incentive to invest because of diminishing returns to scale, which puts downward pressure on cash and increases the likelihood of requiring external financing. Together, these factors increase the demand for cash when a firm is small.

As the small firm's productive capital stock grows, initially, the demand for cash declines steeply because the fixed component of issuance costs and the fixed operating costs have their largest effect when a firm is small. The U-shape emerges because as the firm's productive capital stock grows, the scale of the cash flow shocks increases according to $k^{\alpha}\sigma$. (While the volatility of the productivity shock $\sigma$ is constant across firm sizes, the scale of the cash flow shocks increases with the size of the productive capital stock $k$.) Eventually, the effect of volatility on the firm's demand for cash dominates, and the firm's demand for cash increases with firm size.

The prediction that a firm's cash demand is U-shaped in firm size is a novel contribution of this paper. Older models of the demand for cash that consider cash holdings and firm size predict a monotonic relation (e.g., Baumol, 1952). More recent examinations of a firm's cash policies assume homogeneity in firm size (e.g., Bolton, Chen and Wang, 2011). By modeling *heterogeneous* size, we can uniquely model fixed costs and diminishing returns to scale, which together contribute to the U-shaped demand for cash.

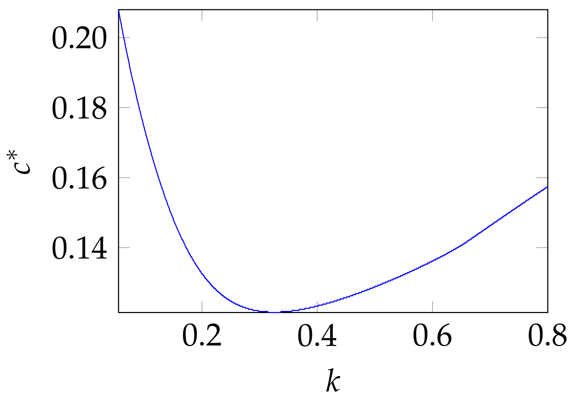
*3.3. Cash holdings and issuance amounts (payout rates) are U-shaped (hump-shaped) in firm size.*

An implication of the prediction that a firm's demand for cash is U-shaped in firm size is that a firm's cash holdings are predicted to be U-shaped in firm size. Figure 4(a) shows a firm's cash position at the highest density location in the simulations of Figure 2, when conditioning on the firm's capital size. It shows that a firm's cash position is U-shaped in its size.

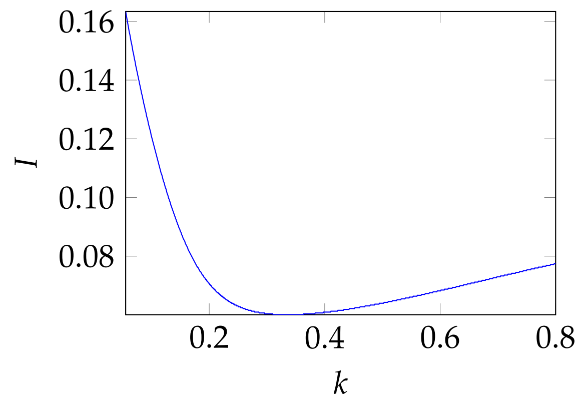
A related prediction is that when firms issue equity, the amount issued is U-shaped in firm size. Figure 4(b) plots the issuance target from Figure 1 to highlight that a firm's optimal issuance amount is U-shaped in a firm's size.

Figure 4(c) shows that a firm's dividend payout rates are hump-shaped in firm size. To translate the U-shape in a firm's dividend payout boundary in Figure 1 into a firm's payout rate, we simulate our model firm's payout activity. Specifically, we fix the cash level below the dividend boundary and then, for each level of capital, simulate the paid dividends over a short interval of time. We scale the predicted payouts from our simulation by a firm's size to arrive at a payout rate. Payout rates are hump-shaped because of firms' U-shaped demand for cash. Also, payout rates decline in firm size when a firm is large because we assume diminishing returns to scale. That is, the incremental cash flows from growth decline as firm size increases.

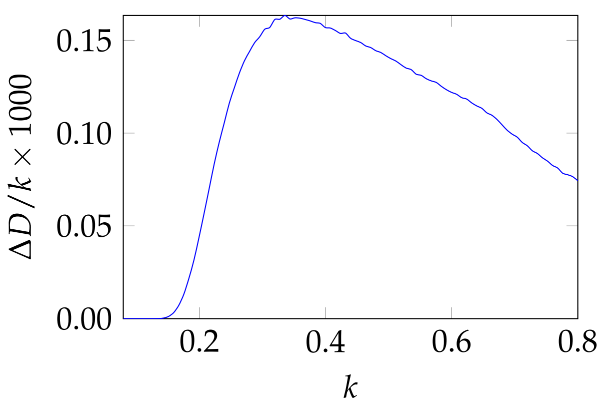
Internet Appendix C.2.5 shows that as the fixed operating cost $b$ increases, the U-shaped (hump-shaped) patterns of cash holdings and issuance amounts (payout rates) in firm size become more convex (concave).




(a) Cash holdings are U-shaped in firm size (cash hoarding)




(b) Issuance amounts are U-shaped in firm size (raising cash)




(c) Payout rates are hump-shaped in firm size (paying out cash)

Figure 4: **A firm's cash policies and firm size**

*Panel (a) plots the most frequent cash position for each k in the simulations in Figure 2. Panel (b) plots the issuance target in Figure 1. In panel (c), the vertical axis shows, for a given initial cash level, the amount paid out to equity holders (normalized by k) over a fixed interval of time. To construct the figure, we fix the cash level, which is lower than the dividend boundary, and then, for each level of capital, we simulate the paid dividends over a short interval of time. The horizontal axis is the size of the firm's capital stock. The parameters used are summarized in Table 1.*

# 4. Empirical Evidence

This section examines the statistical significance and robustness of the predictions in Figure 4 that cash holdings and issuance amounts (payout rates) are U-shaped (hump-shaped) in firm size.

*4.1. Data*

Our primary data source is the annual Compustat data file, which provides detailed financial statement information on public firms from 1971 through 2017.[14] After filtering the data, we have 9,690 firms and 114,481 firm years. (See Internet Appendix Table D.1 for details on the filtering process.)

The two primary state variables in the model are a firm's cash holdings and a firm's productive capital stock. To proxy for the cash state variable, we use a firm's cash and cash equivalents (*che*) from the year-end balance sheet. To proxy for the productive capital stock state variable, we use a firm's market equity less its cash position at the end of year $t$. We also show in the Internet Appendix D robustness to two additional measures: (1) the sum of a firm's tangible and intangible capital on the balance sheet and the intangible capital not on the balance sheet, as estimated by Peters and Taylor (2017)[15]; and (2) a firm's sales. Our primary proxy for firm size is a firm's market capitalization because these other measures are accounting numbers that may not reflect well the economic reality.

To proxy for a firm's issuance activity, we use the firm's total sales of common stock listed on the cash flow statement (*sstk*).

To proxy for a firm's payout activity, we use the firm's annual common stock dividends (*dvt*) and amount of common stock repurchases (*prstkc*).

To proxy for the firm's fixed costs when operating, we follow the measure of operating leverage in Novy-Marx (2010), which is the annual operating costs divided by assets (Compustat item *at*). The annual operating costs are the cost of goods sold (*cogs*) and the

[14]While Compustat data starts in 1950, the flow of funds data (cash flow statement) is not available until 1971. Thus, issuance data are not available prior to 1971. Our sample ends in 2017 because an alternative proxy for firm size incorporates a firm's intangible assets as estimated by Peters and Taylor (2017), which is only available through 2017.

[15]Considering intangible capital is important as Crouzet et al. (2022) observes that intangible assets are a large and growing part of firms' capital stocks and are also accumulated via investment (Eisfeldt and Papanikolaou, 2014).

selling, general, and administrative expenses (*xsga*). Because Novy-Marx (2010) shows that operating leverage and financial leverage are substitutes, we augment the measure in Novy-Marx (2010) to also include a firm's interest expense (*xint*) scaled by assets.

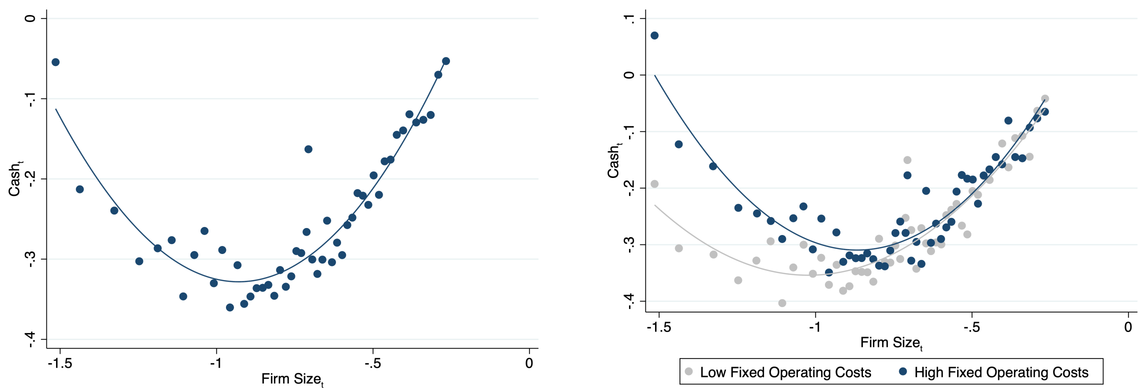
*4.2. Results*

Consistent with the prediction in Figure 4(a), Figure 5(a) shows empirically that firms' cash holdings are U-shaped in firm size. The results in Figure 5(a) are within a firm. To construct the bin scatter plot, we first determine a firm's market capitalization at the end of year $t$ and subtract the firm's cash holdings at the end of year $t$. Then, we standardize this quantity within a firm.[16] We also standardize within a firm, the firm's cash holdings. Lastly, we sort firm years into bins based on a firm's standardized size and plot the average standardized cash holdings. Because fixed operating costs have a larger effect on a firm's demand for cash when the firm is small, in Figure 5, we only show firm years when a firm is below its median size to better illustrate the U-shaped relation. For this reason, the horizontal axis shows negative values of standardized firm size. In Internet Appendix Figure D.7, we repeat the figure for all firm sizes. In that figure, the U-shape emerges when a firm is small.

Figure 5(b) shows that the U-shaped pattern of cash holdings in firm size is stronger when the firm's average measure of fixed operating costs is high (above the sample median), which is consistent with the mechanism in the model. Internet Appendix Figure D.1 examines the relation by industry and finds strong U-shaped relations in the manufacturing, mining, construction, and wholesale industries, which tend to have high fixed operating costs. The weakest U-shaped relation is in the services industry, which tends to use more labor-intensive production. In the Internet Appendix, we show the U-shaped relation is robust to measuring firm size with the sum of a firm's tangible and intangible capital (Figure D.2) and a firm's sales (Figure D.3) and also present other robustness tests.[17]

[16]When conducting a within-firm analysis, the proper standardization is to subtract the firm averages and divide by the firm-specific standard deviation so that the magnitudes and coefficients reflect the actual variability of the regressor after accounting for fixed firm differences (Mummolo and Peterson, 2018; deHaan, 2021). Standardizing within a firm removes between-firm differences in the variances of variables of interest in addition to differences in their means to facilitate the interpretation of the magnitudes.

[17]In the Internet Appendix, we show that the results in Figure 5(a) are robust to using cash holdings at the end of year $t + 1$ instead of at the end of year $t$ (Figure D.4), to using min-max scaling instead of standardizing within a firm (Figure D.5), and to splitting the sample at the median sample year of 1996




(a) Cash holdings and firm size

(b) Cash holdings, firm size, and fixed operating cost

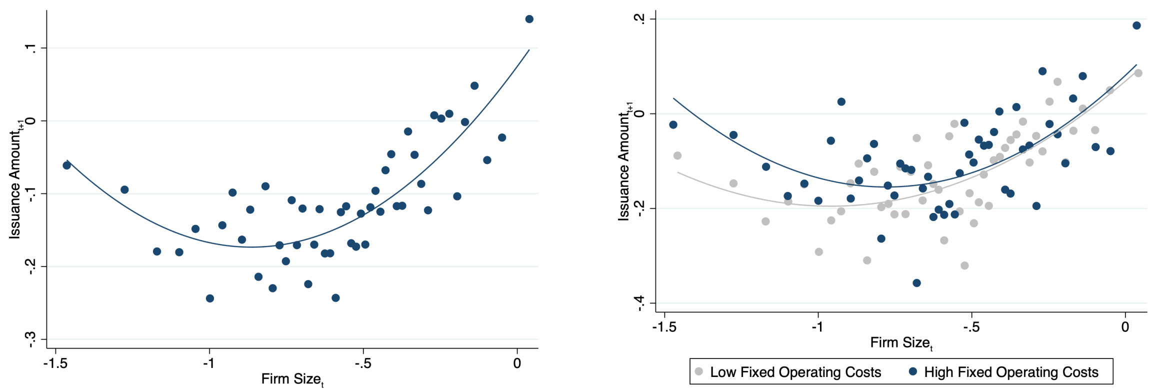
Figure 5: **Cash holdings are U-shaped in firm size.**

*The vertical axis is a firm's level of cash holdings at the end of year t, standardized within a firm. The horizontal axis is a firm's market equity less its cash reserve at the end of year t, standardized within a firm. To better illustrate the U-shape relation, we restrict the sample to firm years when the firm is below its median size. For this reason, the standardized values of firm size on the horizontal axis are always negative. In Internet Appendix Figure D.7, we show the figures for the full range of firm size. In panel A, we sort the firm-year observations into 50 bins based on a firm's standardized size. We plot the average of the standardized cash holdings for each bin. Each bin has about 1,100 firm-year observations. In panel B, we first sort firms by whether their average fixed operating costs in the sample are above the sample median.*

Consistent with the prediction in Figure 4(b), Figure 6(a) shows empirically that firms' issuance amounts in year $t+1$ are U-shaped in firm size. We require that offerings exceed \$5 million to focus on more material capital raises. We also drop the firm's IPO year. Again, because fixed operating costs are relatively more important when a firm is small, it is not surprising that the U-shaped relation is stronger when a firm is below its median size. Figure 6(b) shows that this pattern is stronger when the firm has high fixed operating costs, which is consistent with the mechanism in the model. Internet Appendix Figure D.8 shows that the U-shaped relation is more evident in the wholesale and manufacturing industries, which tend to have more fixed costs. The U-shaped relation is less evident in the lower-fixed-cost services industry. Importantly, we control for a firm's cash holdings at the end of year $t$ to account for differences in firms' ex-ante need for external financing. In the Internet Appendix, we show the U-shaped relation is robust to measuring firm size with the sum of a firm's tangible and intangible capital (Figure D.11) and a firm's sales (Figure D.12) and also present other robustness tests.[18]

(Figure D.6). We also show in Figure D.7 the results in Figure 5 using all firm sizes.

[18] The results in Figure 6(a) are robust to varying the minimum offering threshold from the default of \$5




(a) Equity issuance amounts and firm size

(b) Equity issuance amounts, firm size, and fixed operating costs

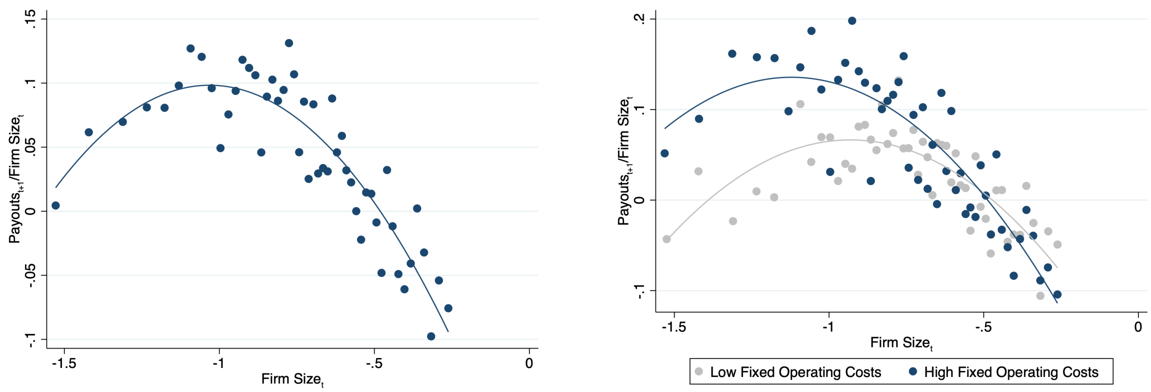
Figure 6: **Equity issuance amounts are U-shaped in firm size.**

*The vertical axis is a firm's amount of issuances during the year $t+1$, standardized within a firm. Before standardizing the issuance amount, we limit the sample to firm years with common stock sales exceeding \$5 million during the year $t+1$. The IPO year is also dropped. For this reason, only firms with multiple years of equity issuances above the specified threshold after the IPO offering are in the sample. The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. To better illustrate the U-shape relation, we restrict the sample to firm years when the firm is below its median size. For this reason, the standardized values of firm size on the horizontal axis are always negative. In Internet Appendix Figure D.15, we repeat the figures for the full range of firm size. In panel A, we sort firm-year observations into 50 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. Each bin has about 270 firm-year observations. In panel B, we first sort firms by whether their average fixed operating costs in the sample are above the sample median. We control for a firm's cash holdings at the end of year $t$.*

Consistent with the prediction in Figure 4(c), Figure 7(a) shows empirically that firms' payouts rates are hump-shaped in firm size. It is less obvious in Figure 7(b) that fixed operating costs affect the concavity of the hump-shaped relation. One issue with analyzing payouts is that few firms make payouts when small. Internet Appendix Figure D.18 shows the results by industry. The hump-shaped relation appears to be strongest for the mining, construction, and manufacturing industries, which tend to have more fixed operating costs. In the Internet Appendix, we show the hump-shaped relation is robust to measuring firm size with the sum of a firm's tangible and intangible capital (Figure D.19) and a firm's sales (Figure D.20) and also present other robustness tests.[19]

million (Figure D.9), to splitting the sample before and after the median sample year of 1996 (Figure D.10), to controlling for the size of the past offering (Figure D.13), and to min-max scaling instead of standardizing within a firm (Figure D.14). We also show in Figure D.15 the results in Figure 6 using all firm sizes.

[19]In the Internet Appendix, we show that the results in Figure 7(a) are robust to splitting the sample before and after the median sample year of 1996 (Figure D.17). We also show in Figure D.16 the results in Figure 7 using all firm sizes.




(a) Payout rates and firm size (b) Payout rates, firm size, and fixed operating costs

Figure 7: **Payout rates are hump-shaped in firm size.**

*The vertical axis is a firm's total payouts (common stock dividends and repurchases) during the year $t+1$, standardized within a firm. The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. To better illustrate the hump-shaped relation, we restrict the sample to firm years when the firm is below its median size. For this reason, the standardized values of firm size on the horizontal axis are always negative. In Internet Appendix Figure D.16, we repeat the figures for the full range of firm size. In panel A, we sort firm-year observations into 50 bins based on a firm's relative size and plot the average of the standardized payout amount for each bin. Each bin has about 1,000 firm-year observations. In panel B, we first sort firms by whether their average fixed operating costs in the sample are above the sample median. We control for a firm's cash holdings at the end of year $t$.*

We examine the statistical significance of these patterns in Table 2. To do so, we estimate the following empirical specification:

$$\begin{aligned}\text{Outcome} = {} & \beta_1 \text{Firm Size}_{i,t} + \beta_2 \text{Firm Size}^2_{i,t} \\ & + \beta_3 \text{Fixed Operating Costs}_i + \beta_4 \text{Firm Size}_{i,t} \times \text{Fixed Operating Costs}_i \\ & + \beta_5 \text{Firm Size}^2_{i,t} \times \text{Fixed Operating Costs}_i + \delta_{j,t} + \epsilon_{i,t}.\end{aligned}$$

We examine the following outcomes: a firm's cash holdings at the end of year $t$, a firm's total equity issued during year $t+1$, and a firm's payout rate during year $t+1$. *Firm Size*$_{i,t}$ denotes a firm's market capitalization at the end of year $t$ less the firm's cash holdings at the end of year $t$. We standardize *Firm Size*$_{i,t}$ within a firm. Thus, *Firm Size*$_{i,t}$ is positive (negative) when a firm is above (below) its average size in the sample. $\beta_2$ multiplies the quadratic form of standardized capital. The degree of non-monotonicity (convexity or concavity) in capital will depend on a firm's size relative to that same firm's average size in the sample. We interact the capital measures with *Fixed Operating Costs*$_i$, which is firm $i$'s average sum of the cost of goods sold; selling, general, and administrative expenses;

and interest expenses, scaled by a firm's total assets (Novy-Marx, 2010). The regressions control for annual industry trends, $\delta_{j,t}$, using SIC-1–by–year fixed effects. Consistent with the figures, we limit the regression to firm years when a firm is below its median size to better assess the statistical significance of the non-monotonicities.

[Table 2 Here]

The results in column (1) of Table 2 provide significant evidence that cash holdings are U-shaped in firm size. Column (2) shows that the U-shape is more significant for firms with higher fixed operating costs. Column (3) shows that issuance amounts are significantly U-shaped in firm size, and column (4) indicates that the U-shape is significantly stronger for firms with higher fixed operating costs. Column (5) shows that the payout rates are significantly hump-shaped. However, column (6) does not show significant evidence that the hump shape is significantly stronger for firms with higher fixed operating costs. Nevertheless, the coefficient being negative is in the correct direction.

## 5. Other Empirical Predictions

The model provides many additional predictions to guide future research beyond those examined carefully in this paper.

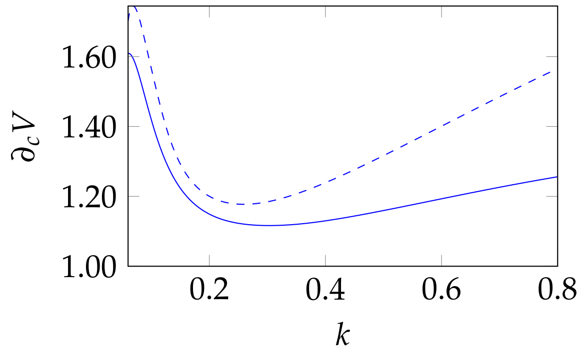
A large literature explores costly financing and firm behavior. Less attention has been paid to how changes in issuance costs differentially affect firms of different sizes. Figure 8 shows the predicted impact of a higher proportional issuance cost $\lambda_p$ on a firm's dynamics for different firm sizes. Figure 8(a) shows that when $\lambda_p$ increases, the marginal value of cash increases more for large values of $k$. This pattern arises because a large firm primarily accumulates cash to manage the proportional issuance costs $\lambda_p$. By contrast, when a firm is small, the marginal value of cash is more determined by the fixed costs when operating $b$, the fixed component of issuance costs $\lambda_f$, and the strong investment incentives. Accordingly, figure 8(b) shows that when $\lambda_p$ increases, the firm's cash holdings increase more for large $k$ than for small $k$. Figure 8(e) shows that when $\lambda_p$ increases, the reduction in investment is at its highest percentage at the lower boundary of the investment regions $B$ and $C$ in Figure 1. After $\lambda_p$ increases, the firm may choose not to invest; hence, the reduction is -100%. This result is consistent with the empirical evidence of Duchin, Ozbas and Sensoy (2010), who document that firm investment declined significantly in the 2008 crisis, and the decline was most significant among firms with low cash reserves.[20]

A long line of research examines cash flow volatility and firm behavior. Interestingly, Internet Appendix Figure C.5 also shows that our model predicts that increases in cash flow volatility more strongly affect the behavior of a firm when it is larger in size. Again, when a firm is small, its demand for cash is high even when cash flow volatility is low. This prediction builds on and extends the extant literature. Several papers provide cross-sectional evidence that firms facing more volatility hold more cash (e.g., Opler et al., 1999; Almeida, Campello and Weisbach, 2004; Palazzo, 2012). Few studies, however, have examined changes in both cash holdings and volatility. Duchin (2010) finds that acquisitions that generate more diversification lead to lower cash holdings. Bates, Kahle and Stulz (2009) finds that cash holdings increase more over time for industries experiencing larger increases in idiosyncratic volatility. Only a few studies have touched on how the sensitivity of cash holdings to volatility varies with firm size. Pinkowitz, Stulz and Williamson (2013) and Song and Lee (2012) provide some evidence that large firms increase their cash reserves more after a financial crisis; however, firms may exit the sample during a crisis, and government intervention may differentially affect small and large firms.

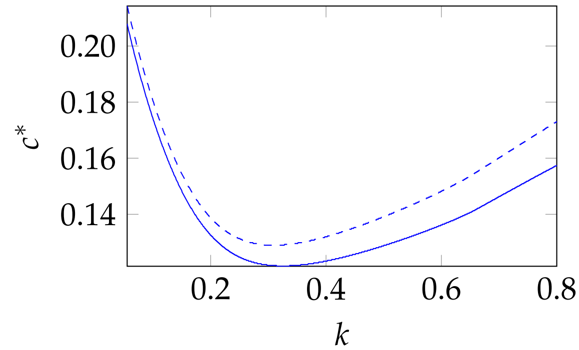
Another large stream of the literature centers on agency costs and firm behavior. Internet Appendix Figure C.6 shows how firm behavior changes with the liquidity premium a firm pays on cash, which may be interpreted as a reduced-form agency cost. If higher agency problems increase the costs of carrying cash, then the model suggests the firm optimally holds less cash, invests less, and issues less equity.

Lastly, Internet Appendix Figure C.7 shows how changing a firm's fixed operating cost $b$ affects the firm's decisions. Because $b$ is a fixed cost, as $b$ increases, a small firm's investments, payouts, and issuances change more. Investment rates fall as $b$ increases because firms forgo investment opportunities to save cash to avoid issuance costs. Because fixed operating costs and fixed debt costs both leverage a firm's cash flows (Lev, 1974), our model sheds light on why profitable firms with high fixed operating costs may use

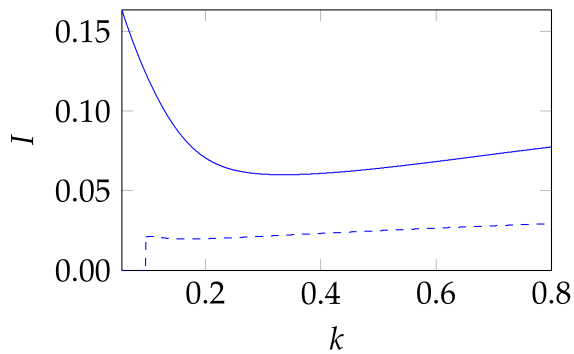
[20]Internet Appendix Figure C.4 shows firm dynamics when the fixed component of issuance costs changes.



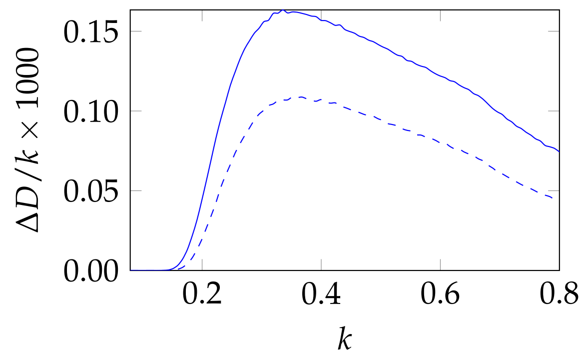

(a) Marginal value of cash and firm size when $c = 0.05$



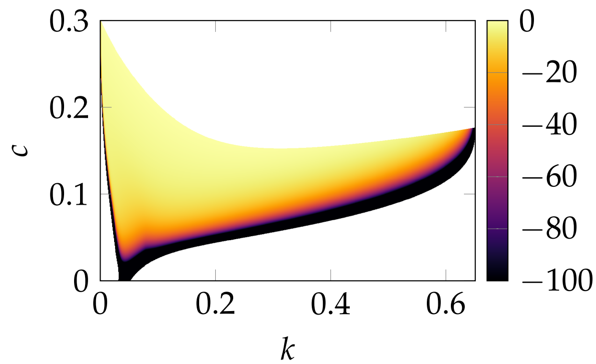

(b) Cash holdings are U-shaped in firm size (cash hoarding)




(c) Issuance amounts are U-shaped in firm size (raising cash)




(d) Payouts are hump-shaped in firm size (paying out cash)




(e) Percent reduction in investment with increase in $\lambda_p$

Figure 8: **Issuance costs and a firm's cash policies**

*When the proportional issuance cost increases by 10 times compared to its value in Table 1, the first four panels show the changes in the marginal value of cash, the most frequent cash position for each size in the simulations in Figure 2, the issuance target, and the amount paid out to equity holders (normalized by k) over a fixed interval of time starting from the same initial cash level. Panel (e) shows the* $(i_{high\ \lambda_p} - i_{low\ \lambda_p})/i_{low\ \lambda_p}$ *below the dividend payout boundary in Figure 1. Parameters used are summarized in Table 1.*

long-term debt conservatively (Opler et al., 1999; Graham, 2000).

## 6. Conclusion

Characterizing a firm's demand for financial flexibility or cash is important for many economic questions and has captured the attention of economists for decades. A surprising new insight in this paper is that fixed costs and firm size interact so that a firm's demand for cash is U-shaped in firm size. Consequently, a firm's cash holdings and issuance amounts may be U-shaped in firm size, and a firm's payout rates may be hump-shaped in firm size. We provide empirical support for these new predictions. And consistent with the mechanism in the model, the non-monotonicities in the data are generally more non-monotonic when the firm faces higher fixed costs.

To arrive at this new insight, we develop and solve a new model of firm dynamics that allows for *heterogeneous* size. By solving a model with the size of a firm's productive capital stock and cash holdings as separate state variables, we can model the effects of fixed costs and diminishing returns to scale on firms' cash policies. Importantly, allowing for heterogeneity in size enables us to examine how these effects vary with a firm's size. The interaction of fixed costs and firm size drives the new non-monotonicities in a firm's cash policies.

More generally, allowing for heterogeneity in firm size in models of firm dynamics promises to yield new insights applicable to a variety of research topics. We discuss several additional predictions that our current model provides to guide future research in Section 5. Moreover, there are several possible extensions of our model that would be fruitful to explore. For example, adding business-cycle risk to our model may help explain the empirical findings in Crouzet and Mehrotra (2020) that show small and large firms behave differently over the business cycle. Overall, relaxing the common assumption of homogeneity in firm size opens up many opportunities for theory to rationalize existing empirical puzzles and to identify surprising new relations.

## Appendix A. Omitted proofs

In Equation (9), the firm is allowed to issue at any point $(k, c)$ in the state space. The following result shows that it is suboptimal to issue before the cash reserve reaches zero. Delaying issuance until it is absolutely necessary allows the firm to defer issuance costs to the future, hence reducing its present value.

**Proposition 1.** *Firms only consider issuance when the cash reserve reaches zero.*

***Proof of Proposition 1.*** Because of the term $(r-\lambda_c)c$ appearing in the dynamics for $c$, the proof of Akyildirim et al. (2014) does not extend to this case. Instead, we define an equivalent control problem by letting

$$\tilde{c}_t = e^{-(r-\lambda_c)t}c_t, \quad \tilde{D}_t = \int_0^t e^{-(r-\lambda_c)s}dD_s, \quad \tilde{I}_t = \int_0^t e^{-(r-\lambda_c)s}dI_s.$$

Then,

$$d\tilde{c}_t = e^{-(r-\lambda_c)t}dY_t - e^{-(r-\lambda_c)t}(b + i_t + g(k_t, i_t))dt - d\tilde{D}_t + d\tilde{I}_t,$$

and the optimization problem is

$$V(k,c) = \sup_{i\geq 0, \tilde{D}, \{\sigma_j, \tilde{I}_j\}} \mathbb{E}\Big[\int_0^\tau e^{-\lambda_c t}d\tilde{D}_t - \sum_{\sigma_j \geq 0} e^{-\lambda_c \sigma_j}\Big(\tilde{I}_j + \lambda_p \tilde{I}_j + e^{-(r-\lambda_c)\sigma_j}\lambda_f\Big)\Big) \\ + 1_{\{\tau<\infty\}}e^{-r(\tau-t)}(\ell k_\tau - b/r_{\text{debt}})_+\Big],$$

where we note that $\tau$ can be defined equivalently for $c$ or $\tilde{c}$.

For this alternative formulation of the control problem, we note that issuance can be delayed at a discount. To make things clear, we fix an investment strategy $i$ and a dividend strategy $D$, index $\tilde{c}^\nu$ by an issuance strategy $\nu = \{\sigma_j, \tilde{I}_j\}_{j\in\mathbb{N}}$ with at least one issuance time $\sigma_i$ at which $\tilde{c}^\nu_{\sigma_i} > 0$. Let $\tilde{I}_i$ be the corresponding issuance amount. Consider another issuance strategy $\nu^-$ that omits this issuance, but keep the rest of the issuance strategy as $\nu$. Finally, construct a third strategy $\nu'$ like $\nu^-$ but with an additional issuance of size $\tilde{I}_i$ at a time $\sigma' = \inf\{t > \sigma_i : \tilde{c}^{\nu^-}_t < 0\}$.[21] Note that for the same $(i, D)$, the increment between $\tilde{c}^\nu$ and $\tilde{c}^{\nu'}$ is the same as between $\sigma_i$ and $\sigma'$. With the same issuance size, we have $\tilde{c}^\nu_{\sigma'} = \tilde{c}^{\nu'}_{\sigma'}$, so the continuation values must coincide, because the strategies are identical after $\sigma'_i$.

[21] If $\tilde{c}^{\nu^-}_t$ fell below zero due to a lump-sum dividend payout, we can balance out the dividend payout and the issuance to obtain the same result in the next step. If there are multiple issuances in $\nu$ between $\sigma_i$ and $\sigma'$, we omit all of them in $\nu^-$ and issue the sum of all missed size at $\sigma'$ in $\nu'$.

Moreover, dividends and issuance until $\sigma'$ have been identical, with one exception, for which $\nu'$ has resulted in a larger discounting factor and a smaller discounted fixed cost.

We therefore conclude that the original strategy is dominated by the one issuing equity only at $c = 0$. As this is true for any strategy, we may consider only strategies that issue equity when $c = 0$. □

## Appendix B. Comparison result and numeric algorithm

Proposition 1 lets us simplify the HJB equation where $c \neq 0$. We further restrict ourselves to the case where there is a maximal permitted investment rate $i_{\max} < \infty$. The proof here only relies on the boundedness of $i_{\max}$, but not on its size. We are now ready to state the resulting HJB equation. Let $\mathcal{O} = (0, \infty) \times (0, k_{\max})$.

$$
\begin{aligned}
0 = \min \Bigg\{ & rV - \sup_{i \in [0, i_{\max}]} \bigg( \Big[ i - \delta k \Big] \partial_k V + \Big[ (r - \lambda_c) c + k^\alpha \mu - b - i - g(k,i) \Big] \partial_c V \\
& \qquad\qquad\qquad + \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V \bigg), \\
& \partial_c V - 1 \Bigg\} \quad \text{in } \mathcal{O}.
\end{aligned}
\tag{B.1}
$$

At $k = 0$, $g(k,i) = \infty$ for any $i > 0$, i.e., investment is infinitely costly, so $k$ will remain zero forever. Hence, at the boundary, the value function satisfies

$$
0 = \min\{ rV - (r - \lambda_c)c + b, \partial_c V - 1 \},
$$

which has the solution $V = c$. Consider, therefore, the following boundary condition (with precedence to the first one in the corner):

$$
\begin{aligned}
c &= V && \text{at } k = 0, \\
0 &= \min\{ V - (\ell k - b / r_{\text{debt}})_+, V - \mathcal{I} V \} && \text{at } c = 0.
\end{aligned}
\tag{B.2}
$$

Here, $\mathcal{I}(V) = V(0,k) - \sup_{I \geq 0} \Big[ V(I,k) - I - \lambda(I) \Big]$.

**Theorem 2.** *Let $u$ and $v$ be, respectively, possibly discontinuous viscosity sub- and supersolutions to (B.1) with the above boundary conditions. Assume further that $u$ and $v$ are both of linear growth in $c$ and polynomial growth in $k$, i.e., they take values in $[c, c + M + p(k)]$ for some constant $M > 0$ and polynomial $p$. Then, $u \leq v$ everywhere in $\mathcal{O}$.*

*Proof.* Suppose there exists a point at which $u > v$. Fix some $\eta > 0$ and consider a maximizing sequence $(c_n, k_n)_{n \geq 1}$ to $\sup_{\mathcal{O}} e^{-\eta k}(u - v) > 0$. By the growth condition, $k_n$

is bounded by some $k^*$, where $k^*$ depends only on $\eta$. Now, for any $\zeta > 0$ small enough, there exists a point $(\bar{c}, \bar{k})$ such that $e^{-\eta\bar{k}}(u-v)(\bar{c},\bar{k}) = \delta_\zeta \geq \sup_{\mathcal{O}} e^{-\eta k}(u-v) - \zeta > 0$. We emphasize that $\bar{k}$ remains bounded, irrespective of $\zeta$. In particular, for any $\eta$, $\delta_\zeta/(\zeta + \sqrt{\zeta})$ can be chosen arbitrarily large.

We begin by showing that if such a point lies on the boundary $c = 0$, then there is another with the same property on the interior. Consider points $(\bar{c}, \bar{k})$ such that $\bar{c} = 0$. Then, depending on whether $u(0,\bar{k}) \leq \bar{k} - \ell/r_{\text{debt}}$ or $u(0,\bar{k}) \leq \mathcal{I}u$, we have

$$(u-v)(0,\bar{k}) \leq \ell\bar{k} - b/r_{\text{debt}} - \max\{\ell\bar{k} - b/r_{\text{debt}}, \mathcal{I}v\} \leq 0$$

or

$$(u-v)(0,\bar{k}) \leq \mathcal{I}u - \max\{\ell\bar{k} - b/r_{\text{debt}}, \mathcal{I}v\} \leq \sup_{I>0}\Big[u(I,\bar{k}) - v(I,\bar{k})\Big].$$

The first case contradicts $\delta_\zeta > 0$, and the second shows that another point with the same properties exists in the interior. Similarly, for $\bar{k} = 0$, we also get $(u-v)(\bar{c},0) \leq \bar{c} - \bar{c} = 0$. Hence, without loss of generality, we may assume $(\bar{c},\bar{k})$ lies away from $c = 0$ and $k = 0$.

Define

$$\begin{aligned}\Phi^{\epsilon,\gamma}(c,k,d,\ell) = (1-\gamma)&e^{-\eta k}u(c,k) - e^{-\eta\ell}v(d,\ell) \\ &- \beta(c-\bar{c})^4 - \frac{1}{2\epsilon}\Big((c-d)^2 + (k-\ell)^2\Big) \quad \text{in } \mathcal{O}\times\mathcal{O}.\end{aligned}$$

Clearly,

$$\sup_{\mathcal{O}\times\mathcal{O}} \Phi^{\epsilon,\gamma} \geq \Phi^{\epsilon,\gamma}(\bar{c},\bar{k},\bar{c},\bar{k}) = e^{-\eta\bar{k}}\Big((1-\gamma)u(\bar{c},\bar{k}) - v(\bar{c},\bar{k})\Big) > \delta_\zeta,$$

when $\gamma > 0$ is small enough. In particular, for any $\gamma > 0$ and $\eta > 0$, $\Phi^{\epsilon,\gamma}$ has a maximizer $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})$ because of the growth conditions on $u$ and $v$. Moreover, the growth conditions give an upper bound for this maximizer, depending only on $\gamma$ and $\eta$. Therefore, $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})$ converges along a subsequence as $\epsilon \to 0$. From here on, let us only consider $\epsilon$ along this subsequence. Because the lower bound at the maximum above is independent of $\epsilon$,

$$0 < \delta_\zeta < \liminf_{\epsilon\to 0} \Phi^{\epsilon,\gamma}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}),$$

which implies

$$\limsup_{\epsilon\to 0} \frac{1}{2\epsilon}\Big((c_{\epsilon,\gamma} - d_{\epsilon,\gamma})^2 + (k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2\Big) < \infty,$$

so $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}), (d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}) \to (c_\gamma, k_\gamma)$. Note that $k_\gamma \leq k^*$, again because of the growth condition.

Rearranging terms and letting $\epsilon \to 0$,

$$
\begin{aligned}
\lim_{\epsilon\to 0} \beta(c_{\epsilon,\gamma}-\bar{c})^4 + \lim_{\epsilon\to 0}\frac{1}{2\epsilon}\Big((c_{\epsilon,\gamma}-d_{\epsilon,\gamma})^2+(k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma})^2\Big)& \\
\leq \limsup_{\epsilon\to 0} e^{-\eta k_{\epsilon,\gamma}}(1-\gamma)u(c_{\epsilon,\gamma},k_{\epsilon,\gamma}) - e^{-\eta \ell_{\epsilon,\gamma}}v(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma}) - \delta_\zeta& \\
\leq e^{-\eta k_\gamma}\Big((1-\gamma)u(c_\gamma,k_\gamma)-v(c_\gamma,k_\gamma)\Big)-\delta_\zeta& \\
\leq \zeta.&
\end{aligned}
$$

That is,

$$
\lim_{\epsilon\to 0} \beta(c_{\epsilon,\gamma}-\bar{c})^4 + \lim_{\epsilon\to 0}\frac{1}{2\epsilon}\Big((c_{\epsilon,\gamma}-d_{\epsilon,\gamma})^2+(k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma})^2\Big)\leq \zeta. \tag{B.3}
$$

As $\beta$ may be taken arbitrarily large, we ensure that $\zeta < \beta\bar{c}^4$, so that $c_\gamma > 0$.

If $k_\gamma = 0$, we directly obtain $u(c_\gamma, 0) \leq c_\gamma \leq v(c_\gamma, 0)$, which is a contradiction. Hence, $(c_\gamma, k_\gamma)$ must lie in the interior, and so will $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma})$ and $(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})$ for sufficiently small $\epsilon$.

Because the maxima are attained in interior points, we proceed to use Ishii's lemma, from which we obtain $(p^u_\gamma, X) \in \bar{J}^{2,+}(e^{-\eta k_{\epsilon,\gamma}}(1-\gamma)u(c_{\epsilon,\gamma},k_{\epsilon,\gamma}))$ and $(p^v_\gamma, Y) \in \bar{J}^{2,-}(e^{-\eta \ell_{\epsilon,\gamma}}v(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma}))$ (Crandall, Ishii and Lions, 1992, Theorem 3.2), satisfying

$$
p^u_\gamma = (p^u_c, p^u_k) = (p^v_c + 4\beta(c_{\epsilon,\gamma}-\bar{c})^3, p^v_k), \quad p^v_\gamma = (p^v_c, p^v_k) = \left(\frac{c_{\epsilon,\gamma}-d_{\epsilon,\gamma}}{\epsilon}, \frac{k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma}}{\epsilon}\right)
$$

and

$$
k^{2\alpha}_{\epsilon,\gamma}X - \ell^{2\alpha}_{\epsilon,\gamma}Y \leq k^{2\alpha}_{\epsilon,\gamma}12\beta(c_{\epsilon,\gamma}-\bar{c})^2 + \frac{(k^\alpha_{\epsilon,\gamma}-\ell^\alpha_{\epsilon,\gamma})^2}{\epsilon} + o(1),
$$

where $o(1)$ denotes a term that converges to 0 as $\epsilon \to 0$.

Because $u$ is a subsolution, $\tilde{u} = (1-\gamma)e^{-\eta k}u$ satisfies

$$
\begin{aligned}
0 \geq \min\Bigg\{ r\tilde{u} - \sup_{i\in[0,i_{\max}]}\Bigg(&\Big[i-\delta_\zeta k_{\epsilon,\gamma}\Big](\eta\tilde{u}+\partial_k\tilde{u}) \\
&+\Big[(r-\lambda_c)c_{\epsilon,\gamma}+k^\alpha_{\epsilon,\gamma}\mu - b - i - g(k_{\epsilon,\gamma},i)\Big]\partial_c\tilde{u} \\
&+\frac{1}{2}k^{2\alpha}_{\epsilon,\gamma}\sigma^2\partial^2_{cc}\tilde{u}\Bigg), \\
&\partial_c\tilde{u}-(1-\gamma)e^{-\eta k_{\epsilon,\gamma}}\Bigg\}.
\end{aligned} \tag{B.4}
$$

Similarly, $\tilde{v} = e^{-\eta k} v$ satisfies

$$0 \leq \min \left\{ r\tilde{v} - \sup_{i \in [0, i_{\max}]} \left( \left[ i - \delta_\zeta \ell_{\epsilon,\gamma} \right] (\eta \tilde{v} + \partial_k \tilde{v}) + \left[ (r - \lambda_c) d_{\epsilon,\gamma} + \ell_{\epsilon,\gamma}^\alpha \mu - b - i - g(\ell_{\epsilon,\gamma}, i) \right] \partial_c \tilde{v} + \frac{1}{2} \ell_{\epsilon,\gamma}^{2\alpha} \sigma^2 \partial_{cc}^2 \tilde{v} \right), \partial_c \tilde{v} - e^{-\eta \ell_{\epsilon,\gamma}} \right\}. \tag{B.5}$$

We split into two cases, depending on which expression is smallest in Equation (B.4). We begin with the simple case of

$$p_c^u \leq (1-\gamma) e^{-\eta k_{\epsilon,\gamma}}.$$

Subtracting the two equations (B.4) and (B.5) thus gives

$$4\beta (c_{\epsilon,\gamma} - \bar{c})^3 = p_c^u - p_c^v \leq (e^{-\eta k_{\epsilon,\gamma}} - e^{-\eta \ell_{\epsilon,\gamma}}) - \gamma e^{-\eta k_{\epsilon,\gamma}}.$$

Letting $\epsilon \to 0$ in the last inequality,

$$4\beta (c_\gamma - \bar{c})^3 \leq -\gamma e^{-\eta k_\gamma},$$

which contradicts Equation (B.3) because $\zeta$ can be chosen arbitrarily small, independently of $k^*$.

In the other case, we subtract the equations and get

$$
\begin{aligned}
r(\tilde{u}-\tilde{v}) \leq \sup_{i\in[0,i_{\max}]} \Big\{ & \Big[i-\delta_\zeta k_{\epsilon,\gamma}\Big](\eta\tilde{u}(c_{\epsilon,\gamma},k_{\epsilon,\gamma})+p_k^u) \\
& + \Big[(r-\lambda_c)c_{\epsilon,\gamma}+k_{\epsilon,\gamma}^\alpha\mu-b-i-g(k_{\epsilon,\gamma},i)\Big](p_c^v+4\beta(c_{\epsilon,\gamma}-\bar{c})^3)+\frac{1}{2}k_{\epsilon,\gamma}^{2\alpha}\sigma^2 X \\
& - \Big[i-\delta_\zeta \ell_{\epsilon,\gamma}\Big](\eta\tilde{v}(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma})+p_k^v) \\
& - \Big[(r-\lambda_c)d_{\epsilon,\gamma}+\ell_{\epsilon,\gamma}^\alpha\mu-b-i-g(\ell_{\epsilon,\gamma},i)\Big]p_c^v-\frac{1}{2}\ell_{\epsilon,\gamma}^{2\alpha}\sigma^2 Y\Big\} \\
\leq \sup_{i\in[0,i_{\max}]} \Big\{ & i\eta(\tilde{u}(c_{\epsilon,\gamma},k_{\epsilon,\gamma})-\tilde{v}(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma})) \\
& + \Big[(r-\lambda_c)c_{\epsilon,\gamma}+k_{\epsilon,\gamma}^\alpha\mu-b-i-g(k_{\epsilon,\gamma},i)\Big]4\beta(c_{\epsilon,\gamma}-\bar{c})^3 \\
& - \delta_\zeta(\ell_{\epsilon,\gamma}-k_{\epsilon,\gamma})p_k^u+\Big[(k_{\epsilon,\gamma}^\alpha-\ell_{\epsilon,\gamma}^\alpha)\mu-(g(k_{\epsilon,\gamma},i)-g(\ell_{\epsilon,\gamma},i))\Big]p_c^v \\
& + 6k_{\epsilon,\gamma}^{2\alpha}\sigma^2\beta(c_{\epsilon,\gamma}-\bar{c})^2+\frac{(k_{\epsilon,\gamma}^\alpha-\ell_{\epsilon,\gamma}^\alpha)^2}{\epsilon}\Big\}+o(1).
\end{aligned}
$$

Let $\eta < (r-\Delta)/i_{\max}$ for $\Delta \in (0,r)$. Then, taking $\limsup$ as $\epsilon \to 0$, and using that $g(\cdot,i)$ and $k \mapsto k^\alpha$ are Lipschitz in the neighborhood of $(c_\gamma,k_\gamma)$, i.e.,

$$
|g(k_{\epsilon,\gamma},i)-g(\ell_{\epsilon,\gamma},i)|+\mu|k_{\epsilon,\gamma}^\alpha-\ell_{\epsilon,\gamma}^\alpha| \leq R|k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma}|,
$$

we get

$$
\begin{aligned}
\limsup_{\epsilon\to 0} & \,\Delta(\tilde{u}(c_{\epsilon,\gamma},k_{\epsilon,\gamma})-\tilde{v}(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma})) \\
& \leq \lim_{\epsilon\to 0}\Big[(\delta_\zeta+R^2)\frac{(k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma})^2}{\epsilon}+R\frac{(c_{\epsilon,\gamma}-d_{\epsilon,\gamma})}{\sqrt{\epsilon}}\frac{(k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma})}{\sqrt{\epsilon}} \\
& \qquad + R'(|c_{\epsilon,\gamma}-\bar{c}|^2+|c_{\epsilon,\gamma}-\bar{c}|^3)+o(1)\Big]
\end{aligned}
$$

for some constant $R'$, depending on $k^*$ (i.e., $\eta$), $i_{\max}$, $\beta$, and the model parameters. In other words, $R'$ is independent of $\zeta$. By Equation (B.3), the right-hand side is bounded by $R''(\zeta+\sqrt{\zeta})$, for some constant $R'' > 0$ that is also independent of $\zeta$. Finally, because $\Delta > 0$,

$$
\delta_\zeta \leq e^{-\eta k_\gamma}((1-\gamma)u-v)(c_\gamma,k_\gamma) \leq \limsup_{\epsilon\to 0}(\tilde{u}(c_{\epsilon,\gamma},k_{\epsilon,\gamma})-\tilde{v}(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma})) \leq \frac{R''}{\Delta}(\zeta+\sqrt{\zeta}),
$$

which is a contradiction because $\delta_\zeta/(\zeta+\sqrt{\zeta})$ can be chosen arbitrarily large. Hence, there cannot exist a point $(c,k)$ such that $(u-v)(c,k)>0$.

□

Following the steps of the proof of Proposition 5 in the Internet Appendix, it can be easily proven that $V(c,k) \leq M + c + k$ for some $M$. As a consequence, $V$ satisfies the assumptions of Theorem 2. The following results are standard consequences of the comparison of viscosity solutions.

**Corollary 3.** *The value function $V$ is the (continuous) unique solution to Equation (B.1) with its boundary conditions.*

We now present our numerical algorithm to solve the model. Equation (B.1) is solved in a square domain $[0,c_{\max}] \times [0,k_{\max}]$ via policy iteration, which produces the value function $V(k,c)$ and investment policy function $i(k,c)$, in addition to the regions of dividend and equity issuance. The singular structure of the dividends is approximated as in (Reppen, Jean-Charles and Soner, 2020, Section 4), which also describes the policy iteration algorithm and the impulse control issuance as in (Reppen, Jean-Charles and Soner, 2020, Section 6.1.2).

In addition to (B.2), the boundary conditions where $c = c_{\max}$ and $k_{\max}$ are given by

$$\begin{aligned}
0 &= \partial_c V - 1 && \text{at } c = c_{\max} \\
0 &= \min\left\{ rV + \delta k \partial_k V - \left[ rc + k^\alpha \mu - b \right] \partial_c V - \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V, \partial_c V - 1, \mathcal{I} V \right\} && \text{at } k = k_{\max}
\end{aligned}$$

At the corners, the $c$-conditions are used.

Another consequence of the comparison result in Theorem 2 is the convergence of the numerical scheme (see Barles and Souganidis (1991)).

**Corollary 4.** *Numerical solutions converge to the value function as the discretization gets finer.*

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Table 1: Model Parameters

| Parameter | Name | Values | Comments |
|---|---|---|---|
| $r$ | Interest rate | 6% | In line with long-term average yield to maturity on 30 year U.S. Treasuries. |
| $\lambda_c$ | Cash holding cost or liquidity premium | 1% | Cash may earn low returns because interest earned on a firm's cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate (Graham, 2000; Faulkender and Wang, 2006). Also, agency problems may lower cash returns (Jensen, 1986; Harford, 1999; Dittmar and Shivdasani, 2003; Pinkowitz, Stulz and Williamson, 2006; Dittmar and Mahrt-Smith, 2007; Harford, Mansi and Maxwell, 2008; Caprio, Faccio and McConnell, 2011; Gao, Harford and Li, 2013). |
| $\mu$ | Expected productivity shock | 0.18 | In line with the estimates of Eberly, Rebelo and Vincent (2009) for large U.S. firms. |
| $\sigma$ | Volatility of productivity shock | 0.09 | In line with the estimates of Eberly, Rebelo and Vincent (2009) for large U.S. firms. |
| $\theta$ | Degree of adjustment cost | 1.5 | See Whited (1992). |
| $\delta$ | Depreciation rate | 10.07% | In line with the estimates of Eberly, Rebelo and Vincent (2009) for large U.S. firms. |
| $b$ | Fixed operating cost | 0.02 | For $k = 0.5$, $\alpha = 0.7$, and $\mu = 0.18$, the fixed operating cost represents 18% of expected cash flows ($\mu \times k^{\alpha} = 0.11$). |
| $\lambda_p$ | Variable issuance cost | 6.4% | In line with the estimates of Altınkılıç and Hansen (2000). |
| $\lambda_f$ | Fixed issuance cost | 0.05 | The fixed component of issuance costs encourages lumpy issuances. |
| $\alpha$ | Curvature of the production function. When $\alpha < 1$, then diminishing returns to scale | 0.7 | $\alpha = 0.75$ in Riddick and Whited (2009) and $\alpha = 0.627$ for the full sample of firms in Hennessy and Whited (2007). |
| $\ell$ | Recovery rate in the liquidation of capital | 90% | The choice of $\ell$ is consistent with Hennessy and Whited (2007), where the recovery rate is estimated to be 0.896 for the full sample of firms. |
| $r_{\text{debt}}$ | Cost of financing for long-term debt | 9% | Represents a 300-basis-point spread over $r = 6\%$. Since 1986, the Moody's seasoned Baa corporate bond yield relative to the yield on 10-year Treasury constant maturity generally fluctuates between 200 and 300 basis points. Source: St. Louis Fed. |

Table 2: Cash holdings and issuance amounts (payout rates) are U-shaped (hump-shaped) in firm size

The main explanatory variable is *Firm Size*$_{i,t}$, which is firm $i$'s market capitalization less its cash holdings at the end of year $t$, standardized within a firm. In columns (1) and (2), the outcome variable is firm $i$'s cash holdings at the end of year $t$, standardized within a firm. In columns (3) and (4), the outcome variable is the issuance amount in year $t+1$, standardized within a firm. We restrict the sample of firm years to those with offerings exceeding $5 million. In columns (5) and (6), the outcome variable is the total common stock dividends and share repurchases in year $t+1$ scaled by a firm's size at the end of year $t$, standardized within a firm. Thus, firms with no payouts are excluded from the sample. In columns (2), (4), and (6), we interact *Firm Size*$_{i,t}$ with *Fixed Operating Costs*$_i$, which is the firm's average annual sum of cost of goods sold; selling, general, and administrative expenses; and interest expenses scaled by its total assets (Novy-Marx, 2010). *Cash*$_{i,t}$ is a firm's cash and cash equivalents at the end of year $t$. To examine the non-monotonicities in Figures 5, 6, and 7, we likewise limit the sample to the firm years when a firm is below its median size in the sample. We control for SIC-1–by–year industry trends. Because the variables are all standardized within a firm, the results are within-firm results. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | Cash$_{i,t}$ | | Issuance Amt.$_{i,t+1}$ | | Payout Rate$_{i,t+1}$ | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Firm Size$_{i,t}$ | 1.05*** | 0.86*** | 0.37*** | 0.38*** | -0.45*** | -0.45*** |
| | (0.09) | (0.16) | (0.09) | (0.09) | (0.12) | (0.12) |
| Firm Size$^2{}_{i,t}$ | 0.58*** | 0.40*** | 0.18*** | 0.18*** | -0.23*** | -0.23*** |
| | (0.06) | (0.09) | (0.06) | (0.06) | (0.08) | (0.08) |
| Fixed Operating Costs$_i$ | | 0.04 | | 0.05** | | -0.02 |
| | | (0.03) | | (0.02) | | (0.03) |
| Firm Size$_{i,t}\times$Fixed Operating Costs$_i$ | | 0.13 | | 0.14** | | -0.05 |
| | | (0.09) | | (0.07) | | (0.09) |
| Firm Size$^2{}_{i,t}\times$Fixed Operating Costs$_i$ | | 0.12** | | 0.11** | | -0.03 |
| | | (0.06) | | (0.05) | | (0.05) |
| Cash$_{i,t}$ | | | 0.01 | 0.01 | 0.20*** | 0.20*** |
| | | | (0.01) | (0.01) | (0.01) | (0.01) |
| SIC-1$\times$Year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| % Adjusted R$^2$ | 4.4 | 4.5 | 3.7 | 3.8 | 7.2 | 7.2 |
| Observations | 57,163 | 57,163 | 12,713 | 12,713 | 45,339 | 45,339 |

# Internet Appendix to Cash Policies and Firm Size

This Internet Appendix contains supplementary analyses. These include the following:

1. Appendix C provides additional model outputs.
   (a) Figure C.1 shows a heat map of net investment $i - \delta k$.
   (b) Appendix C.1 discusses the determinants of investment in terms of $\partial_k V$ and $\partial_c V$ (Figure C.2).
   (c) Appendix C.2 presents comparative statics for Figure 1.
      - Appendix C.2.1 varies the drift $\mu$ (Figure C.3).
      - Appendix C.2.2 varies the fixed component of issuance costs $\lambda_f$ (Figure C.4).
      - Appendix C.2.3 varies the volatility $\sigma$ (Figure C.5).
      - Appendix C.2.4 varies the liquidity premium $\lambda_c$ (Figure C.6).
      - Appendix C.2.5 varies the firm's fixed operating cost $b$ (Figure C.7).
   (d) Appendix C.3 provides sufficient conditions for a strategic default region to exist.
2. Appendix D provides additional empirical work.
   (a) Table D.1 shows the sample selection criteria.
   (b) Section Appendix D.1 provides robustness for the empirical fact that cash holdings are U-shaped in firm size.
      - Figure D.1 repeats Figure 5(a) by SIC-1 industry.
      - Figure D.2 repeats Figure 5(a) using a firm's tangible and intangible capital to proxy for a firm's size.
      - Figure D.3 repeats Figure 5(a) using a firm's sales to proxy for a firm's size.
      - Figure D.4 repeats Figure 5(a) using cash at the end of year $t + 1$.
      - Figure D.5 repeats Figure 5(a) using min-max scaling instead of standardizing within a firm.
      - Figure D.6 shows that the U-shaped relation in Figure 5(a) exists before and after the median sample year of 1996.
      - Figure D.7 repeats Figure 5 using the full sample rather than only when firms are below their median size.
   (c) Section Appendix D.2 provides robustness for the empirical fact that issuance amounts are U-shaped in firm size.
      - Figure D.8 repeats Figure 6(a) by SIC-1 industry.
      - Figure D.9 repeats Figure 6(a) using different cutoffs for offering size.
      - Figure D.10 shows that the U-shaped relation in Figure 6(a) exists before and after the median sample year of 1996.

- Figure D.11 repeats Figure 6(a) using a firm's tangible and intangible capital to proxy for a firm's size.
- Figure D.12 repeats Figure 6(a) using a firm's sales to proxy for a firm's size.
- Figure D.13 repeats Figure 6(a) controlling for past acquisition amounts.
- Figure D.14 repeats Figure 6(a) using min-max scaling instead of standardizing within a firm.
- Figure D.15 repeats Figure 6 using the full sample rather than only when firms are below their median size.
- Table D.2 shows that a firm's cash and capital improve the explanatory power of the specification in Chang, Dasgupta and Hilary (2006).
- Tables D.3 and D.4 show that a firm's cash and capital matter for the tests in Hovakimian, Opler and Titman (2001).

(d) Section Appendix D.3 provides robustness for the empirical fact that payout rates are hump-shaped in firm size.

- Figure D.16 repeats Figure 7 using the full sample rather than only when firms are below their median size.
- Figure D.17 shows the hump shape in Figure 7 exists before and after the median sample year of 1996.
- Figure D.18 repeats 7(a) by SIC-1 industry.
- Figure D.19 repeats 7(a) using a firm's tangible and intangible capital to proxy for a firm's size.
- Figure D.20 repeats 7(a) using a firm's sales to proxy for a firm's size.
- Table D.5 shows that a firm's cash and capital improve the explanatory power and significance of the tests in Bliss, Cheng and Denis (2015).

(e) Section Appendix D.4 provides evidence that the default region expands with leverage so that more levered firms running out of cash are more likely to choose to default rather than issue equity.

- Figure D.21(a) shows a reduction in offerings when capital is low and leverage is high.
- Figure D.21(b) shows an increase in default when capital is low and leverage is high.

(f) Section Appendix D.5 provides support in the data for the diminishing returns to scale assumption.

- Figure D.22(a) shows that the model predicts a convex relation between the investment-to-depreciation ratio and capital. Figure D.22(b) provides empirical support.
- Table D.6 provides evidence of the convex relation between the investment-to-depreciation ratio and capital.
- Table D.7 shows that firms in the data generally exhibit diminishing returns to scale.

## Appendix C.

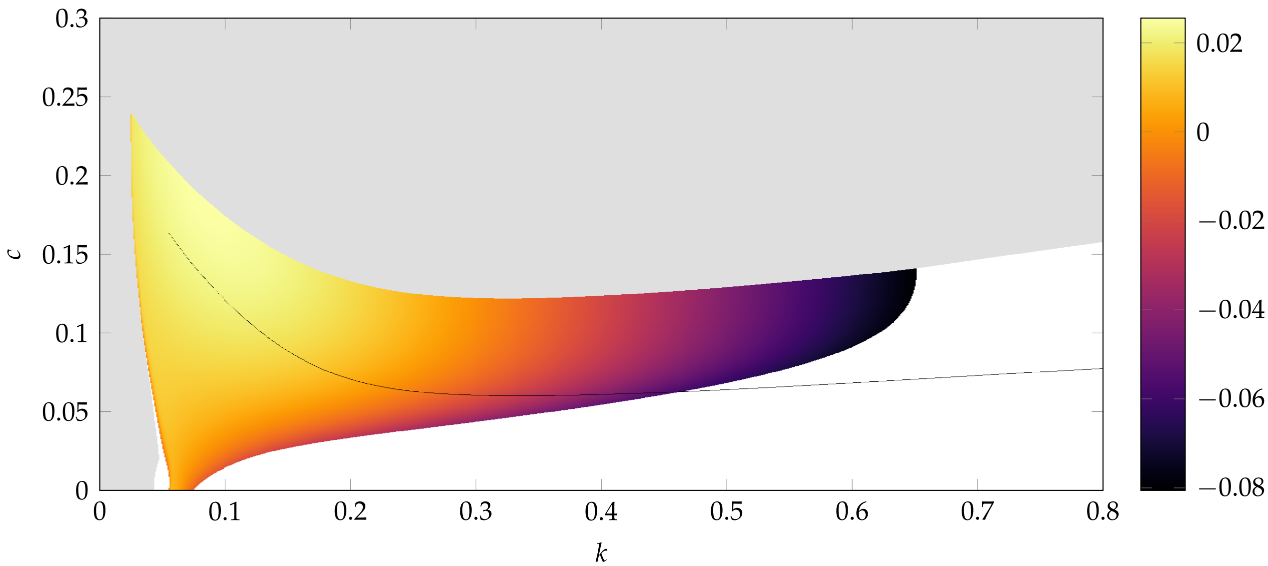



Figure C.1: **Net investment heat map**

*This figure replaces the investment regions B and C in Figure 1 with a color gradient representing the net investment rate ($i - \delta k$). For example, a -0.04 on the net investment scale means that capital drops at a rate of 0.04. Parameters used are summarized in Table 1.*

*Appendix C.1. Determinants of Investment*

To better understand the determinants of investment, let us examine the tradeoff between the marginal cost of cash, $\partial_c V$, and the marginal benefit of capital, $\partial_k V$, for the baseline result in Figure 1. For a fixed level of capital, Figure C.2(a) shows that $\partial_c V$ is particularly high when cash is close to zero; moreover, $\partial_c V$ is decreasing in $c$, implying the concavity of $V$ in $c$ and the firm's effective risk aversion in cash. For different levels of capital, Figure C.2(a) shows that $\partial_c V$ does not scale with $k$ linearly. As we discussed for Equation (10), investment is optimal only when $\partial_c V < \partial_k V$. Comparing $\partial_c V$ and $\partial_k V$ in Figure C.2 (a), we see that the minimum cash reserve level needed for positive investment is lower when $k = 0.1$ than when $k = 0.4$. This observation is consistent with Figure C.1. For fixed cash levels, Figure C.2(b) plots $\partial_k V$ and $\partial_c V$ for different values of $k$. The marginal benefit of capital $\partial_k V$ is non-monotonic in $k$ in general. But when $k$ is higher than a threshold, C.2(b) shows that $\partial_k V$ decreases with $k$, reflecting the firm's decreasing returns to scale. The strategic default also induces the non-monotone pattern of $\partial_k V$ in Figure C.2(b): when $k$ is low, strategic default is imminent. Rather than using cash to invest, equity holders prefer to hold back investments and wait for payout and strategic default. This strategic consideration yields a low marginal benefit of capital $\partial_k V$ when $k$ is low in Figure C.2(b).

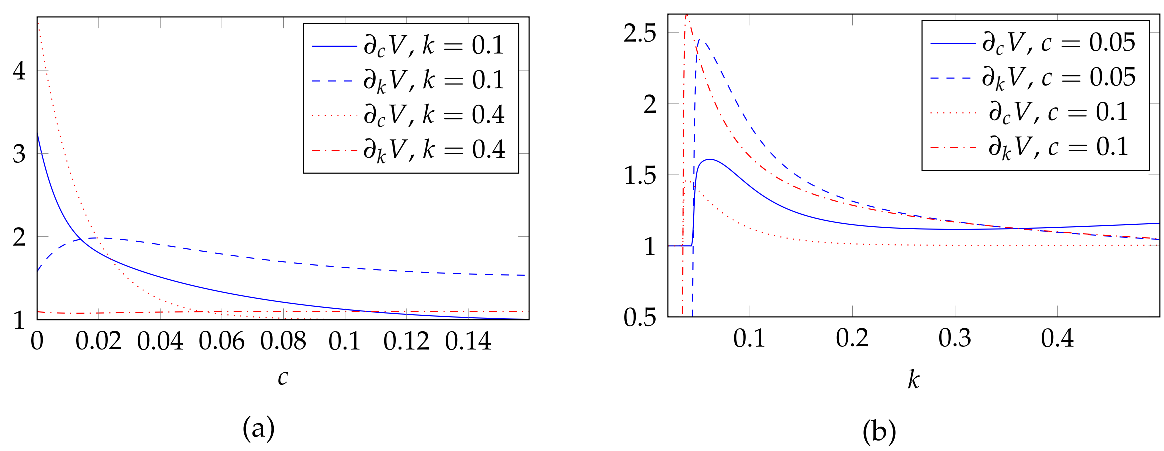



Figure C.2: **Marginal cost of cash and marginal benefit of capital**

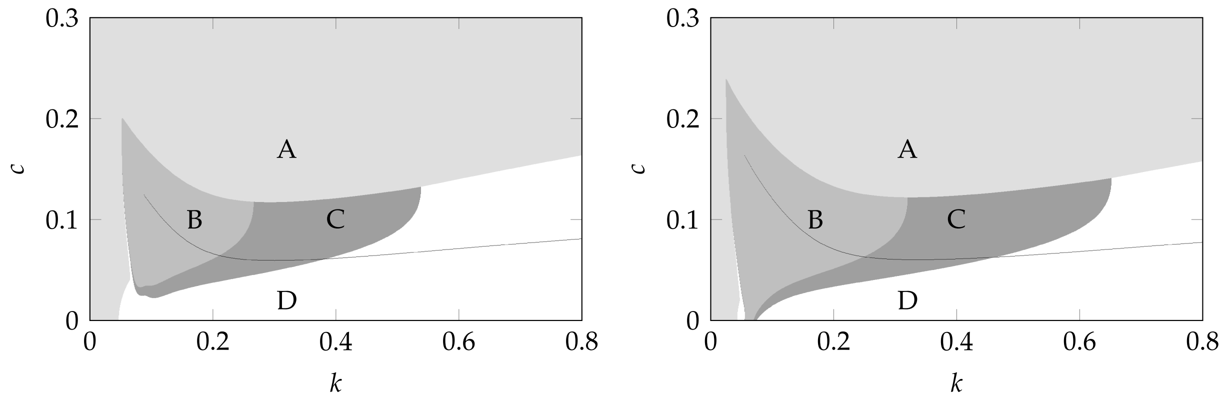
*Parameters used are summarized in Table 1.*

*Appendix C.2. Comparative statics*

This section examines how a firm's choices vary when we adjust the assumed values of specific parameters in Table 1.

*Appendix C.2.1. Impact of expected productivity*

As $\mu$ increases, the expected cash flows of the firm increase. Figure C.3 shows how higher expected cash flows change the firm's choices (moving from C.3(a) to C.3(b)).




(a) $\mu = 0.17$ (b) $\mu = 0.18$

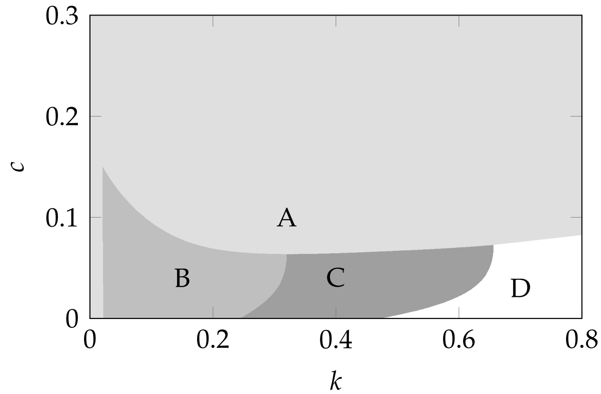
Figure C.3: **Impact from $\mu$**

*The grey ■-colored region (labeled A) indicates dividend payouts; the ■-colored region (labeled B) indicates positive net investment; the ■-colored region (labeled C) indicates negative net investment; and the white region is the no-investment region (labeled D), where the firm neither pays dividends nor invests in capital. The solid black line indicates the state after equity issuance. The other parameters used are summarized in Table 1.*

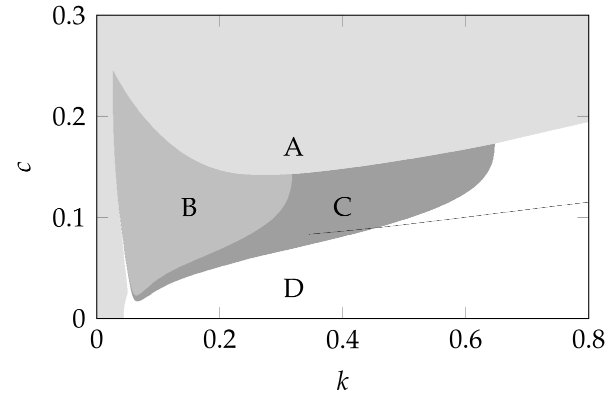
When expected cash flows increase, the investment regions (labeled $B$ and $C$) grow in size. Second, when expected cash flows increase, the default region shrinks as the issuance target shifts to the left. Intuitively, when profitability is higher, the benefits of issuing equity to continue investing increase. Consequently, equity holders find it more attractive to issue equity when cash reaches zero instead of defaulting. Relatedly, when capital is low, increases in expected profitability increase the optimal cash balance due to the drop in endogenous default risk.

*Appendix C.2.2. Impact of the fixed component of issuance costs*

As $\lambda_f$ increases, the fixed cost of issuing equity increases. Figure C.4 (moving from (a) to (b)) shows how a higher fixed issuance cost changes the firm's choices.




(a) $0 \times \lambda_f$




(b) $7 \times \lambda_f$

Figure C.4: **Impacts of fixed issuance costs**

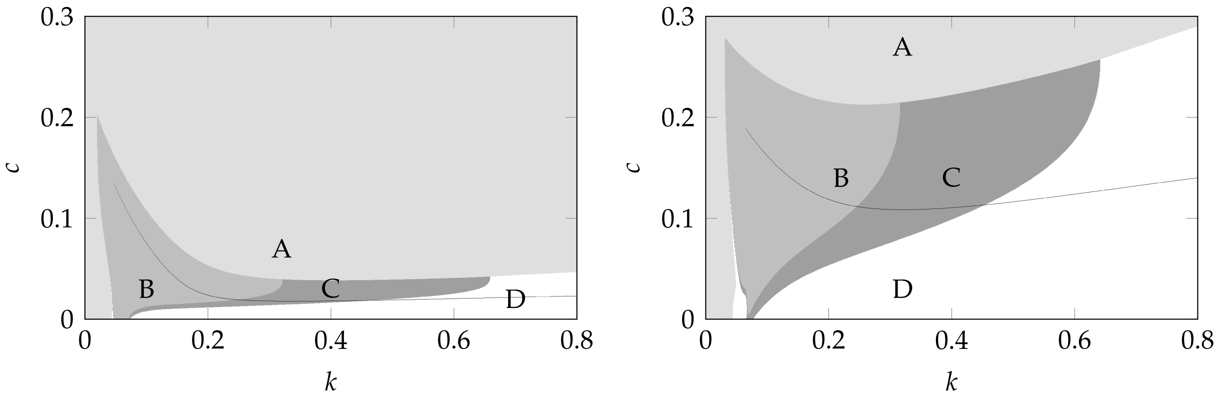
*The grey -colored region (labeled A) indicates dividend payouts; the -colored region (labeled B) indicates positive net investment; the -colored region (labeled C) indicates negative net investment; and the white region is the no-investment region (labeled D) where the firm neither pays dividends nor invests in capital. The solid black line indicates the state after equity issuance. The other parameters used are summarized in Table 1.*

First, Figure C.4 illustrates that the fixed component of issuance costs drives the issuance boundary of the firm. Specifically, Figure C.4(a) shows that when there are no fixed issuance costs, the issuance boundary collapses to the horizontal axis. Intuitively, firms still value a cash reserve to hedge against costly issuance (there is still a proportional cost), but when the cash reserve reaches zero, firms raise just enough cash, leading the cash reserve back into the positive domain. In the presence of fixed issuance costs, the firm optimally issues equity in lumps to economize over the fixed issuance costs.

Second, higher fixed issuance costs move the equity issuance target to the right, expanding the default region. In the default region, when the firm runs out of cash, there is no optimal issuance amount (no issuance target). Instead, the firm liquidates. The default region expands because higher issuance costs increase the expected costs of operating the firm, which reduces the incentive for equity holders to invest and build up the capital.

*Appendix C.2.3. Impact of production volatility*

Figure C.5 shows how a change in the volatility of the cash flow shock, $\sigma$, alters a firm's choices.




(a) $\sigma = 0.04$

(b) $\sigma = 0.14$

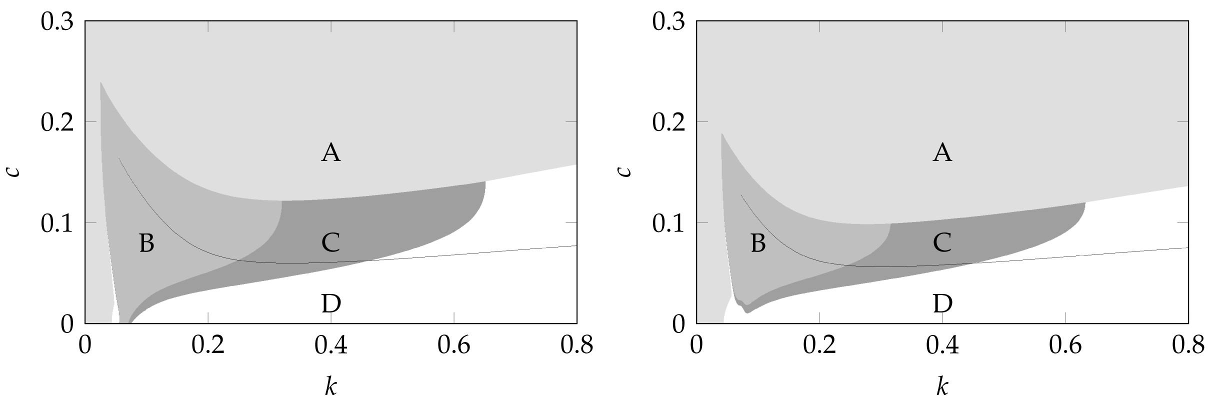
Figure C.5: **Different** $\sigma$

*The grey ■-colored region (labeled A) indicates dividend payouts; the ■-colored region (labeled B) indicates positive net investment; the ■-colored region (labeled C) indicates negative net investment; and the white region is the no-investment region (labeled D), where the firm neither pays dividends nor invests in capital. The solid black line indicates the state after equity issuance. The other parameters used are summarized in Table 1.*

When cash flow shocks are more volatile, the dividend payout boundary increases and the equity issuance boundary increases. Equity holders facing more volatility demand higher cash reserves to hedge against the possibility of larger negative cash flow shocks that could lead to costly issuance. Relatedly, conditional on issuance, equity holders raise more capital. This effect is more pronounced when capital $k$ is high because the scale of the cash flow shock is larger for higher capital levels. Also, when a firm is small, the demand for cash is already high because of high issuance costs, high default risk, strong investment opportunities, and low cash flows. Thus, raising the volatility of the cash flow shock has a relatively smaller impact on the marginal value of cash when a firm is small.

*Appendix C.2.4. Varying the liquidity premium of cash $\lambda_c$.*

When the liquidity premium is lower (panel (a) of Figure C.6), the dividend boundary is higher. Intuitively, lower liquidity premiums reduce the difference between the net rate of return for cash and the equity holders' discounting rate. Hence, early dividend payout becomes less attractive, and the dividend payout boundary increases.




(a) $\lambda_c = r/6$ (b) $\lambda_c = r/2$

Figure C.6: **Different $\lambda_c$ (Cash liquidity premium)**

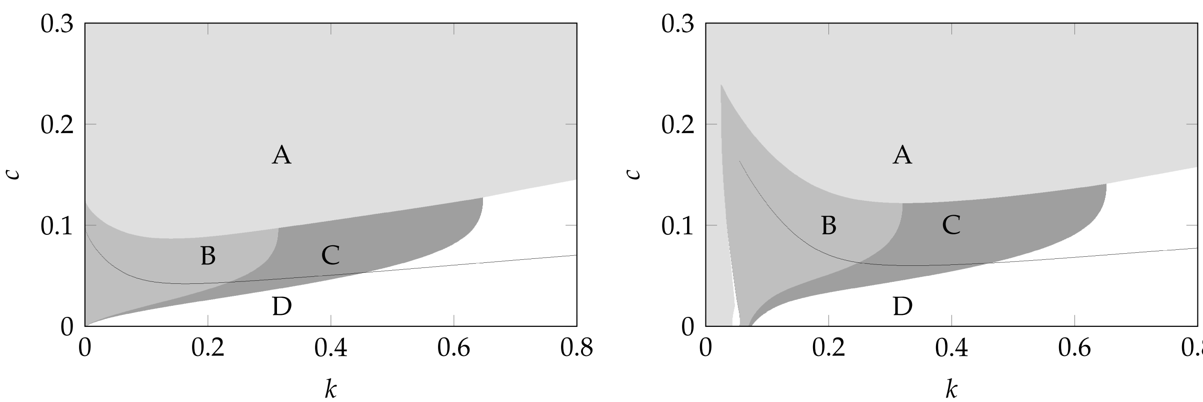
*The grey ■-colored region indicates dividend payouts; the ■-colored region indicates positive net investment; the ■-colored region indicates negative net investment; and the white region is the continuation region, where the firm neither pays dividends nor invests in capital. The solid black line indicates the state after equity issuance. The other parameters used are summarized in Table 1.*

*Appendix C.2.5. Impact of fixed operating costs*

Figure C.7 shows how varying the fixed operating cost alters the firm's choices in a few ways.

First, when $b$ increases, the default region expands. Specifically, increasing the fixed operating cost shifts the issuance target to the right. For example, in Figure C.7(b), the issuance target only begins to appear around $k = 0.05$. For the region $0 \leq k < 0.05$, if the firm runs out of cash, the equity holders optimally choose to default rather than to issue new equity. Despite this intuitive prediction, prior studies mostly examine equity offerings and liquidation separately and do not consider the impact of fixed operating costs on the decision of whether to issue or liquidate. In Internet Appendix D.4, we find empirical support for the prediction that low-capital firms with more leverage are more likely to default than issue new equity.

Second, when capital $k$ is low, Figure C.7, panels (a) to (b) show that increases in the fixed operating cost sharply raise the dividend payout boundary and the equity issuance target. By contrast, when $k$ is high, the dividend payout boundary and issuance target are less sensitive to the fixed operating cost $b$. The intuition is that when $k$ is low, there is less cash flow generated, so the firm needs a larger cash buffer to fund the higher fixed operating cost. When $k$ is high, the cash flows generated by production can better support the fixed operating cost, reducing the need for a cash buffer.




(a) $b = 0$

(b) $b = 0.02$

Figure C.7: **Impact from** $b$

*The grey -colored region (labeled A) indicates dividend payouts; the -colored region (labeled B) indicates positive net investment; the -colored region (labeled C) indicates negative net investment; and the white region is the no-investment region (labeled D), where the firm neither pays dividends nor invests in capital. The solid black line indicates the state after equity issuance. The parameters used are summarized in Table 1.*

*Appendix C.3. Strategic default*

The HJB equation also implicitly incorporates the option of default at $c > 0$, in which case the firm's value is the sum of $c$ and the firm's liquidation value after debt payment. We call this strategic default. This case is incorporated into the second group of terms, $\partial_c V - 1$, in Equation (9). The following result provides a sufficient condition for a strategic default region to exist. This result shows that a strategic default region exists when the fixed operating cost $b$ (i.e. modeled as a fixed coupon payment on long-term debt) is higher than a given threshold, which is an increasing function of the expected productivity $\mu$. Therefore, firms with lower expected productivity are more likely to have a strategic default region. This result also provides for the existence of a strong type of strategic default region, for which strategic default is optimal regardless of initial cash reserves.

**Proposition 5.** *For $b > \mu(1-\alpha)(\mu\alpha/(r+\delta))^{\alpha/(1-\alpha)}$, there is a region $[0,\infty) \times [0,\underline{k}]$ where strategic default is optimal for all $c$, i.e, $V(c,k) = c$ for $k \in [0,\underline{k}]$.*

*Proof.* Consider the following optimization problem:

$$\begin{aligned} V_R(k,c) &= c + V_R(k) \\ &= c + \sup_{\tau, i \geq 0} \mathbb{E}\left[ \int_0^\tau e^{-rt} \Big( k^\alpha \mu - b - i_t - g(k_t, i_t) \Big) dt + 1_{\{\tau < \infty\}} e^{-r(\tau - t)} (\ell k_\tau - b/r_{\text{debt}})_+ \right]. \end{aligned}$$

This is the optimization problem for a firm that is subject to neither external financing costs nor a cash liquidity premium. The firm can invest and choose its default time optimally, i.e., a real option. This value function $V_R$ dominates the value function $V$ in Equation (8), i.e., $V(k,c) \leq V_R(k,c)$ for any $(k,c)$. The equation for $V_R(k)$ is given by dynamic programming:

$$0 = \min\{rV_R - \sup_{i \geq 0}((i - \delta k)V_R' + k^\alpha \mu - b - i - g(k,i)), V_R - (\ell k - b/r_{\text{debt}})\}. \tag{C.1}$$

We assume that this equation satisfies the comparison principle. This can be proven as in Theorem 2, which proves the comparison principle for a more complicated but related case.

Let

$$v(k) = (k - \underline{k})_+$$

for some $\underline{k} > 0$. We show that this is a viscosity supersolution to Equation (C.1) for some sufficiently small $\underline{k}$. First, we restrict $\underline{k}$ so that $\underline{k} \leq b/r_{\text{debt}}$; then, clearly $v - (\ell k - b/r_{\text{debt}})_+ \geq 0$. Let $\phi$ be any $C^1$ function such that $v - \phi$ attains a local minimum at $k$. Then,

$$\phi'(k) \in \begin{cases} \{0\}, & k < \underline{k} \\ [0,1], & k = \underline{k} \\ \{1\}, & k > \underline{k} \end{cases}.$$

For all these cases, because $\phi' \geq 1$, we obtain

$$r(k-\underline{k})_+ - k^\alpha \mu + b - \sup_{i\geq 0}((i-\delta k)\phi' - i - g(k,i)) = r(k-\underline{k})_+ + \delta k\phi' - k^\alpha \mu + b. \quad \text{(C.2)}$$

When $k < \underline{k}$, the previous expression is

$$b - k^\alpha \mu,$$

which is nonnegative for $\underline{k} \leq (b/\mu)^{1/\alpha}$.

When $k > \underline{k}$, the right-hand side of Equation (C.2) is $r(k-\underline{k}) + \delta k - k^\alpha \mu + b$. This expression is minimized at $((r+\delta)/\mu\alpha)^{1/(\alpha-1)}$, with the minimum value

$$(r+\delta)(1-1/\alpha)(\mu\alpha/(r+\delta))^{1/(1-\alpha)} - r\underline{k} + b.$$

If

$$b > (r+\delta)(1/\alpha - 1)(\mu\alpha/(r+\delta))^{1/(1-\alpha)},$$

we can choose a sufficiently small $\underline{k} > 0$ such that the previous minimum value is still nonnegative.

Finally, when $k = \underline{k}$,

$$\delta\underline{k}\phi' - (\underline{k})^\alpha \mu + b \geq -(\underline{k})^\alpha \mu + b \geq 0$$

is satisfied if we choose $\underline{k} \leq (b/\mu)^{1/\alpha}$. Combining the previous three cases, we confirm that $v$ is a viscosity supersolution to Equation (C.1).

By the comparison principle, $V_R \leq v$, so we have

$$c \leq V(c,k) \leq c + V_R(k) \leq c + v(k) = c$$

for $k \leq \underline{k}$. The first inequality above holds because for Equation (8) in the paper, the firm can always pay out the remaining cash and default. Hence, $V(c,k) = c$ for all $k \leq \underline{k}$, which is attained by the immediate payout of all cash as dividends. $\square$

## Appendix D. Empirical Appendix

Table D.1: Sample Selection

| Criteria | Obs. Lost | Obs. Remaining |
|---|---|---|
| COMPUSTAT, 1971 – 2017 | | 495,968 |
| Less: | | |
| Missing intangible capital data | (42,097) | 453,871 |
| Pre-IPO data | (39,894) | 413,977 |
| Firms headquartered outside of USA | (72,960) | 341,017 |
| Firms incorporated outside of USA | (3,888) | 337,129 |
| Financials (SIC-1=6) | (91,576) | 245,553 |
| Utilities (SIC-2=49) | (16,375) | 229,178 |
| Public administration (SIC-1=9) | (4,566) | 224,612 |
| Missing or zero assets | (6,589) | 218,023 |
| Missing common stock price at close of fiscal year | (29,086) | 188,937 |
| Missing shares outstanding | (550) | 188,387 |
| Missing or zero book value of equity | (105) | 188,282 |
| Missing cash and cash equivalents | (22) | 188,260 |
| Missing or zero total liabilities | (329) | 187,931 |
| Missing net income | (376) | 187,555 |
| Missing operating income | (1) | 187,554 |
| Missing retained earnings | (1,684) | 185,870 |
| Missing working capital | (4,365) | 181,505 |
| PP&E less than $5M or missing PP&E | (62,525) | 118,980 |
| Negative cash and cash equivalents | (9) | 118,971 |
| Less than $1M in sales | (983) | 117,988 |
| Zero market equity | (34) | 117,954 |
| Singleton firms | (1,132) | 116,822 |
| SIC-4 industries with one firm | (2,341) | 114,481 |
| Final sample (9,690 firms) | | 114,481 |

*Appendix D.1. Robustness test for the empirical finding that cash holdings are U-shaped in firm size*

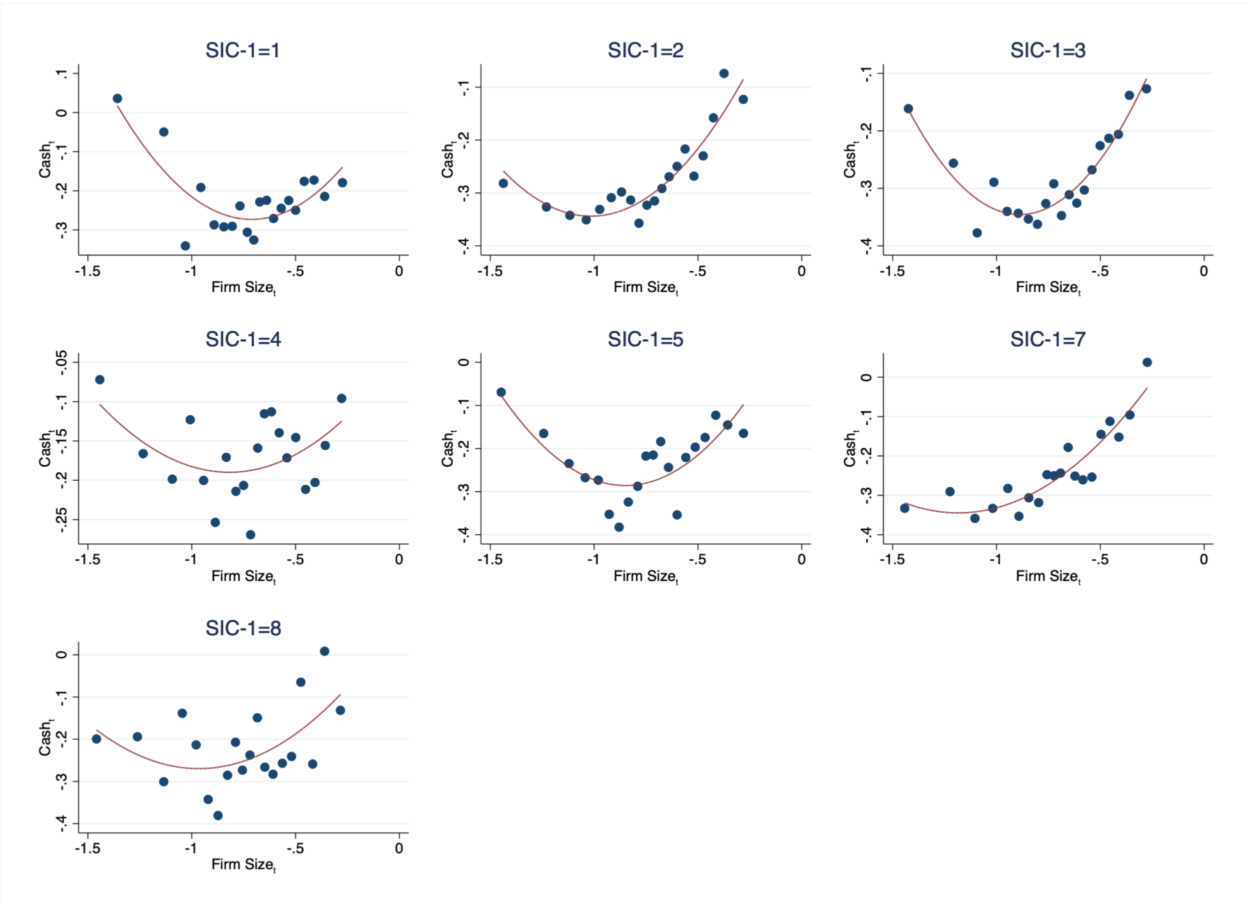



Figure D.1: **Repeats Figure 5(a) by industry**

*The vertical axis is a firm's level of cash holdings at the end of year t, standardized within a firm. The horizontal axis is a firm's market equity less its cash reserve at the end of year t, standardized within firm. For each SIC-1 industry, we sort firm-year observations into 20 bins based on a firm's relative size. We plot the average of the standardized cash holdings for each bin. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. The industries are SIC-1=1 is Mining & Construction, SIC-1=2 is Manufacturing (Food, Apparel, Furniture, Chemicals, Petroleum), SIC-1=3 is Manufacturing (Rubber, Leather, Stone, Metals, Transportation Equipment), SIC-1=4 is Transportation and Public Utilities, SIC-1=5 is Wholesale and Retail Trade, SIC-1=7 is Services (Hotel, Personal, Business, Auto Repair), and SIC-1=8 is Services (Health, Legal, Educational, Social, Museums).*

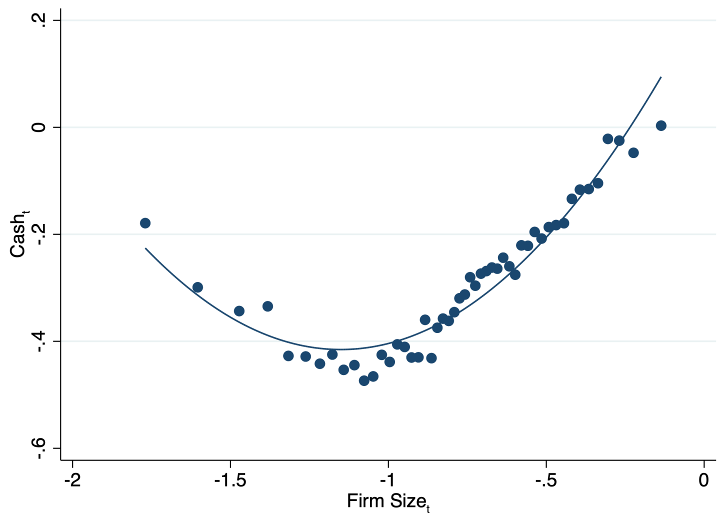



Figure D.2: **Repeats Figure 5(a) using a firm's tangible and intangible capital to proxy for a firm's size**

*The vertical axis is a firm's level of cash holdings at the end of year t, standardized within a firm. The horizontal axis is the sum of a firm's tangible and intangible capital at the end of year t, standardized within a firm (Peters and Taylor, 2017). Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort the firm-year observations into 50 bins based on a firm's relative size. We plot the average of the standardized cash holdings for each bin.*

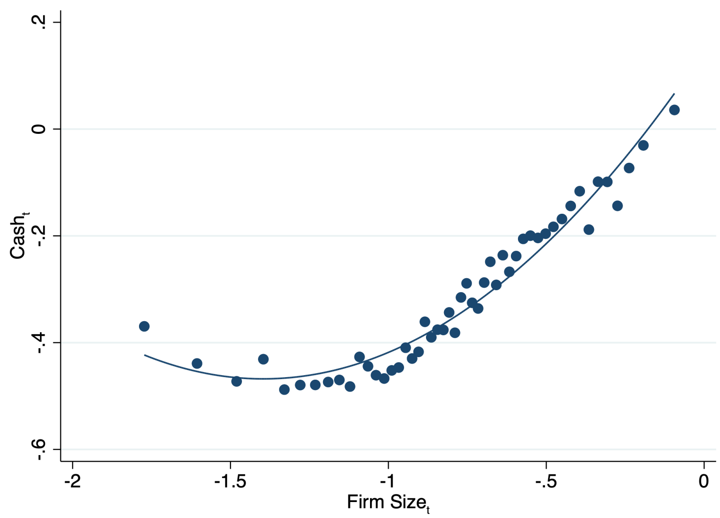



Figure D.3: **Repeats Figure 5(a) using a firm's sales to proxy for a firm's size**

*The vertical axis is a firm's level of cash holdings at the end of year $t$, standardized within a firm. The horizontal axis is a firm's sales in year $t$, standardized within a firm. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort the firm-year observations into 50 bins based on a firm's relative size. We plot the average of the standardized cash holdings for each bin.*

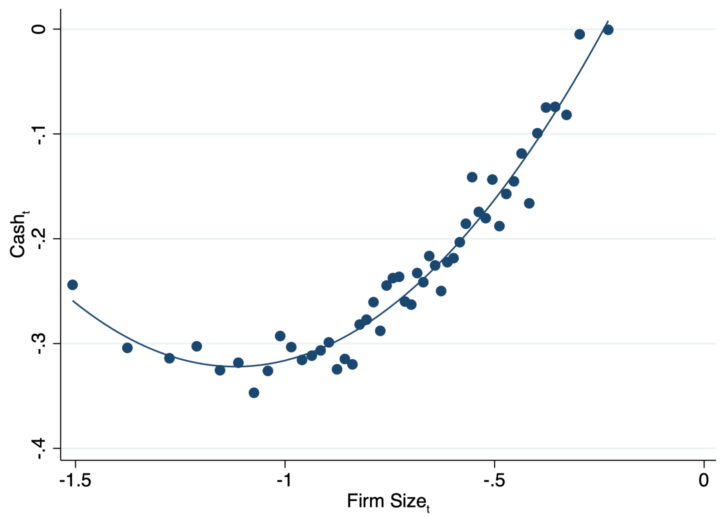



Figure D.4: **Repeats Figure 5(a) measuring cash at year** $t + 1$

*The vertical axis is a firm's level of cash holdings at the end of year* $t + 1$*, standardized within a firm. The horizontal axis is a firm's market equity less its cash reserve at the end of year* $t$*, standardized within a firm. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort the firm-year observations into 50 bins based on a firm's relative size. We plot the average of the standardized cash holdings for each bin.*

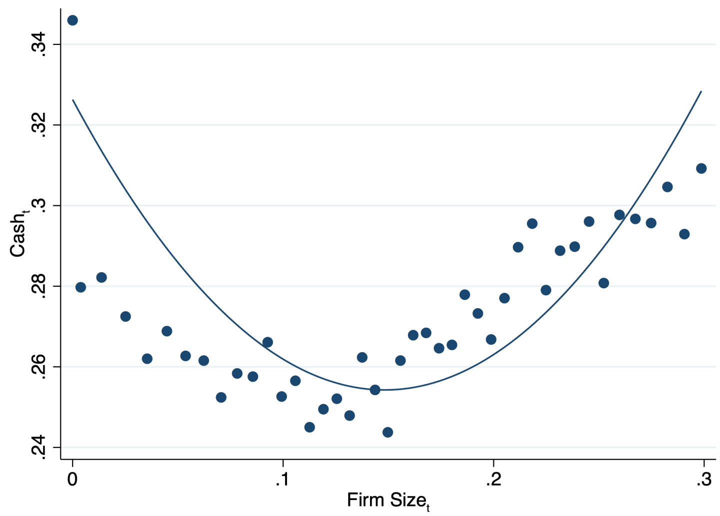
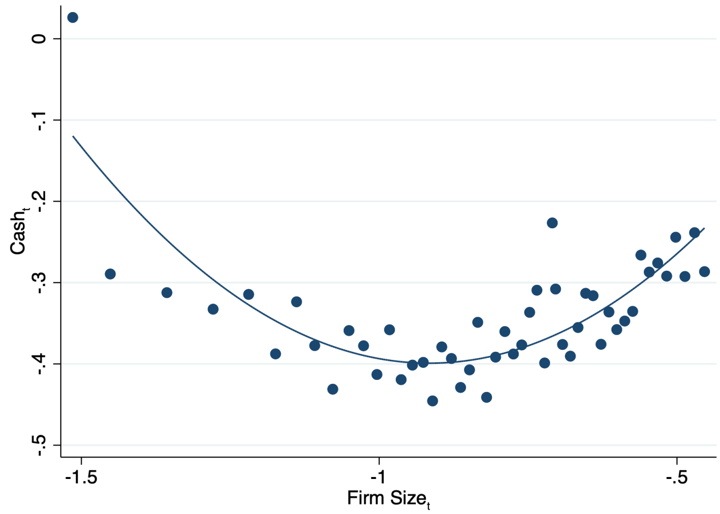
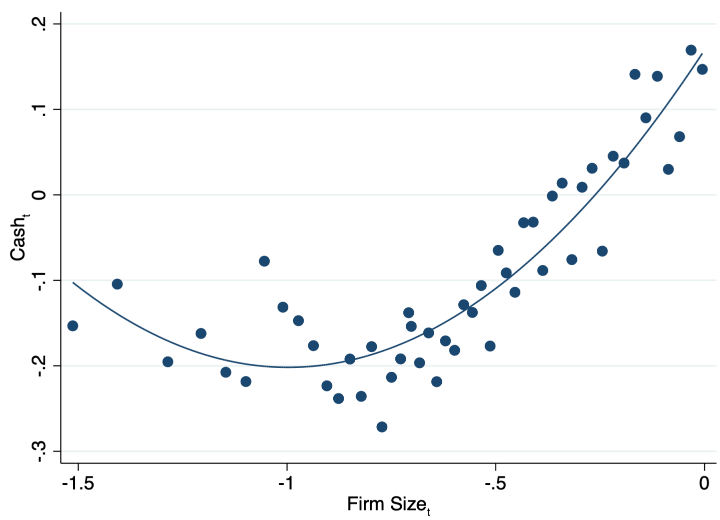



Figure D.5: **Repeats Figure 5(a) using min-max scaling instead of standardizing within a firm**

*The vertical axis is a firm's level of cash holdings at the end of year t minus the minimum cash holdings for that firm in the sample scaled by the range of cash holdings in the sample. The horizontal axis is a firm's market capitalization less cash holdings at the end of year t, minus the minimum of that quantity for that firm in the sample scaled by the range of that quantity in the sample. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median min-max scaled size. For this reason, the horizontal axis only has min-max-scaled values of firm size less than 0.5. We sort firms into 50 bins by this min-max scaling variable. Then, we plot the average of the standardized cash holdings for each bin.*



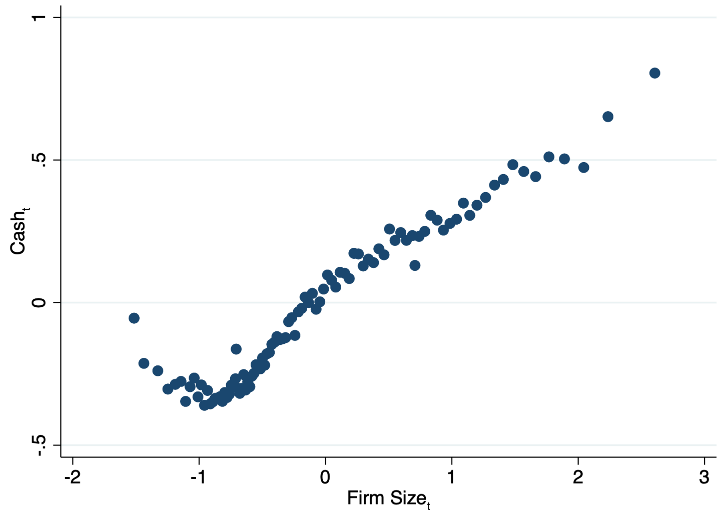

(a) **Cash holdings and firm size before 1996**



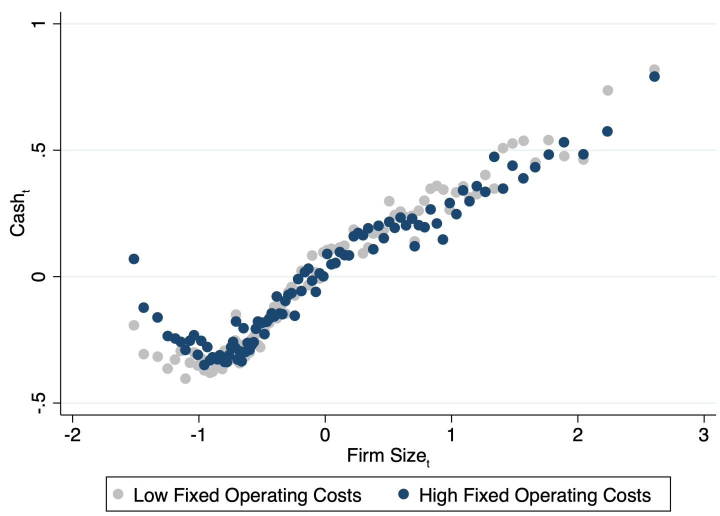

(b) **Cash holdings and firm size after 1996**

Figure D.6: **Cash holdings and firm size before and after the median sample year of 1996.**

*The vertical axis is a firm's level of cash holdings at the end of year t, standardized within a firm. The horizontal axis is a firm's market equity less its cash reserve at the end of year t, standardized within a firm. For each sample period, we sort the firm-year observations into 50 bins based on a firm's relative size. Then, we plot the average of the standardized cash holdings for each bin. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero.*




(a) **Cash holdings**




(b) **Cash holdings and Fixed Operating Costs**

Figure D.7: **Repeats Figure 5 for all firm years.**

*The vertical axis is a firm's level of cash holdings at the end of year t, standardized within a firm. The horizontal axis is a firm's market equity less its cash reserve at the end of year t, standardized within a firm. In panel A, we first sort the firm-year observations into 100 bins based on a firm's relative size. Then, we plot the average of the standardized cash holdings for each bin. In panel B, we first sort firms by whether their average fixed costs in the sample are above the sample median.*

*Appendix D.2. Robustness test for the empirical fact that issuance amounts are U-shaped in firm size*

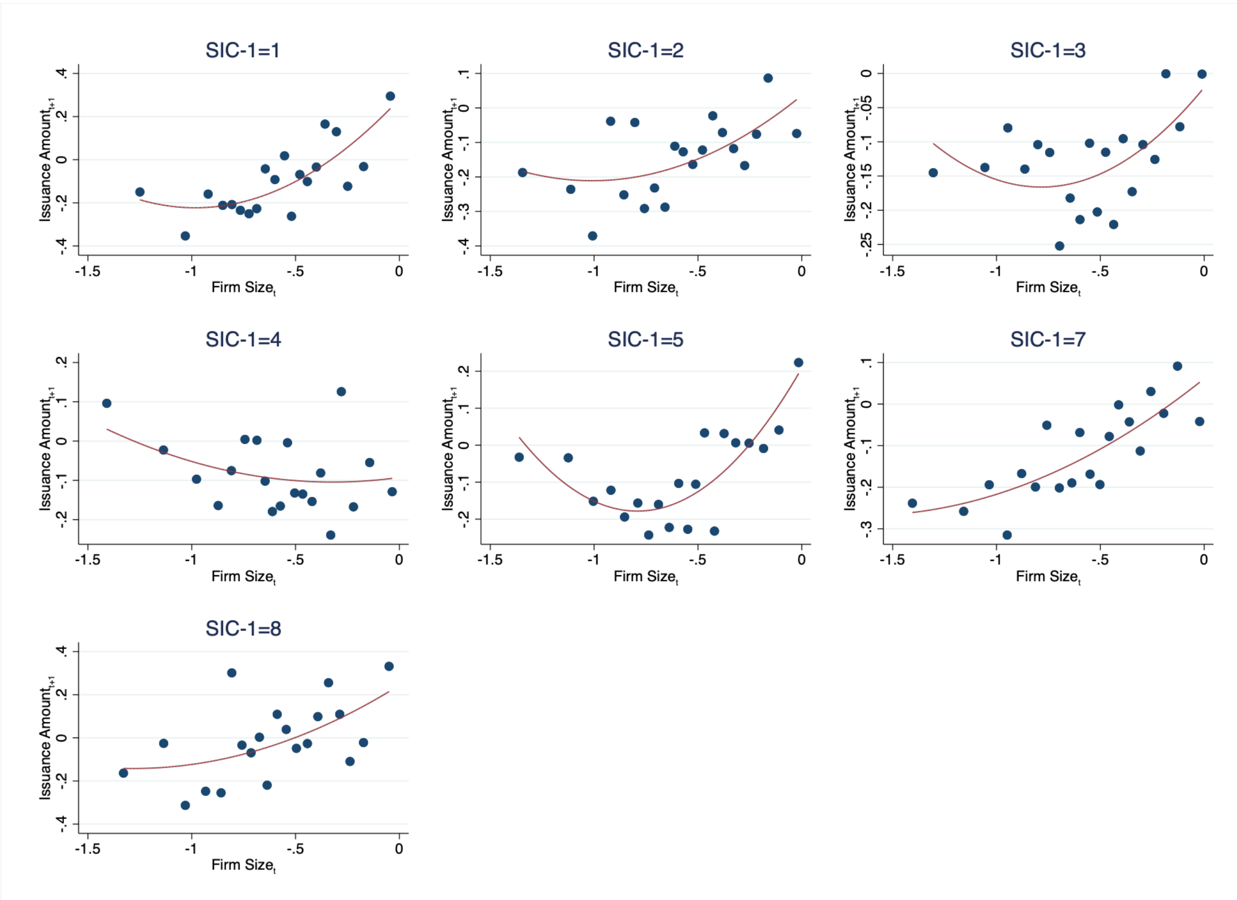
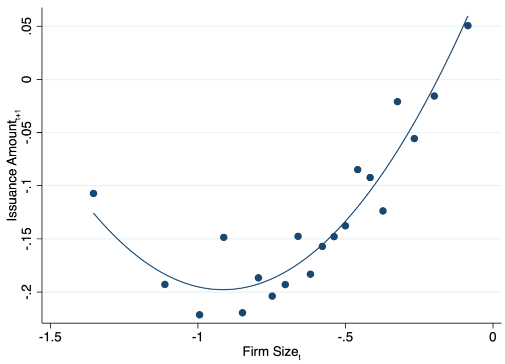
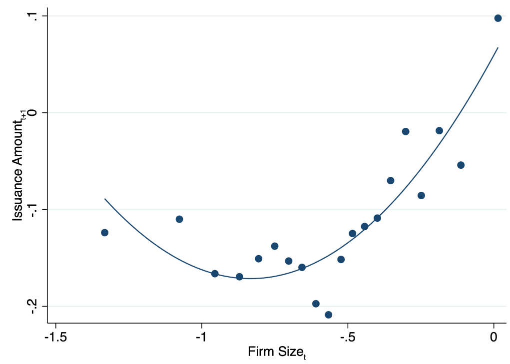



Figure D.8: **Repeats Figure 6(a) by industry**

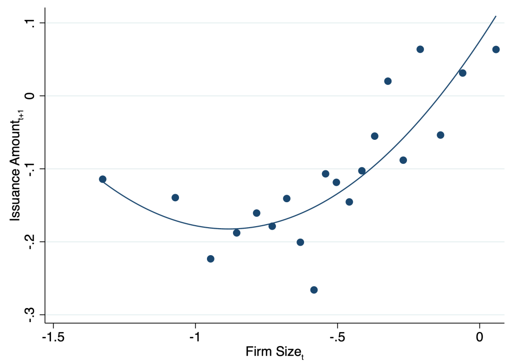
*The horizontal axis is a firm's market capitalization less its cash holdings at the end of year t, standardized within a firm. The sample is limited to firm years with common stock sales exceeding \$5 million in year $t+1$. After this restriction, we calculate a firm's common stock sold during year $t+1$, standardized within a firm (the vertical axis). Also, because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. For each SIC-1 industry, we sort firm-year observations into 20 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. We control for a firm's cash holdings at the end of year t. The industries are SIC-1=1 is Mining & Construction, SIC-1=2 is Manufacturing (Food, Apparel, Furniture, Chemicals, Petroleum), SIC-1=3 is Manufacturing (Rubber, Leather, Stone, Metals, Transportation Equipment), SIC-1=4 is Transportation and Public Utilities, SIC-1=5 is Wholesale and Retail Trade, SIC-1=7 is Services (Hotel, Personal, Business, Auto Repair), and SIC-1=8 is Services (Health, Legal, Educational, Social, Museums).*




(a) **$1 million threshold**



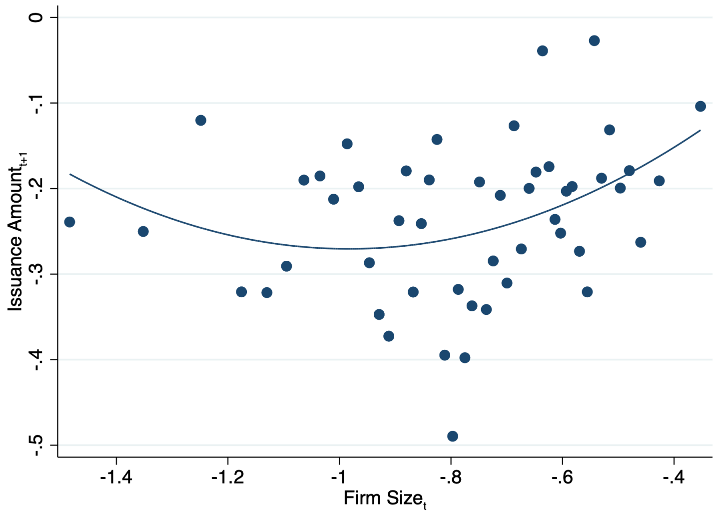

(b) **$10 million threshold**



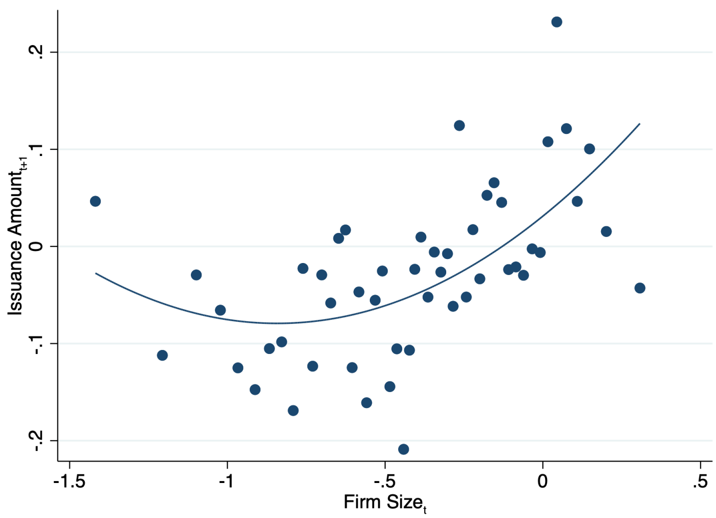

(c) **$25 million threshold**

Figure D.9: **Repeats Figure 6(a) for different cutoffs for offering size**

*The horizontal axis is a firm's market capitalization less its cash holdings at the end of year $t$, standardized within a firm. The vertical axis is the amount of common stock sold in year $t+1$, standardized within a firm. Before standardizing the issuance amount, we limit the sample to firm years with common stock sales exceeding the specified thresholds during the year $t+1$. The IPO year is also dropped. For this reason, only firms with multiple equity issuances above the specified threshold after the IPO offering are in the sample. Because fixed costs are more consequential when a firm is small, we also limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort firm-year observations into 20 bins based on a firm's relative size. We plot the average of the standardized issuance amount for each bin. We control for a firm's cash holdings at the end of year $t$ standardized within firm.*




(a) **Equity issuance amounts and firm size before 1996**




(b) **Equity issuance amounts and firm size after 1996**

Figure D.10: **Equity issuance amounts and firm size before and after the median sample year of 1996**

*The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. The sample is limited to firm years with common stock sales exceeding \$5 million in year $t+1$. We drop the IPO year. After these restrictions, we calculate a firm's common stock sold during the year $t+1$, standardized within a firm (the vertical axis). We sort firm-year observations into 50 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. We control for a firm's cash holdings at the end of year $t$, standardized within a firm. Because fixed costs are more consequential when a firm is small, we also limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis mostly has standardized values of firm size less than zero.*

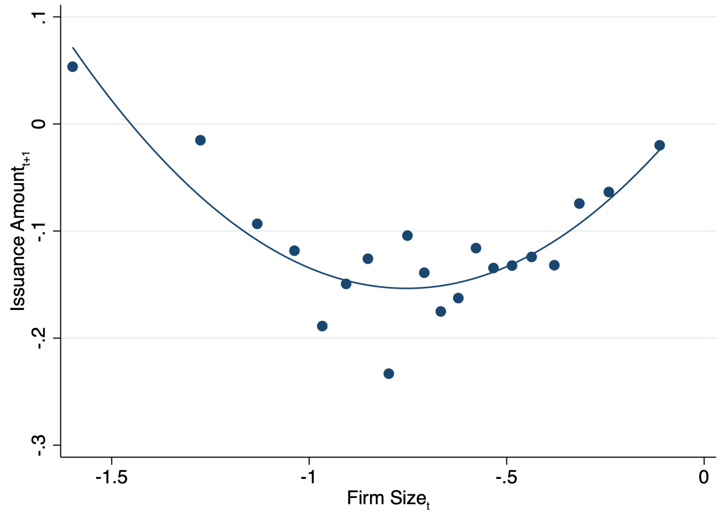



Figure D.11: **Repeats Figure 6(a) using a firm's tangible and intangible capital to proxy for a firm's size**

*The horizontal axis is a firm's physical and intangible capital at the end of year $t$, standardized within a firm. The sample is limited to firm years with common stock sales exceeding \$5 million in year $t + 1$. After this restriction, we calculate a firm's common stock sold during year $t + 1$, standardized within a firm (the vertical axis). Also, because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort firm-year observations into 20 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. We control for a firm's cash holdings at the end of year $t$.*

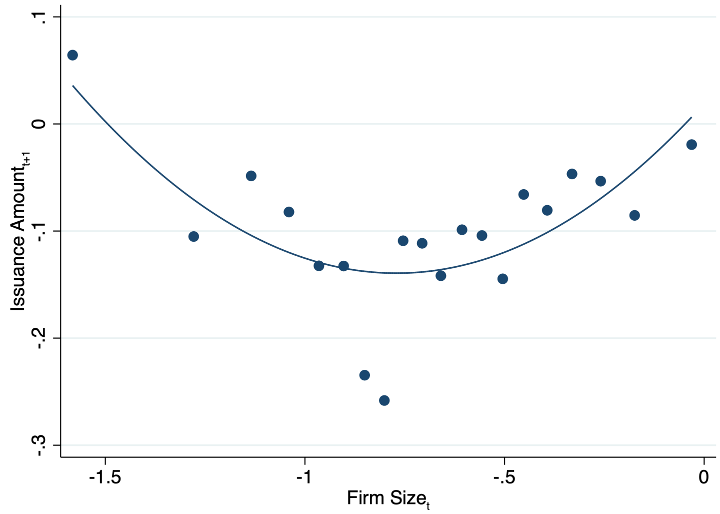



Figure D.12: **Repeats Figure 6(a) using a firm's sales to proxy for a firm's size**

*The horizontal axis is a firm's revenues during year $t$, standardized within a firm. The sample is limited to firm years with common stock sales exceeding \$5 million in year $t+1$. After this restriction, we calculate a firm's common stock sold during year $t+1$, standardized within a firm (the vertical axis). Also, because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort firm-year observations into 20 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. We control for a firm's cash holdings at the end of year $t$.*

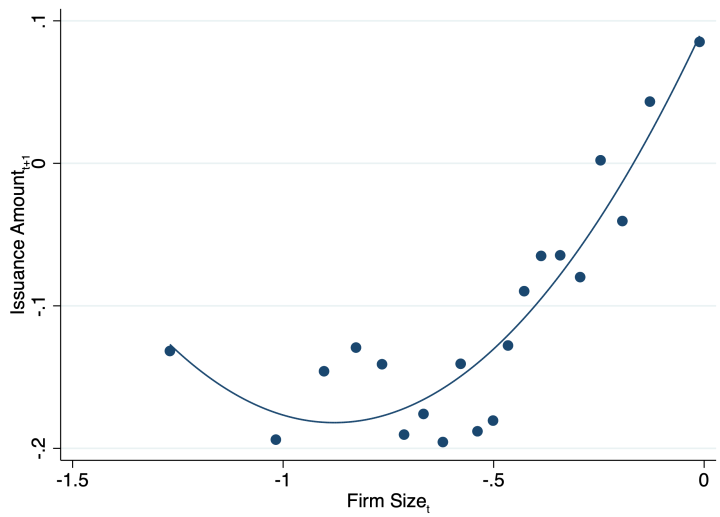



Figure D.13: **Repeats Figure 6(a) controlling for past offering amounts**

*The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. The sample is limited to firm years with common stock sales exceeding \$5 million in year $t+1$. After this restriction, we calculate a firm's common stock sold during year $t+1$, standardized within a firm (the vertical axis). We sort firm-year observations into 20 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. We control for a firm's cash holdings at the end of year $t$, standardized within a firm, and the size of a firm's past issuance amount.*

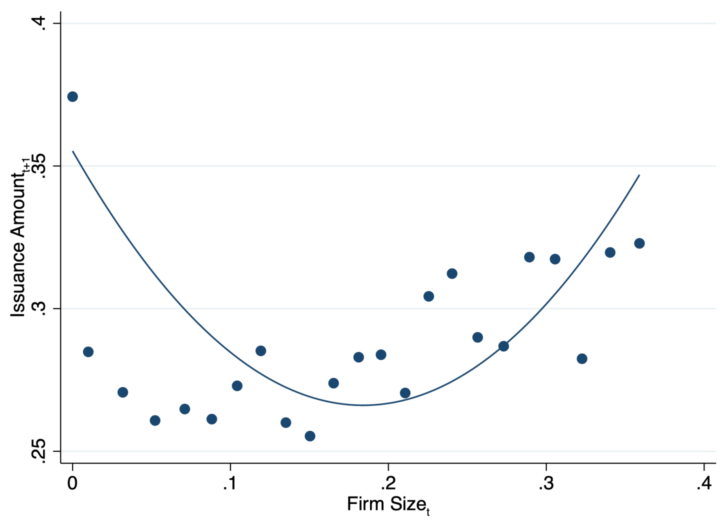
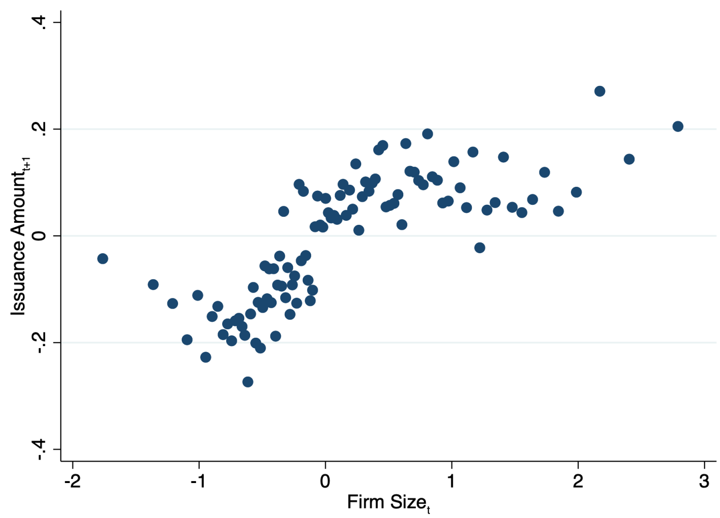
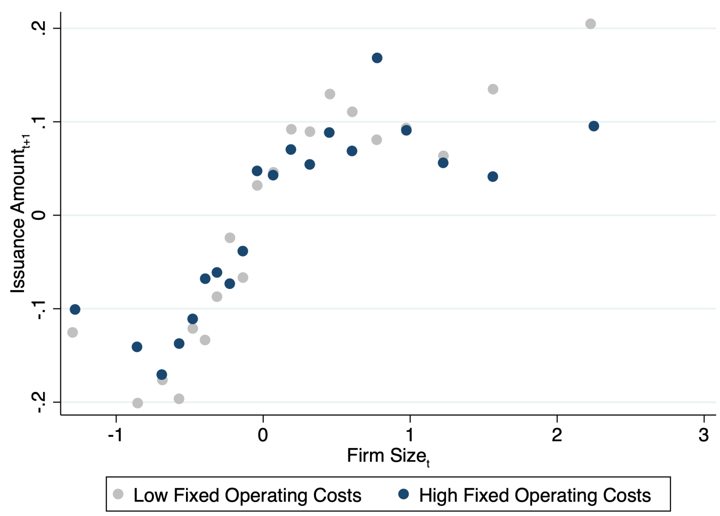



Figure D.14: **Repeats Figure 6(a) using min-max scaling instead of standardizing within a firm**

*The sample is limited to firm years with common stock sales exceeding \$5 million in year $t+1$. The vertical axis is the issuance amount in year $t+1$ minus the minimum issuance amount for that firm in the sample scaled by the range of issuance amounts in the sample. The horizontal axis is a firm's market capitalization less cash holdings at the end of year $t$, minus the minimum of that quantity for that firm in the sample scaled by the range of that quantity in the sample. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median min-max scaled size. For this reason, the horizontal axis only has min-max-scaled values of firm size less than 0.5. We sort firms by this min-max scaling variable. Then, we plot the average of min-max scaled issuance amounts for each bin.*




(a) **Equity issuance amounts and firm size**




(b) **Equity issuance amounts, firm size, and fixed operating costs**

Figure D.15: **Equity issuance amounts, firm size, and fixed operating costs.**

*The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. The sample is limited to firm years with common stock sales exceeding \$5 million in year $t+1$. After this restriction, we calculate a firm's common stock sold during the year $t+1$, standardized within a firm (the vertical axis). In panel A, we sort firm-year observations into 100 bins based on a firm's relative size and plot the average of the standardized issuance amount for each bin. In panel B, we first sort firms by whether their average fixed operating costs in the sample are above the sample median. We control for a firm's cash holdings at the end of year $t$, standardized within a firm.*

## Table D.2: Re-examines the results on issue size in table VI of Chang, Dasgupta and Hilary (2006), accounting for our predicted determinants of issue size

**Discussion:** This table shows that accounting for the novel non-linearities in issue size predicted by our model improves the explanatory power of the regressions in Chang, Dasgupta and Hilary (2006) and does not overturn their results. Specifically, the coefficient on *PSTKRTN (t)×NbrAnal (t-3)* remains negative and significant, suggesting that issuances by firms with more analyst coverage are less sensitive to past stock price performance. One takeaway is that the adjusted within-$R^2$ from their baseline specification in column (1) increases from 5.73% to 10.48%, or by 83%, when we add controls for a firm's relative cash and capital positions. A second takeaway is that the non-linearity in issuance size and capital is significant as in Table D.2 after accounting for all of the controls in Chang, Dasgupta and Hilary (2006).

As in Chang, Dasgupta and Hilary (2006), data are collected from Compustat, CRSP, and I/B/E/S for the years 1985 to 2000. Note that the details in Chang, Dasgupta and Hilary (2006) are not precise enough for a perfect replication. The dependent variable is the amount of net equity issued, scaled by the firm's prior-year total capital, which is the sum of a firm's physical and intangible capital per Peters and Taylor (2017). Regressions include issue years in which the net equity issued exceeds 1% of capital. Net equity issued is the sale of common stock minus the purchase of common stock. *NbrAnal* is the maximum number of analysts that make annual earnings forecasts in any month over a 12-month period that ends 3 years prior to the issue decision. *Return on assets* is earnings before depreciation and amortization, divided by total assets. The *market-to-book ratio* is defined as (the market value of equity plus the book value of debt)/the book value of assets. *PSTKRTN* is the compounded monthly stock return over the past-24-month period. *Capital* is the amount of physical and intangible capital per Peters and Taylor (2017) (standardized within firm) in the prior year and its square. *Cash* is the firm's cash reserve (standardized within firm). Standard errors clustered by firm are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | Amt. Raised (t)/Capital (t-1) × 100 | |
|---|---|---|
| | (1) | (2) |
| PSTKRTN (t) | 6.05*** | 5.89*** |
| | (1.45) | (1.41) |
| NbrAnal (t-3) | -1.13*** | -0.92*** |
| | (0.17) | (0.16) |
| PSTKRTN (t) × NbrAnal (t-3) | -0.56*** | -0.59*** |
| | (0.11) | (0.11) |
| $\text{PSTKRTN}^2$ (t) | 1.73** | 1.88** |
| | (0.76) | (0.73) |
| $\text{PSTKRTN}^3$ (t) | -0.01 | -0.04 |
| | (0.75) | (0.73) |
| Return on assets (t) | -4.96 | -16.87*** |
| | (5.18) | (5.13) |
| Market-to-book ratio (t) | 1.80*** | 1.45*** |
| | (0.57) | (0.55) |
| Ln(Assets) (t) | 14.85*** | 21.04*** |
| | (1.58) | (1.91) |
| Cash (t-1) | | -8.42*** |
| | | (0.60) |
| Capital (t-1) | | -9.84*** |
| | | (1.04) |
| $\text{Capital}^2$ (t-1) | | 1.59** |
| | | (0.65) |
| Constant | -64.89*** | -95.53*** |
| | (8.12) | (9.70) |
| Firm FE | Yes | Yes |
| Year FE | Yes | Yes |
| % Adjusted Within $R^2$ | 5.73 | 10.48 |
| Observations | 9,016 | 9,016 |

Table D.3: Predicting a firm's target debt-to-asset ratio per Hovakimian, Opler and Titman (2001), which is used later in Internet Appendix Table D.4

**Discussion:** The purpose of this table is to determine the target capital structure for firms used in the tests in Internet Appendix Table D.4. We replicate as closely as possible the tobit model in Hovakimian, Opler and Titman (2001) used to predict a firm's target debt-to-asset ratio (1979–1997). The details in Hovakimian, Opler and Titman (2001) are not precise enough for a perfect replication. Nevertheless, we achieve a fairly similar sample size, and the coefficients are generally similar. Column (1) shows the coefficients for our sample, and column (2) shows the results in Hovakimian, Opler and Titman (2001) table 3, panel C, which is the main specification used in the subsequent tables in that paper.

All variables are defined as differences from three-digit SIC industry means for a given year. The dependent variable, debt/assets, equals (the book value of debt)/(the book value of debt + the market value of equity). The *tangible assets ratio* is the ratio of property, plant, and equipment to the book value of assets. *Firm size* is the natural log of total assets. Robust standard errors are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | Our Sample<br>(1) | Hovakimian, Opler and Titman (2001) Sample<br>(2) |
|---|---|---|
| R&D expenditures/sales | -0.26*** | -0.27*** |
| | (0.02) | |
| Selling expenses/sales | -0.08*** | -0.06*** |
| | (0.01) | |
| Tangible assets ratio | 0.10*** | 0.20*** |
| | (0.01) | |
| Firm size | -0.02*** | 0.00** |
| | (0.00) | |
| Constant | -0.05*** | -0.01*** |
| | (0.00) | |
| Observations | 36,254 | 39,387 |

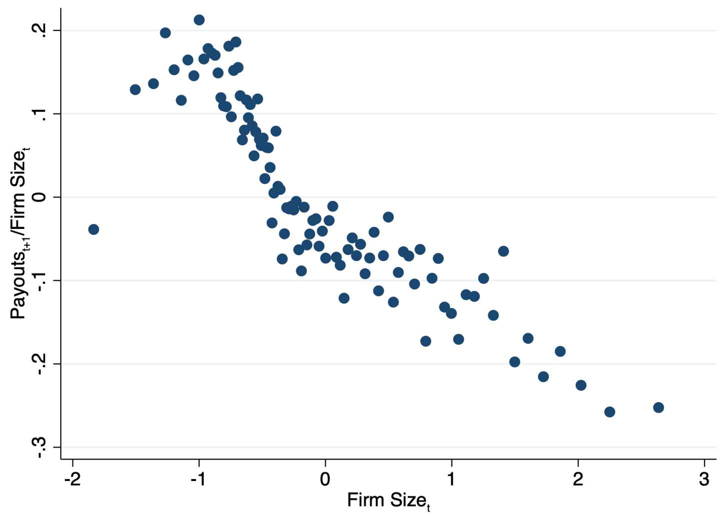
# Table D.4: Re-examining column (1) of table 7 of Hovakimian, Opler and Titman (2001) relating distance to the target capital structure and equity issuance amount

**Discussion:** In Hovakimian, Opler and Titman (2001), table 7, column (1), there is a positive but insignificant relation between the distance of a firm to its target capital structure and equity issuance. That paper measures distance to the target capital structure by comparing the estimated target debt-to-assets ratio (D/A) derived as in Internet Appendix Table D.3 to the industry average D/A and separately by comparing the industry average D/A to the firm's actual D/A. Column (1) in our table shows their estimated coefficients. We are not able to match their sample exactly given the information provided. However, we construct all the same controls and then update the sample through 2017. Many coefficients are similar. Surprisingly, in column (2) for our sample, the positive coefficients on the two measures of distance to the target capital structure become statistically significant. These positive coefficients are the wrong direction because tradeoff theory would predict that firms that are further below their target D/A should issue less equity. However, in column (3), when we control for a firm's relative cash and capital positions and the non-linearity in capital, we find that the positive relations weaken. Also, the adjusted $R^2$ increases from 6.88% to 10.26%, or by 49%. Also, note that the relative cash and capital positions remain statistically significant when controlling for the covariates in that paper. In Column (4), because our model shows that financing costs are related to issuance size, we add controls for the number of analysts and the standard deviation of analyst estimates as proxies of a firm's financing costs. The coefficients related to target capital structure are insignificant, and the coefficients on a firm's relative cash and capital positions remain statistically significant. Also, the adjusted $R^2$ now increases from 6.88% to 11.85%, or by 72%.

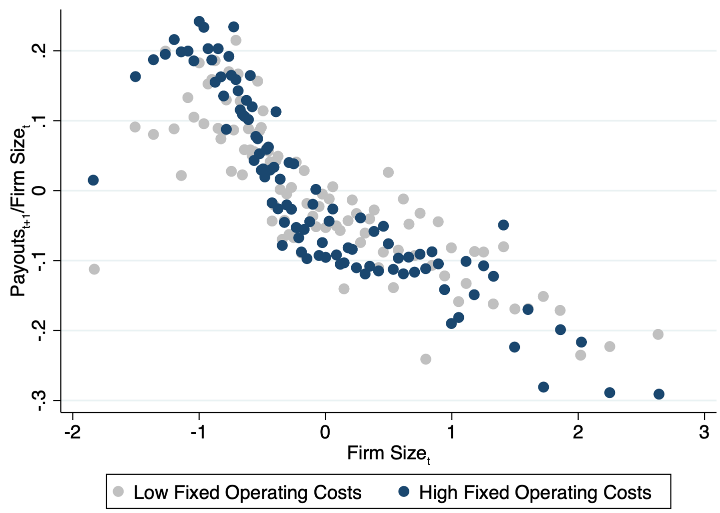
The details in Hovakimian, Opler and Titman (2001) are not precise enough for a perfect replication. As in that paper, we restrict the sample to firm-years with issuances, which is when the net amount issued divided by the book value of assets exceeded 5%. Cases where firms issued both debt and equity in a given fiscal year are omitted. *D/A* is the debt/assets measured with equity at market in the year prior to the issuance period. *Target D/A* is estimated as the fitted value from the regression in their table 3, panel C. Our Table D.3 shows a similar specification. *ROA* is earnings before interest, taxes, depreciation, and amortization divided by the book value of assets. *NOLC* is the net operating loss carryforwards scaled by the book value of assets. The *two-year stock return* is defined as the split- and dividend-adjusted percentage return from the beginning of the pre-issue year until the close of the issue year. The *market-to-book ratio* is defined as (the market value of equity + the book value of debt)/total assets. *Dilution dummy* captures whether an equity issue could dilute earnings and is set to zero except when one minus the assumed tax rate times the yield on Moody's Baa-rated debt is less than a firm's after-tax earnings-price ratio. The tax rate is assumed to be 50% before 1987 and 34% afterward. Robust standard errors are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | Common Stock Issue (Net Amount Issued (t) / Capital (t-1)) | | | |
|---|---|---|---|---|
| | Hovakimian, Opler and Titman (2001) Table 7 Column (1) | Our Sample | Adding Cash | Adding Analyst |
| | (1) | (2) | (3) | (4) |
| Target D/A-industry mean D/A | 0.24 | 0.13** | 0.10* | 0.07 |
| | | (0.06) | (0.06) | (0.06) |
| Industry mean D/A-actual D/A | 0.06 | 0.04* | 0.02 | 0.04 |
| | | (0.02) | (0.02) | (0.02) |
| Three-year mean ROA | -0.34*** | -0.34*** | -0.34*** | -0.27*** |
| | | (0.04) | (0.04) | (0.04) |
| NOLC | 0.03** | -0.00 | -0.01 | -0.01 |
| | | (0.01) | (0.01) | (0.01) |
| Two-year stock return | 0.06*** | 0.04*** | 0.03*** | 0.03*** |
| | | (0.01) | (0.01) | (0.01) |
| Market-to-book ratio | 0.05*** | 0.01* | 0.00 | 0.01* |
| | | (0.00) | (0.00) | (0.00) |
| Dummy for M/B > 1 | 0.04 | 0.04*** | 0.04*** | 0.04*** |
| | | (0.01) | (0.01) | (0.01) |
| Dilution dummy | -0.05*** | 0.01 | 0.01 | 0.00 |
| | | (0.01) | (0.01) | (0.01) |
| FD3 | 0.04* | 0.04** | 0.03** | 0.02 |
| | | (0.02) | (0.02) | (0.02) |
| Loss dummy $\times$ FD3 | -0.03 | 0.01 | 0.01 | 0.01 |
| | | (0.02) | (0.02) | (0.02) |
| Cash (t) | | | -0.02*** | -0.02*** |
| | | | (0.00) | (0.00) |
| Capital (t) | | | -0.05*** | -0.04*** |
| | | | (0.01) | (0.01) |
| Capital$^2$ (t) | | | 0.01* | 0.01** |
| | | | (0.01) | (0.01) |
| Number of Analyst Estimates | | | | -0.00*** |
| | | | | (0.00) |
| Standard Deviation of Analyst Estimates | | | | 0.68** |
| | | | | (0.32) |
| Constant | 0.10*** | 0.22*** | 0.21*** | 0.21*** |
| | | (0.02) | (0.02) | (0.02) |
| Sample Period | '79-'97 | '79-'17 | '79-'17 | '79-'17 |
| Year FE | Yes | No | No | No |
| SIC-2–by–Year FE | No | Yes | Yes | Yes |
| % Adjusted Within $R^2$ | | 6.88 | 10.26 | 11.85 |
| Observations | 2,231 | 3,452 | 3,452 | 3,452 |

*Appendix D.3. Robustness for the empirical fact that payout rates are hump-shaped in firm size*



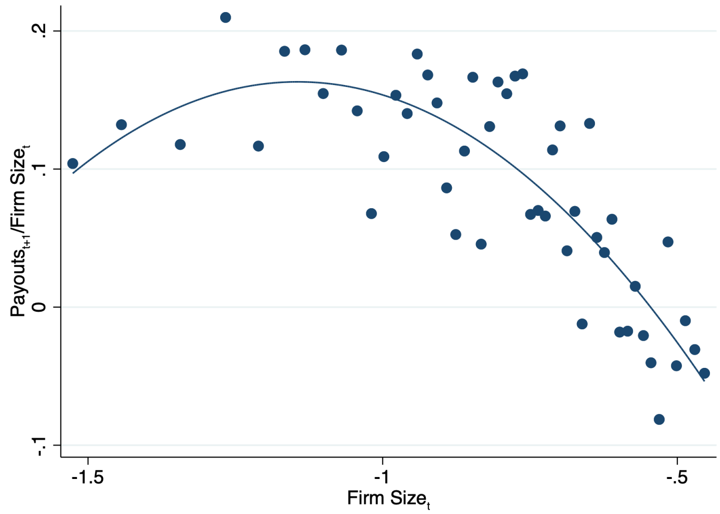

(a) **Payout rates and firm size**



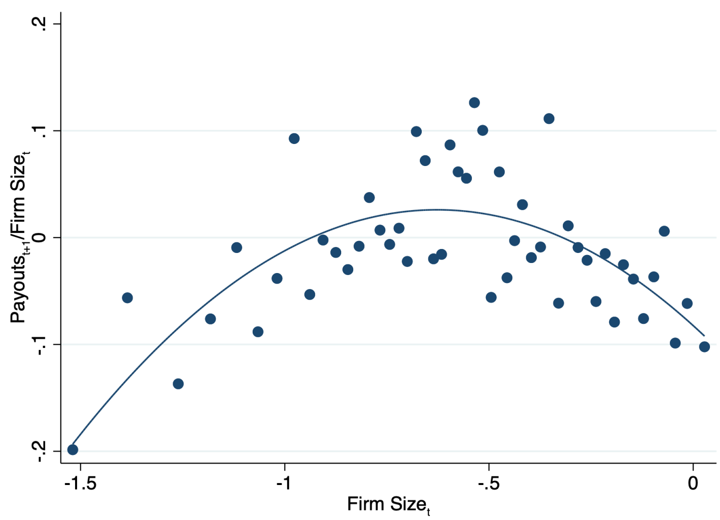

(b) **Payout rates, firm size, and fixed operating costs**

Figure D.16: **Payout rates, firm size, and fixed operating costs.**

*The vertical axis is a firm's total payouts (common stock dividends and repurchases) during the year $t+1$, standardized within a firm. The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. In panel A, we sort firm-year observations into 100 bins based on a firm's relative size and plot the average of the standardized payout amount for each bin. Each bin has about 1,000 firm-year observations. In panel B, we first sort firms by whether their average fixed operating costs in the sample are above the sample median. We control for a firm's cash holdings at the end of year $t$.*




(a) **Payout rates and firm size before 1996**




(b) **Payout rates and firm size after 1996**

Figure D.17: **Payout rates and firm size before and after the median sample year of 1996.**

*The vertical axis is a firm's total payouts (common stock dividends and repurchases) during the year $t+1$, standardized within a firm. The horizontal axis is a firm's market equity less its cash holdings at the end of year $t$, standardized within a firm. We sort firm-year observations into 50 bins based on a firm's relative size and plot the average of the standardized payout amount for each bin. We control for a firm's cash holdings at the end of year $t$.*

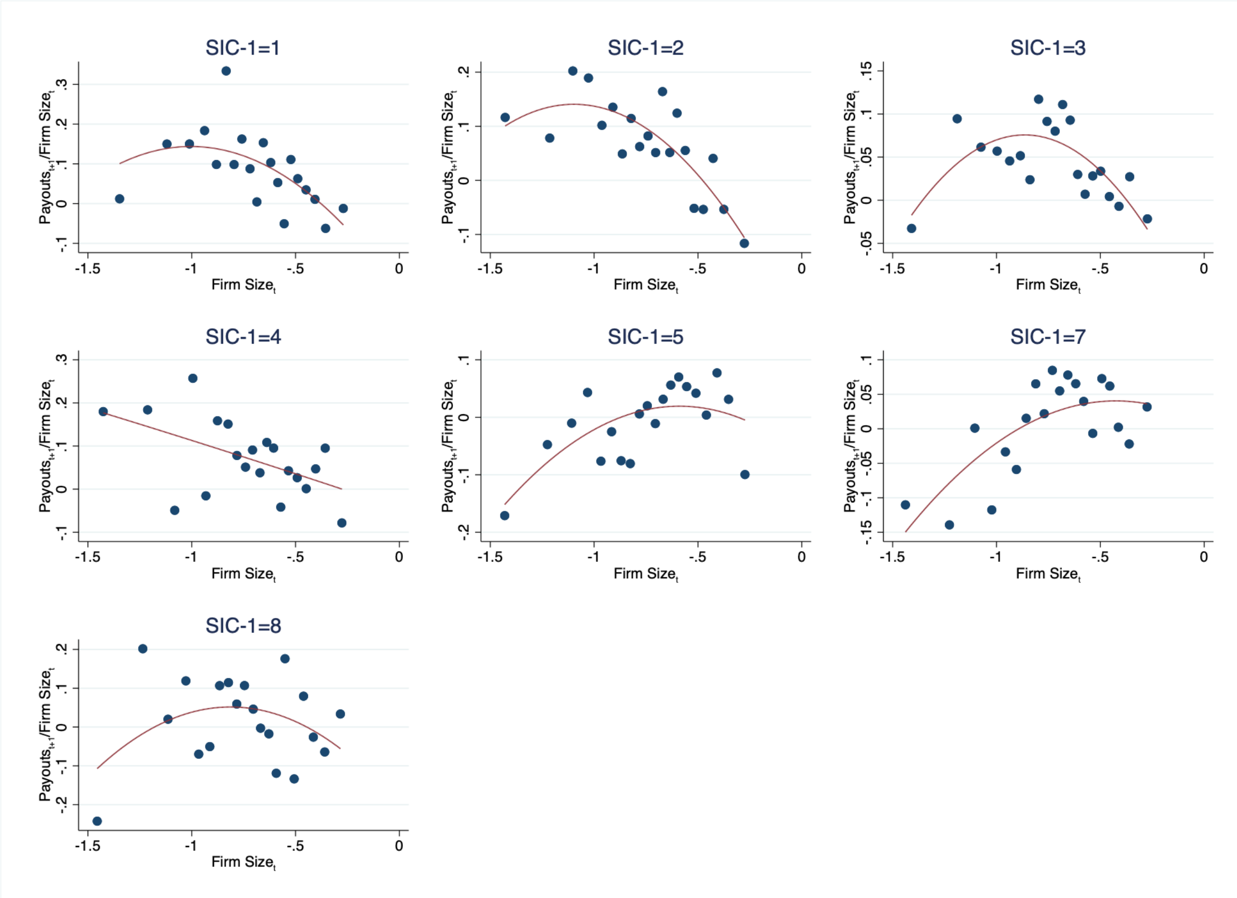



Figure D.18: **Repeats Figure 7(a) by industry**

*The vertical axis is a firm's payout rate during year $t+1$. Payouts include common stock dividends and share repurchases. We scale payouts by a firm's market capital less its cash holdings at the end of year $t$. We standardize payout rates within a firm. The horizontal axis is a firm's market capitalization less its cash holdings at the end of year $t$, standardized within a firm. To better illustrate the hump-shape relation, we restrict the sample to firm years when the firm is below its median size. For this reason, the standardized values of firm size on the horizontal axis are always negative. We first sort firms by SIC-1 industry classification. Then, we sort firm-year observations into 20 bins based on a firm's relative size. We plot the average of the standardized payout rates for each bin. We control for a firm's cash holdings at the end of year $t$ standardized within a firm. The industries are SIC-1=1 is Mining & Construction, SIC-1=2 is Manufacturing (Food, Apparel, Furniture, Chemicals, Petroleum), SIC-1=3 is Manufacturing (Rubber, Leather, Stone, Metals, Transportation Equipment), SIC-1=4 is Transportation and Public Utilities, SIC-1=5 is Wholesale and Retail Trade, SIC-1=7 is Services (Hotel, Personal, Business, Auto Repair), and SIC-1=8 is Services (Health, Legal, Educational, Social, Museums).*

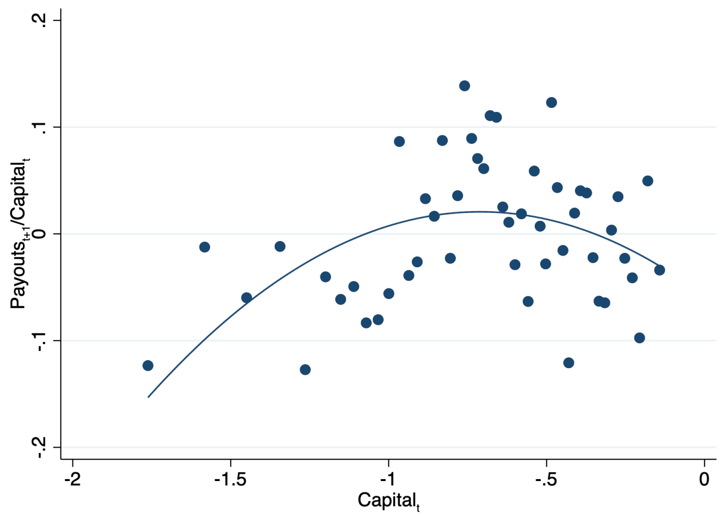



Figure D.19: **Repeats Figure 7(a) using a firm's tangible and intangible capital to proxy for a firm's size**

*The vertical axis is a firm's payout rate during the year $t+1$. Payouts include common stock dividends and share repurchases. We scale payouts by a firm's physical and intangible capital at the end of year $t$ (Peters and Taylor, 2017). We standardize payout rates within a firm. The horizontal axis is a firm's physical and intangible capital at the end of year $t$, standardized within a firm. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort firm-year observations into 50 bins based on a firm's relative size and plot the average of the standardized payout rates for each bin. We control for a firm's cash holdings at the end of year $t$.*

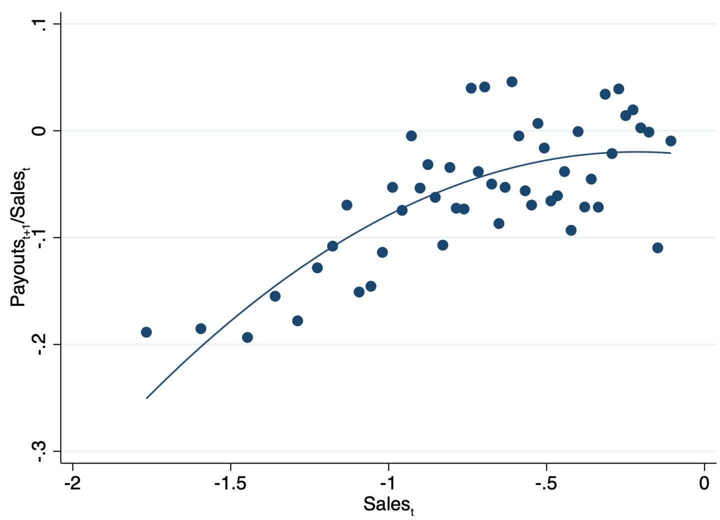



Figure D.20: **Repeats Figure 7(a) using a firm's sales to proxy for a firm's size**

*The vertical axis is a firm's payout rate during the year* $t+1$*. Payouts include common stock dividends and share repurchases. We scale payouts by a firm's sales during the year* $t$*. We standardize payout rates within a firm. The horizontal axis is a firm's sales in year* $t$*, standardized within a firm. Because fixed costs are more consequential when a firm is small, we limit the sample to firm years when a firm is below its median size. For this reason, the horizontal axis only has standardized values of firm size less than zero. We sort firm-year observations into 50 bins based on a firm's relative size and plot the average of the standardized payout rates for each bin. We control for a firm's cash holdings at the end of year* $t$*.*

## Table D.5: Examining column (1) of table 3 of Bliss, Cheng and Denis (2015), looking at dividend reductions during the 2008–2009 financial crisis

**Discussion:** The results in this table show a similar increase in the probability that a firm reduces payouts during a crisis as in Bliss, Cheng and Denis (2015). One takeaway is that when we go from column (2) to column (3) and control for the relation between payouts and a firm's cash and capital stock, the coefficient on *Crisis* increases in size by 46% (0.73/0.50) in our sample. A second takeaway is that the pseudo $R^2$ increases by 14.5% (10.77/9.41) when accounting for a firm's cash and capital stock. A third takeaway is that the relations between cash and capital and the non-linearity in capital hold after controlling for the other variables in Bliss, Cheng and Denis (2015). It is important to note that this test looks at reductions in payouts, so the signs on *Cash (t)* and *Capital (t)*$^2$ are flipped.

The details in Bliss, Cheng and Denis (2015) are not precise enough for a perfect replication. We focus on their table 3, column (1). As in that paper, the sample is restricted to firms with positive average payouts over the prior two years. The sample period is 2005 to 2009. The outcome is a binary variable that equals one if the firm reduces the total payout (dividends plus repurchases) by at least 5%. The main explanatory variable is *Crisis*, which is a binary variable equal to one if the fiscal year is 2008 or 2009, and zero otherwise. The next 10 control variables are constructed as in the appendix to Bliss, Cheng and Denis (2015). We add the following based on H2 of our paper: *Capital* is the total capital of the firm (standardized within firm) and its square. *Cash* is the cash and cash equivalents of the firm (standardized within firm). The independent variables are lagged except contemporaneous *Cash flow/Lag TA* and *Tobin's Q*. Industry fixed effects are based on the Fama-French 48-industry definitions. Robust standard errors are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | $\mathbb{1}$(Reduction in Total Payout>5%) | | |
|---|---|---|---|
| | Bliss, Cheng and Denis (2015) Table 3 Column (1) | Our Sample | Adding Cash&Capital |
| | (1) | (2) | (3) |
| Crisis | 0.11** | 0.50*** | 0.73*** |
| | (0.05) | (0.19) | (0.22) |
| Crisis × Market leverage | 0.14** | -0.04 | -0.06 |
| | (0.07) | (0.06) | (0.06) |
| Crisis × Cash/TA | -0.34*** | -0.30 | -0.24 |
| | (0.08) | (0.43) | (0.47) |
| Crisis × Tobin Q | 0.08*** | 0.04 | -0.06 |
| | (0.03) | (0.13) | (0.13) |
| Crisis × Cash flow/TA | -0.00 | -1.63** | -1.59** |
| | (0.13) | (0.76) | (0.77) |
| Crisis × (R&D+CapEx)/TA | -0.54*** | 0.61 | 0.26 |
| | (0.15) | (0.80) | (0.81) |
| Crisis × Capital (t) | | | 0.01 |
| | | | (0.14) |
| Crisis × Capital (t)$^2$ | | | -0.16 |
| | | | (0.10) |
| Crisis × Cash (t) | | | -0.16* |
| | | | (0.08) |
| Ln(Assets) | -0.01*** | -0.14*** | -0.09*** |
| | (0.00) | (0.02) | (0.03) |
| Losses | 0.02** | 0.02 | 0.01 |
| | (0.01) | (0.03) | (0.03) |
| (R&D+CapEx)/TA | 0.88*** | 2.05*** | 1.84*** |
| | (0.15) | (0.51) | (0.52) |
| Market leverage | 0.31*** | 0.15*** | 0.16*** |
| | (0.07) | (0.06) | (0.06) |
| Cash flow/TA | -0.88*** | -2.19*** | -2.13*** |
| | (0.12) | (0.51) | (0.51) |
| Cash/TA | 0.00 | -0.17 | 0.78** |
| | (0.07) | (0.31) | (0.37) |
| Tobin Q | -0.15*** | -0.46*** | -0.51*** |
| | (0.02) | (0.09) | (0.09) |
| Cash flow volatility | 0.29** | 1.96** | 1.70* |
| | (0.13) | (0.91) | (0.90) |
| Total Payout/TA | 3.27*** | 13.55*** | 13.73*** |
| | (0.17) | (1.41) | (1.44) |
| Payout Reduction | 0.32*** | 0.35*** | 0.33*** |
| | (0.01) | (0.07) | (0.08) |
| Capital (t) | | | -0.13 |
| | | | (0.10) |
| Capital (t)$^2$ | | | 0.17*** |
| | | | (0.06) |
| Cash (t) | | | -0.27*** |
| | | | (0.06) |
| Industry FE | Yes | Yes | Yes |
| % Psuedo $R^2$ | 21.9 | 9.41 | 10.77 |
| Observations | 6,886 | 5,354 | 5,354 |

*Appendix D.4. The default region and leverage*

The literatures on offerings and defaults are generally distinct and concerned with the timing of offerings and defaults, respectively. Few studies examine how firms facing costly financing and distress choose whether to issue more equity or to default and how that decision interacts with leverage.

To examine default behavior, we compile bankruptcy information from various sources. First, we retrieve bankruptcy years from SDC Platinum. We drop a firm's observations following the bankruptcy year. Because firms may delist before declaring bankruptcy, we designate the final year a firm is in the sample as a bankruptcy year if SDC says the bankruptcy year is within two years of the end of a firm's time in the sample. Second, we use the status identifier in Compustat (*stalt*), which equals "TL" for companies in bankruptcy or liquidation. We drop observations after the first year the status changes to "TL." Third, we examine the deletion code (*dlrsn*), which equals two for bankruptcy and three for liquidation. Fourth, we retrieve delisting information from the Center for Research in Security Prices (CRSP, codes between 450 and 490 and between 550 and 587). Collectively, these bankruptcy data are most complete starting in 1984. In total, our data set contains 2,080 bankruptcies.

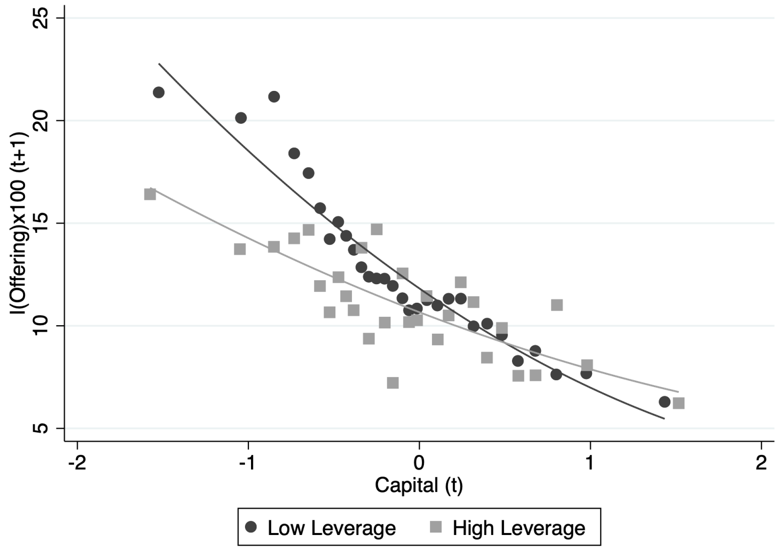
Figure C.7 shows that, in the presence of costly financing, more levered firms facing distress are less likely to issue equity and are more likely to default. Specifically, the target issuance boundary shifts to the right, expanding the default region. Intuitively, when capital is low, cash flows are low. Thus, equity holders have to decide whether to default or pay the issuance costs (which are larger for small firms because of the fixed component) to raise cash to fund the fixed operating cost and invest over time to rebuild the capital stock (because of the convex adjustment costs).

This reasoning leads to the following hypothesis:

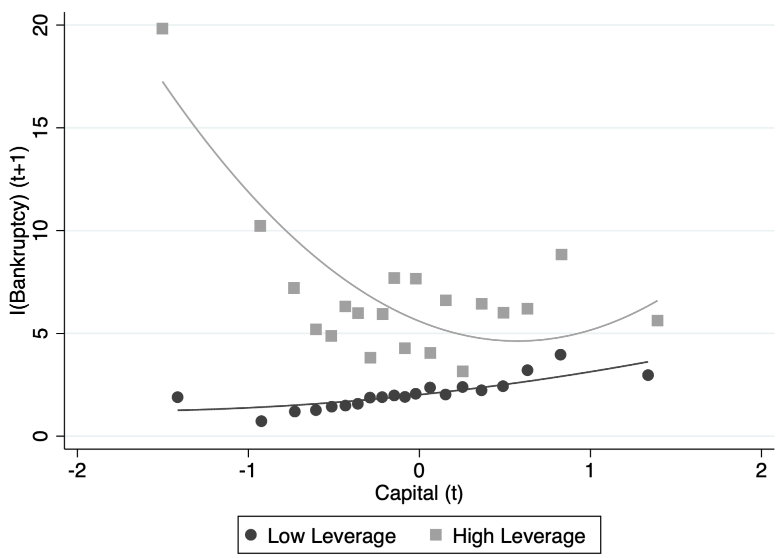
**H:** When capital is low, levered firms are less likely to conduct offerings and more likely to default.

Figure D.21(a) provides visual evidence consistent with this hypothesis. The vertical axis plots the propensity to conduct an offering for both firms with high leverage (above the 90th percentile of the sample distribution of debt to market equity) and low leverage. The horizontal axis represents a firm's total capital standardized within a firm. The figure shows that the propensity to conduct an offering is increasing as capital decreases. More importantly, the figure shows that the propensity to conduct an offering is lower for highly leveraged firms with low capital than for low-leveraged firms with low capital. By contrast, for firms with high capital, there is no difference between offering propensity and leverage.

Figure D.21(b) provides additional visual evidence consistent with this hypothesis. The vertical axis plots the propensity to declare bankruptcy in year $t + 1$. The horizontal axis shows a firm's total capital standardized within a firm. The figure shows that the propensity to declare bankruptcy is higher for highly leveraged firms with low capital than for low-leverage firms with low capital. By contrast, when capital is high, there is less of a difference in the bankruptcy propensity for high- and low-leverage firms.




(a) The vertical axis plots the percentage of firms issuing shares worth at least 5% of their capital stock in year $t + 1$. The horizontal axis shows a firm's total capital (tangible and intangible) standardized within a firm. We control for year and firm fixed effects. *High leverage* indicates a debt-to-market equity ratio above the 90th percentile in the firm-year sample.

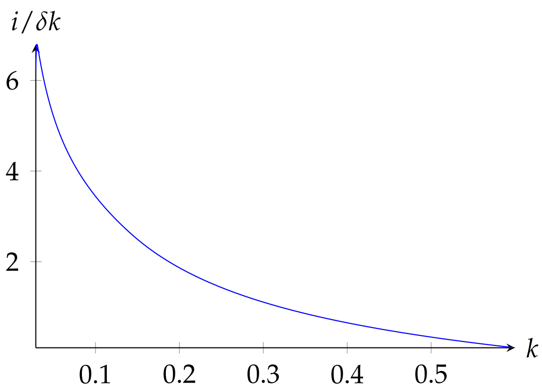



(b) The vertical axis plots the percentage of firms declaring bankruptcy in year $t + 1$. The horizontal axis shows a firm's total capital (tangible and intangible) standardized within a firm. Controls for year and firm fixed effects are included. *High leverage* indicates a debt-to-market equity ratio above the 90th percentile in the firm-year sample.

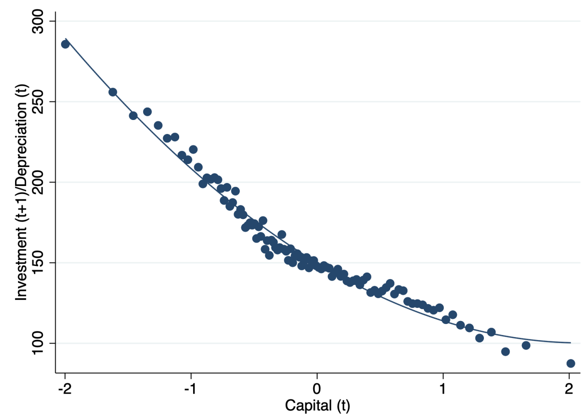
Figure D.21: **Leverage, Offerings, and Bankruptcy**

*Appendix D.5. Investment-to-depreciation is declining and convex in capital*

In our model, the assumption that capital exhibits diminishing returns creates convexity in the investment-to-depreciation ratio. Figure D.22(a) shows that the model-implied investment-to-depreciation ratio is declining and convex in capital. Intuitively, as capital increases, the marginal returns to investing decrease, discouraging investment. As capital increases, depreciation increases linearly in capital at the rate $\delta$. Together, the ratio of investment to depreciation should be declining and convex in capital.




(a) Figure D.22a plots the amount a firm invests scaled by depreciation at $c = 0.15$ from Figure 1. Here, $b = 0.02$, and the other parameters used are summarized in Table 1.




(b) The vertical axis plots a firm's investment in physical and intangible capital in year $t+1$. Depreciation is the depreciation of tangible and intangible capital in year $t$. The horizontal axis represents a firm's total capital (tangible and intangible) standardized within a firm.

Figure D.22: Predicted investment from Figure 1 versus actual investment

This reasoning leads to the following hypothesis:

**H:** The ratio of investment to depreciation is declining and convex in capital.

Examining the data, Figure D.22(b) provides initial support for this hypothesis. The bin-scatter plot shows a similarly convex pattern in the investment-to-depreciation ratio and a firm's capital stock, standardized within a firm.

To evaluate this hypothesis more formally, we estimate the following empirical specification:

$$\frac{\text{Investment}_{i,t+1}}{\text{Depreciation}_{i,t+1}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}^2_{i,t} + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}. \tag{D.1}$$

The outcome is firm $i$'s investment in physical and intangible capital in year $t+1$ scaled by the depreciation of physical and intangible capital in year $t$.[22] The main explanatory variable is a firm's total capital at the end of year $t$ standardized within a firm and winsorized at the 1% level. $\beta_2$ captures the sensitivity of the investment-to-depreciation ratio to the quadratic form of capital (standardized). We control for a firm's cash and equivalents, firm fixed effects $\mu_i$, and SIC-2–by–year industry trends ($\delta_{j,t}$). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double-clustered by firm and year.

Table D.6 presents the results. Column (1) uses the full sample of firms. The coefficient $\beta_1$ on a firm's capital position is -45.56 and is highly significant. The coefficient $\beta_2$ on a firm's capital squared, which captures the relation between the investment-to-depreciation ratio and the quadratic form of a firm's capital, is +18.33 and is also highly significant. Together, these results provide strong evidence that payouts are declining in capital in a convex pattern. Columns (2) and (3) show the relation is similar when splitting the sample for the investment analyses at the median year of 1995. Columns (4) to (10) show the convexity exists across SIC-1 industry classifications.

[22] To capture a firm's net investment activity, we use proxies for a firm's investment in physical and intangible capital. We use a firm's capital expenditures on property, plant, and equipment from the cash flow statement (*capx*) to measure expenditures on physical capital. We follow Hulten and Hao (2008), Eisfeldt and Papanikolaou (2014), and Peters and Taylor (2017) in measuring expenditures on intangible capital. Specifically, We calculate a firm's expenditures on intangible capital as its R&D (*xrd* plus *rdip*) plus 30% of a its SG&A (*xsga*) minus R&D expenses (*xrd*) minus in-process R&D (*rdip*). When *xrd* exceeds *xsga* but is less than the cost of goods sold (*cogs*), or when *xsga* is missing, then we measure SG&A as *xsga* with no further adjustments, or set it to zero if missing. R&D is subtracted from SG&A because Compustat adds R&D expenses to SG&A. Also, the literature interprets the remaining 70% of SG&A as operating costs that support the current period's profits. Both R&D and SG&A are set to zero if the corresponding data elements on Compustat are missing. We compare the investment amount to the depreciation on the firm's physical capital (*dp*) and intangible capital. Like the literature, we assume that the depreciation on intangible capital is 20% of the stock of intangible capital.

Table D.6: The ratio of investment to depreciation is convex and declining in capital
The outcome variable is the ratio of investment to depreciation. Investments include spending on physical capital as well as intangible capital on and off the balance sheet. We also determine the depreciation of physical and intangible capital. The main explanatory variable is the total capital of the firm (standardized within firm) and its square. We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 1995, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | Investment (t+1) / Depreciation (t+1) × 100 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| Capital (t) | -45.56*** | -43.43*** | -44.09*** | -64.70*** | -32.79*** | -41.26*** | -52.70*** | -50.66*** | -56.78*** | -39.36*** |
| | (1.70) | (3.41) | (1.91) | (5.38) | (1.89) | (1.89) | (5.02) | (3.13) | (3.87) | (4.44) |
| Capital (t)$^2$ | 18.33*** | 15.81*** | 17.07*** | 23.76*** | 11.89*** | 18.09*** | 17.40*** | 21.84*** | 25.73*** | 15.26*** |
| | (0.99) | (1.86) | (1.17) | (2.82) | (1.10) | (1.18) | (3.21) | (1.67) | (2.31) | (2.90) |
| Cash (t) | 10.35*** | 9.87*** | 11.95*** | 20.47*** | 8.25*** | 9.64*** | 16.24*** | 9.29*** | 7.53*** | 8.99*** |
| | (0.63) | (0.85) | (0.68) | (3.32) | (0.95) | (0.88) | (2.38) | (1.67) | (2.42) | (2.49) |
| Constant | 157.40*** | 140.29*** | 171.57*** | 248.28*** | 135.16*** | 144.45*** | 173.55*** | 169.66*** | 155.89*** | 146.87*** |
| | (0.05) | (0.78) | (0.53) | (0.25) | (0.12) | (0.09) | (0.31) | (0.20) | (0.20) | (0.26) |
| Specification | All | Yr>'95 | Yr≤'95 | SIC1=1 | SIC1=2 | SIC1=3 | SIC1=4 | SIC1=5 | SIC1=7 | SIC1=8 |
| Firm FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| SIC-2×Year FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| % Adjusted R$^2$ | 51.78 | 58.05 | 50.51 | 47.49 | 44.20 | 46.59 | 49.23 | 53.86 | 52.32 | 58.95 |
| % Within R$^2$ | 9.36 | 9.93 | 6.88 | 6.42 | 7.26 | 11.96 | 8.22 | 13.44 | 13.06 | 6.93 |
| Observations | 109,711 | 50,292 | 58,810 | 8,188 | 23,114 | 37,986 | 8,229 | 15,529 | 11,726 | 4,502 |

We provide evidence of diminishing returns to scale in our sample by examining how profitability varies with capital stock. Table D.7 shows that a firm's return on assets and return on equity decline with capital. Note that the specification includes firm and industry-by-year fixed effects.

Table D.7: Decreasing returns to scale
This table provides evidence of decreasing returns to scale. The outcome variable in column (1) is the return on assets (EBITDA/book value of assets), measured in percentage points. The outcome variable in column (2) is the return on equity (net income/book value of equity). All explanatory variables are standardized to facilitate comparisons. Each specification includes firm fixed effects and industry trends. Standard errors are double-clustered by stock and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| | Return on Assets (%) | Return on Equity (%) |
|---|---|---|
| | (1) | (2) |
| Capital (t) | -6.46*** | -1.93*** |
| | (0.66) | (0.25) |
| Cash (t) | 5.32*** | 1.33*** |
| | (0.38) | (0.14) |
| Constant | 6.04*** | 13.16*** |
| | (0.01) | (0.00) |
| Firm FE | Yes | Yes |
| SIC-4 x Year FE | Yes | Yes |
| % Adjusted $R^2$ | 15.25 | 61.46 |
| % Within $R^2$ | 0.48 | 0.88 |
| Observations | 93,916 | 93,840 |