Process Intangibles and Agency Conflicts*

Hui Chen — MIT & NBER
Ali Kakhbod — UC Berkeley
Maziar Kazemi — ASU
Hao Xing — Boston University

April 4, 2024

Abstract

We study how process-focused intangible capital affects compensation and investment at the firm level. We document three connected novel facts. First, executive and skilled labor pay are increasing in process intangibility. Second, the pay-process intangibility association is stronger amongst high physical investment firms. Third, this strengthening is due to the fact that process intangibles and physical investment are complements in improving physical capital efficiency of the firm. We rationalize these connections in a dynamic agency model that ties the optimal contract to process intangibility and investment.

Keywords: Process intangibles, intangible capital, dynamic contracting, investment, compensation

JEL Classification: D21, E22, G31, G32, L22, O31, O34.

*First draft November 2022. We are grateful to Daron Acemoglu, Jan Bena, Gilles Chemla, Nicolas Crouzet, Janice Eberly, Andrea Eisfeldt, Barney Hartman-Glaser, Stefan Hirth, Hossein Kazemi, Amir Kermani, Dmitry Livdan, Deborah Lucas, Dimitris Papanikolaou, Seth Pruitt, Alejandro Rivera, Neng Wang, Colin Ward, Mindy Xiaolan, and seminar and conference participants at UC Berkeley, UC Irvine, the 2022 ASU-UA Junior Conference, the 12th Tepper-LAEF Advances in Macro-Finance, the 8th SDU Finance Workshop, the NBER Summer Institute Capital Markets and the Economy Session, the Summer Institute of Finance (SIF), the UBC Summer Finance Conference, and the AFA Annual Meeting for very helpful comments. We thank Bledi Taska from Lightcast (formerly Burning Glass Technologies) for data.

Hui Chen is with the Sloan School of Management at MIT and NBER. Ali Kakhbod is with the Haas School of Business at UC Berkeley. Maziar Kazemi is with the W.P. Carey School of Business at Arizona State University. Hao Xing is with the Questrom School of Business at Boston University. Emails: Hui Chen (huichen@mit.edu), Ali Kakhbod (akakhbod@berkeley.edu), Maziar Kazemi (maziar.kazemi@asu.edu), and Hao Xing (haoxing@bu.edu).

1 Introduction

Firms innovate in many different ways. The OECD's Oslo Manual (OECD (2005)) defines *process* and *organization innovation* as

> *...implementation of a new or significantly improved production or a new organizational method in the firm's business practices, workplace organization or external relations...*

and defines *product innovation* as

> *...the introduction of a good or service that is new or significantly improved with respect to its characteristics or intended uses.*

It is well known that the importance of innovation for corporate investment and economic development has grown significantly since the 1970s (for example, Belo et al., 2022, show that the total amount of intangible capital has increased from 25% of firm value in the 1970s to 45% in the 2010s). Less known is the fact that the growth in intangible capital is increasingly driven by non-product-related innovations, which we call "process innovation" (the share of process innovations has steadily increased from 26% in 1975 to 37% in 2010; see Bena and Simintzi, 2019). In addition, over roughly the same period, executive pay, particularly the fraction of deferred incentive pay, has also skyrocketed.[1] Given these aggregate trends and the intertwined relationship between intangibles and human capital (Eisfeldt and Papanikolaou (2013)), we ask in this paper "Does *heterogeneity* in intangible capital affect the compensation of executives and skilled employees?"[2]

Our set-up is motivated by a novel empirical stylized fact: higher process intangibility is associated with more physical capital investment.[3] Table 1, Panel A shows this relationship. We assign firms to one of three process intangibility bins each year.[4] Inde-

[1]Figure 5 in Edmans, Gabaix and Jenter (2017) shows that the average fraction of CEO pay that is deferred (stocks and option grants) was about 25% in the 1980s and rose to 60% by the early 2000s.

[2]Bena and Simintzi (2019) identify process focused patents among all patents. We call all other patents non-process focused patents or product focused patents. Throughout the paper, we call non-process focused intangibles product intangibles and use the terms "product (focused) intangible" and "non-process (focused) intangible" interchangeably.

[3]Intangibility refers to intangible capital divided by total capital (physical plus intangible capital). Liu, Sojli and Tham (2022) also normalize variables by total assets, as opposed to just physical capital, for example. Process intensity (Bena and Simintzi (2019) is defined as the number of process focused patent claims divided by total claims across all patents applied for by a firm in a given year. Process intangibility is intangibility times process intensity.

[4]Low, medium, and high are defined as the bottom 30%, middle 40%, and top 30% of firms, respectively. The bins are re-balanced every year.

pendently, we assign firms to product intangibility bins, as well.[5] The table shows that for a given level of product intangibility, increases in process intangibility are associated with increases in physical investment the following year.[6] The pattern is also strong for process intangibility compared to non-process (product) intangibility. There appears to be a distinct connection between process intangibles and physical investment.

Table 1, Panel B shows a similar pattern arises for executive compensation.[7] That is, compensation is strongly related to the process intangibility of the firm. In this paper, we argue that the patterns in the two panels are related. The compensation-process intangibility relationship arises because of the physical investment-process intangibility relationship. We rationalize the connection with a dynamic principal-agent model in which process intangibles are exposed to agency frictions and verify the model's predictions in the data.[8]

Table 1: Process versus Non-Process Intangibility: Investment and Executive Compensation

Panel A: Investment

	Product		
Process	Low	Med	High
Low	11.43	11.18	17.35
Med	12.44	15.21	21.70
High	22.27	20.94	30.74

Panel B: Compensation

	Product		
Process	Low	Med	High
Low	0.65	0.78	1.36
Med	0.57	0.88	1.47
High	2.19	1.37	1.97

This table shows the relationship between physical investment per unit physical capital (Panel A) and compensation per unit total capital (B) with process versus non-process intangibility. Each year, we sort firms into high, medium, or low process intangibility bins (bottom 30%, middle 40%, top 30%). We similarly sort firms into total intangibility bins. Process intangibility is defined as the share of patent claims that are process-focused times intangible capital divided by total capital. Non-process intangibility is the difference between total intangibility and process intangibility. All numbers are multiplied by 100 to be in percent units.

The key feature of our model is the idea that physical investment and process intangibles are complementary in efficiency-adjusted physical capital accumulation. Efficient

[5] Product intangibility is total intangibility minus process intangibility.

[6] In our theoretical model, process intangibility is a state variable and investment is a control, so it makes sense to look at investment a year in advance. The relationship is very similar using contemporaneous investment.

[7] In this table, executive compensation is total compensation divided by total capital.

[8] Our interpretation of process-focused intangibles extends beyond the one proposed in the model. For example, a firm that streamlines its data handling procedures or optimizes its supply chain could be engaged in process-focused innovations. Reorganizing the management structure or improving company culture might also fit in this paradigm. To that end, the process vs. product intensity question is related to Frésard et al. (2023).

capital goods are an important source of productivity change (see e.g., Greenwood, Hercowitz and Krusell (1997), Jorgenson (2005)). We think of physical capital efficiency as the firm-specific productivity of physical capital in firm output, which is improved by the firm's process intangibles (i.e., the firm with more process intangibles gets more "bang for the buck" per unit of physical capital investment).[9] This modeling choice is consistent with the existing literature. Parisi, Schiantarelli and Sembenelli (2006) and Bena, Ortiz-Molina and Simintzi (2022) both find that process improvements are associated with more capital investment.[10] In Section 5, we provide supporting evidence for our key model feature using a proxy of physical capital efficiency.

To bring in human capital, and, hence, compensation, we assume that process intangibles must be combined with human capital effort to reach their full potential. However, intangibles are in general opaque (e.g., Lev (2000)), and process intangibles, being "internal to the firm", are especially hard for firm outsiders to monitor. The necessity of an agent's effort combined with opaqueness generates a moral hazard problem tied to the implementation of process innovation. Without the agent's effort, the firm's process intangibles do not improve its capital efficiency. This creates a "hold-up" problem through which the agent can extract rent.

We solve for the optimal contract that induces the agent to provide effort and find that there are two channels through which process intangibility and compensation are linked. We call these the *direct* and *indirect effects*. The direct effect can be considered a *level effect*: Holding other variables and parameters fixed, as the process intangibility of the firm increases, so does the promised utility to the agent. This effect arises because the agent's benefit from shirking increases as process intangibility increases, ceteris paribus. Therefore, the owners of the firm must promise the agent more utility to induce them to provide effort.

The indirect effect is a novel effect that is distinct from the expected set of outcomes from an optimal contract. It is akin to a *slope effect*: The process intangibility-compensation association becomes stronger as the firm undertakes more physical investment.[11] As process intangibles become more important to efficiency-adjusted physical capital growth, the agent can extract more rents from the firm. This is most easily seen

[9] When one applies the perpetual inventory method to Compustat physical investment, one finds a large gap between reported and "accumulated" physical capital. Bai et al. (2022) show that this gap can be partly explained through accounting differences and more accurate measures. This further motivates us to model physical capital efficiency rather than physical capital quantities in our model.

[10] Pan and Li (2016) model process innovations as cost reducing. They also state, "[Process innovations] may involve investment in new technology embodied in machinery and equipment..."

[11] Also, the slope effect itself is mediated by the level of complementarity between physical investment and process intangibles.

in the extreme cases. When physical investment and process intangibles are perfect substitutes, any rent extraction by the agent can be perfectly offset by an equivalent increase in physical capital investment. The level of efficiency-adjusted physical capital growth is affected, but the marginal product of investment is not. In the other extreme case, process intangibles and physical investment are perfect complements. In this case, the agent must be induced to provide effort; otherwise, all physical investment is wasted: The agent can block the firm from growing until he is compensated enough. In reality, most firms are somewhere between these two extremes.

Process intangibility is equal to intangibility (intangible capital divided by total capital) times process intensity, which we measure using the data from Bena and Simintzi (2019). They scrape the text of filed patents, looking for phrases like "a process for..." or "a product for..." to determine the type of patent. We focus on the Bena and Simintzi (2019) data and method because it is straightforward and publicly available.

The two key parameters in the model are process intensity and the complementarity of physical investment and process intangibility. The former is chosen according to the average process intensity in the data. Other parameters, including the complementarity parameter, are calibrated to the average physical investment rates and compensation-physical capital ratios across intangible-to-physical capital quintiles.[12] The calibrated model produces the flat physical investment rate and the more convex compensation to physical capital ratio. After splitting the data into low and high process intensity subsamples, the model also matches well the conditional moments of the physical investment rate and compensation in both subsamples. Importantly, as stated above, our key parameters are not calibrated to these subpopulations, making these splits a type of predictive exercise.

We start our empirical analysis by verifying our key model assumption: Physical investment and process intangibles are complements in increasing the growth rate of efficiency-adjusted capital.[13] We use two different measures of efficiency. Following Liu, Sojli and Tham (2022), we study the association between the productivity measures created by İmrohoroğlu and Tüzel (2014) and process intangibility. These productivity measures are inferred using the methods of Olley and Pakes (1996). They are widely used in the finance literature. Second, because İmrohoroğlu and Tüzel (2014) do not use intangible capital in the model upon which their productivity measures are based, we residualize their original productivity measures with respect to intangibles and their

[12]A final set of parameters are taken from the existing literature.

[13]The model can be written in per-capital form, so the relevant empirical test is actually to show that process intangibility increases efficiency growth. This is what we show.

lags. For both measures we find that, first, process intangibility (intensity) forecasts productivity growth, and, in particular, there is a stronger relationship between process intangibility and productivity growth than non-process intangibility and productivity growth. Second, the ability of these process measures to forecast growth increases with the level of physical investment. Once again, the increase in forecastibility is greater for process measures than non-process measures. This empirical evidence supports our key model assumption of complementarity between physical investment and process intangibles.

In our empirical analysis, we measure our main outcome variable, compensation, in two different ways. First, we use executive compensation, both total and deferred, from Execucomp. The argument for using this data is that executives are the most powerful people in a firm and are best positioned to extract rents. However, it is not clear that executive effort actually matters for process intangibles to be effective.[14] Our second measure overcomes that issue. We gather wage data on vacancy postings from Burning Glass Technologies (BGT).[15] BGT is a firm whose competitive advantage is its unique vacancy posting data. The main benefit of this data is that BGT provides a large and standardized set of skills associated with each vacancy posting. Therefore, we can look at the posted wages for workers with skills specific to innovation, process improvement, and research and development (R&D).[16]

We verify the direct and indirect effects identified in the model. We find that a one standard deviation increase in the process intangibility of the firm is associated with an increase in total executive compensation of 0.46% of total capital, a 0.15% of total capital increase in deferred compensation, and a 3% of total compensation increase in deferred compensation. The wage data from BGT is not a total flow, so it must be normalized differently. We normalize with respect to the wage of job postings requiring similar skills that year. A one standard deviation increase in process intangibility is associated with a 10% increase in this relative skilled wage.[17]

The indirect effect, due to complementarity, implies that all of the above effects should be stronger amongst firms undertaking more physical investment. Indeed, we

[14]For example, when Nissan had a break-through in its car production methods (Link), its CEO was embroiled in a serious legal scandal. Exerting effort over process innovations was surely the last thing on his mind.

[15]BGT has since merged with Lightcast, and the merged entity uses the Lightcast name.

[16]Eisfeldt, Falato and Xiaolan (2021) show that skilled labor is increasingly being paid via equity compensation. We focus on wages due to data availability, though our theory also speaks to deferred compensation of skilled labor.

[17]In the Internet Appendix IA.2, we also estimate executive-year level (as opposed to firm-year level) regressions amongst the set of executives who switch firms at least once. Our main results are confirmed.

find evidence for this. Across our main specifications, the point estimate of the coefficient linking process intensity and compensation tends to increase as one moves from low to high physical investment firms. Further, these increases are much stronger for process intangibility as compared to non-process intangibility.

Finally, we consider the policy implications of our model by demonstrating that imposing blanket compensation restrictions on executives and skilled labor can have unintended adverse effects, particularly by depressing physical investment and reducing firm value, especially among high-intangibility firms. We argue for a more nuanced approach that differentiates reasons for high compensation and allows firms the flexibility to incentivize key employees involved in process innovations.

We make four main contributions. First, we present a new finding that heterogeneity in the uses of intangibles is associated with heterogeneity in pay. In particular, higher process intangibility (intensity) is associated with higher pay. Second, we establish new stylized facts with respect to process intangibility and firm characteristics. Namely, high process intangibility is associated with more physical capital investment, via complementarity with process intangibles. This complementarity manifests through more efficiency-adjusted capital growth. Third, we develop a dynamic principal-agent model with heterogeneity in intangibles that can rationalize the empirical phenomena. Importantly, the model shows that there is a direct and indirect effect of process intensity on compensation: The level effect comes from variation in the shirking benefit, and the slope effect comes from variation in the complementarity between process intangibles and physical capital investment. The indirect effect depends on complementarity and is novel to our setup. That is why other dynamic contracting models do not decompose into these two effects. Fourth, we show that both the direct and indirect effects exist in the data, not only for executives but also for the skilled workers whose effort determines the efficacy of process intangibles.

This paper sits at the intersection of three different literatures. First, we contribute to the literature on dynamic agency theory (e.g., DeMarzo and Sannikov (2006), Biais et al. (2007), DeMarzo and Fishman (2007*a*,*b*), Sannikov (2008), DeMarzo et al. (2012)). Our model extends this framework to include heterogeneous forms of intangible capital. This adds new testable predictions (the core of our paper) and a new state variable. More closely related, Ward (2022) and Chen et al. (2023) study the role of agency frictions on intangibles, but do not distinguish between different types of intangibles.[18] In agency models of firms, efficiency-adjusted physical capital is typically used, see e.g.

[18] Grabner (2014) studies the empirical relationship between "creativity-dependent" firms and incentive pay.

Brunnermeier and Sannikov (2014) and Ai, Kiku and Li (2023). This modelling assumption improves model tractability. However, capital efficiency is not directly observable empirically. We empirically and theoretically distinguish between observable capital and its efficiency when testing and calibrating the model.

Second, we contribute to the literature connecting intangible capital and finance (e.g., Lev and Radhakrishnan (2005), Eisfeldt and Papanikolaou (2013), Kung and Schmid (2015), Peters and Taylor (2017), Crouzet and Eberly (2018), Ewens, Peters and Wang (2019), Eisfeldt, Falato and Xiaolan (2021), Crouzet et al. (2022)). These papers do not study agency conflicts, nor do they seek to quantify heterogeneous intangible capital.[19] We fit within the subset of this literature that looks at the relationship between pay and innovation/intangibles (Lerner and Wulf (2007), Lustig, Syverson and Van Nieuwerburgh (2011), Kline et al. (2019), Song et al. (2019), Sun and Xiaolan (2019), Kogan et al. (2020), Bhandari and McGrattan (2021)). These papers also do not look at agency conflicts or the heterogeneous nature of intangibles.

Third, we contribute to the small and growing literature on process versus product innovation and finance. Our measure of process innovation intensity is derived from the patent data of Bena and Simintzi (2019). Ganglmair, Robinson and Seeligson (2022) provide a survey of the empirical evidence on process claims over time and provide their measure of process intensity, and Liu, Sojli and Tham (2022) provide an international analysis of process vs product innovations.[20] To the best of our knowledge, we are the first to explicitly tie a formal model of the firm (with or without agency) to the empirical data on process versus product innovation.[21]

2 Stylized Facts

This section presents the two key empirical stylized facts that we want our model in the next section to replicate. These facts also serve as a summary of our main results on compensation. We rely on simple double sorts in this section and defer the more detailed empirical work to Section 7.[22]

[19] We use the methods of Ewens, Peters and Wang (2019) and Peters and Taylor (2017) to create our firm-level measure of intangible capital and investment.

[20] Angenendt (2018) also estimates process intensity.

[21] Mohnen and Hall (2013) provide an overview of the empirical evidence linking firm outcomes to process and product innovation.

[22] This section focuses on total executive compensation, one of our three compensation and salary measures. We leave the other two, deferred compensation and skilled labor salaries, to Section 7.

Our key variables throughout the paper are process intensity:

$$\text{Process Intensity}_{ft} = \frac{\text{Process focused patent claims filed by firm } f \text{ in year } t}{\text{Total patent claims filed by firm } f \text{ in year } t},$$

and process intangibility:[23]

$$\text{Process Intangibility}_{ft} = \text{Process Intensity}_{ft} \times \frac{\text{Intangible Capital}_{ft}}{\text{Intangible Capital}_{ft} + \text{Physical Capital}_{ft}}.$$

The data to construct process intensity comes from Bena and Simintzi (2019), and our intangibility measure comes from Compustat using the Peters and Taylor (2017) method. As explained in Internet Appendix IA.1, we assume the process intensity at the firm's patent filing level scales to the level of all intangible capital in the firm.[24] Also, we refer to the sum of intangible and physical capital as "total capital."[25]

The facts we present are the following: First, firms with higher process intensity provide higher compensation for their executives. Second, the association between process intensity and compensation increases with the amount of physical capital investment.

Table 1 Panel B (in the Introduction) displays the first stylized fact. To construct this figure, we independently sort firms into three bins based on their process intangibility and three bins based on their non-process intangibility.[26] We see two effects here. In general, fixing one type of intangibility and moving from low to high for the other type increases compensation.[27] However, this increase is considerably steeper when we hold non-process intangibility fixed and increase process intangibility, rather than vice versa. This novel fact suggests that not only does the level of intangibles matter, but that the composition of intangibles matters, too. This empirical fact has not been documented in the data, nor have existing models explained it.

To illustrate the economic importance of our channel, we use Figure 1 to compare high and low process intense firms. In particular, we focus on the subset of firms that had 90% process intensity or higher (high process) or 10% or lower (high product). We split the sample into intangibility quintiles and look at the average compensation per

[23] Associated with process intangibility is non-process (or product) intangibility. This is total intangibility minus process intangibility.

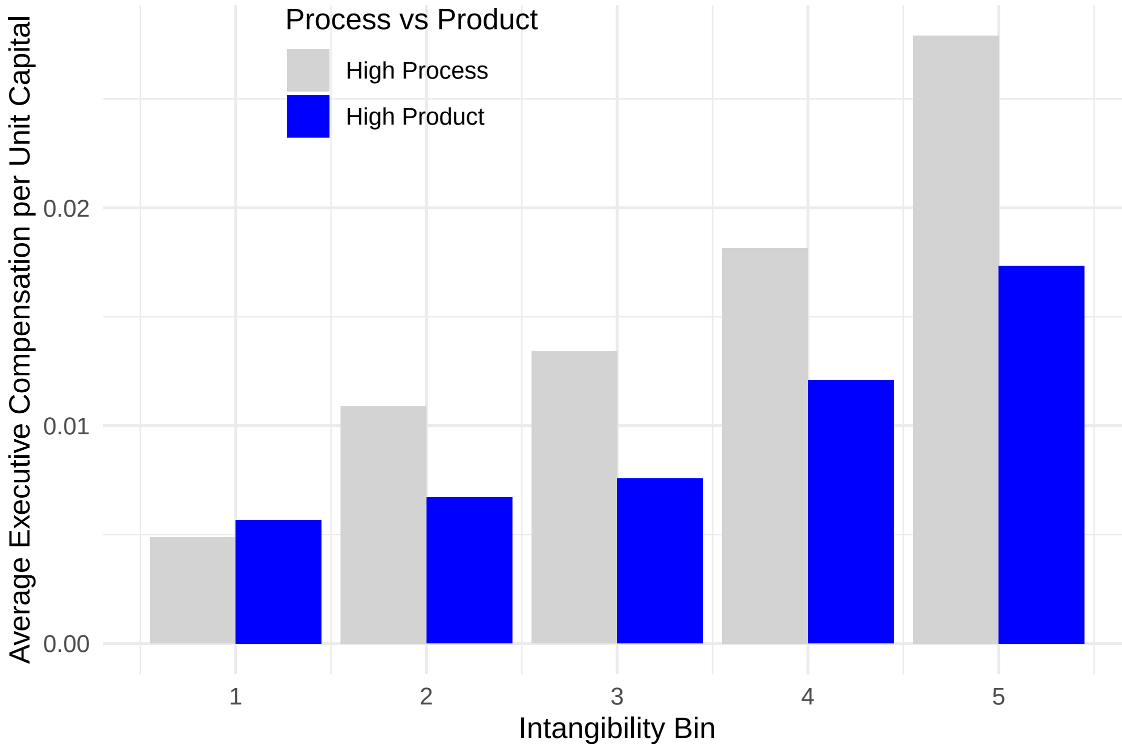
[24] We also provide more details on the Bena and Simintzi (2019) data and on how we construct our measure of process intensity in Internet Appendix IA.1.

[25] This is consistent with our model's definition of the firm's total capital stock.

[26] The bins are rebalanced every year.

[27] This general intangibility effect is consistent with Ward (2022).

Figure 1: High Process vs High Product Firms and Compensation



This figure shows the average executive compensation per unit capital for high process versus high product intense firms within intangibility quintiles. High process intense firms have a process intensity greater than or equal to 90%, while high product intense firms have a process intensity less than or equal to 10%. The intangibility quintiles are rebalanced every year. The average compensation per unit capital is computed by first taking the cross-sectional median within each quintile-date (for high product and high process firms separately) and then taking the time-series average of these medians.

unit total capital for executives at these high process or high product firms.[28] Looking at the top bin, we can do a back-of-the-envelope calculation to show how economically meaningful this process vs product distinction is. Using the average total capital in the high process and high product firms, the average difference in compensation is \$4.3 million.[29] Assuming 5 executives per firm, that gives a per-person difference of \$900,000 to \$1 million.

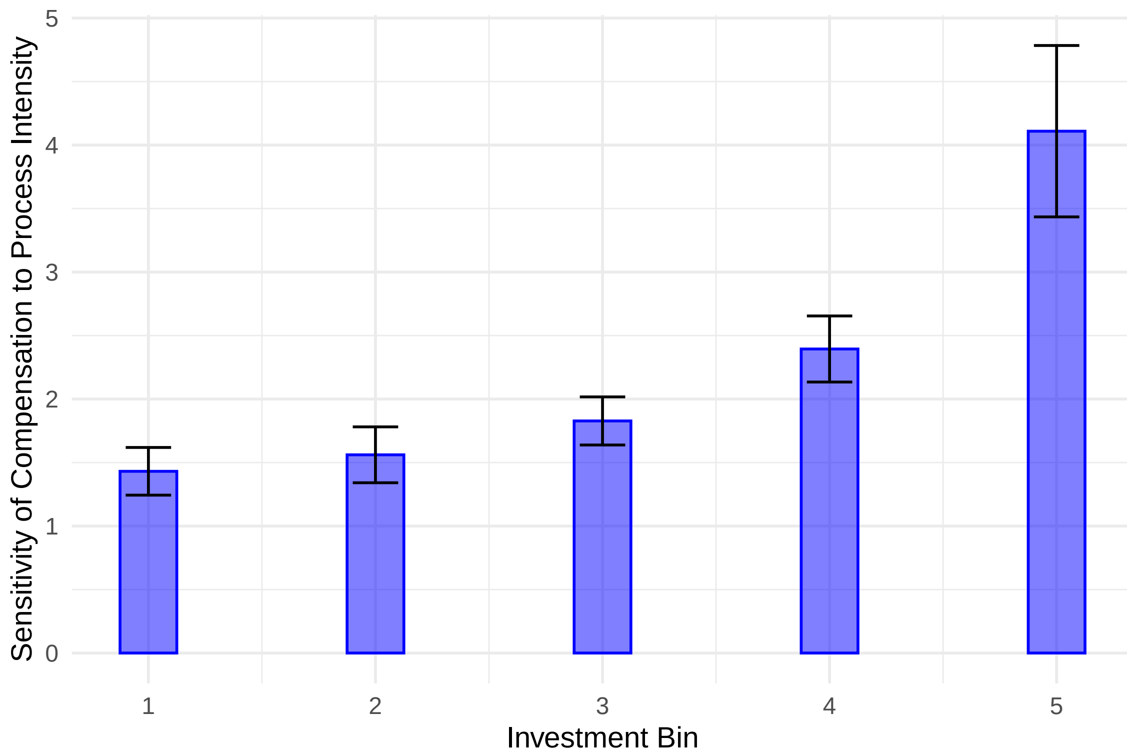
Figure 2 displays our second stylized fact. This figure shows the sensitivity of executive compensation to a one-unit increase in process intensity (i.e., going from 0 to full process intensity) within physical capital investment bins. 95% confidence intervals are also displayed.

[28]Each quintile-date-process intensity bin has anywhere from 4-40 observations. Thus, we caution that this is merely an illustrative exercise. The qualitative aspects of the figure are not sensitive to the choice of 90/10.

[29]Note that this is for a set of executives, not just the CEO.

The key takeaway here is that the sensitivity increases as physical investment increases. This captures our idea of the hold-up problem inherent in process intangibles. Firms undertaking more physical investment are more "dependent" on the efforts of the agents to fully realize the benefits of the investment. The agents can thus extract rents from the firm. This effect is increasing with physical investment. This positive relationship is predicated on the assumption that physical investment and process intangibles are complements. This is our key model assumption, and we find empirical evidence for it in Section 5 later.

Figure 2: Sensitivity of Executive Compensation to Process Intensity by Physical Investment Bin



This figure shows the univariate regression coefficient on process intensity when total compensation over total capital is the dependent variable. The investment bins are based on the physical investment to physical capital ratio of firms. They are rebalanced every year. 95% confidence intervals are displayed using firm level clustering. The vertical axis has been multiplied by 100 so that it can be interpreted as a percent.

3 Model setting

3.1 Capital, investment, and agency

Capital and investment The firm produces output using both physical and intangible capital, whose stock values at time t are K_t and O_t, respectively. The firm determines its investment, I_t, in physical capital and its investment, S_t, in intangible capital.The physical capital, K_t, and the intangible capital, O_t, evolve according to

$$\mathrm{d}K_t = (I_t - \delta_K K_t)\mathrm{d}t, \tag{3.1}$$

$$\mathrm{d}O_t = (S_t - \delta_O O_t)\mathrm{d}t, \tag{3.2}$$

where δ_K and δ_O are the deprecation rate of the physical and intangible capital, respectively.

Technological innovations impact the productivity and efficiency of the physical capital. Greenwood, Hercowitz and Krusell (1997) document that the introduction of new, more efficient capital goods is an important source of productivity change. We denote the efficiency-adjusted physical capital stock by $\widehat{K}$, whose dynamics is driven by process intangibles:

$$\mathrm{d}\widehat{K}_t = \left(D(e_t, I_t, \theta O_t) - \delta_K \widehat{K}_t\right)\ \mathrm{d}t + \sigma \widehat{K}_t \mathrm{d}Z_t^e. \tag{3.3}$$

Efficiency shocks are modeled by increments of a Brownian motion Z^e and σ represents the volatility of physical capital efficiency.[30] Accumulation of efficiency-adjusted physical capital depends on both investment, I_t, and process intangibles, θO_t, which is a fraction, θ, of total intangibles. The production function D takes a CES form:[31]

$$D(e_t, I_t, \theta O_t) = \frac{A}{a^{1/\rho}} \left[a\, I_t^{\rho} + e_t(1-a)\left(\theta O_t\right)^{\rho}\right]^{1/\rho}. \tag{3.4}$$

In this production function, the agent's effort e_t is either 0 or 1. When $e_t = 1$, the agent exerts full effort and works efficiently; when $e_t = 0$, the agent shirks his effort. When $e_t = 1$, process intangibles and physical investment jointly expand the efficiency-adjusted physical capital. The level of $\widehat{K}$ increases with process intangibles θO_t for a

[30] The process Z^e is a Brownian motion under $\mathbb{P}^e$ which is the probability induced by agent's effort e.

[31] The function D can be rewritten as $D(e_t, I_t, \theta O_t) = A\left[I_t^{\rho} + e_t \frac{1-a}{a}(\theta O_t)^{\rho}\right]^{1/\rho}$. Therefore the function depends on a and θ via the product $\frac{1-a}{a}\theta^{\rho}$. The parameter θ is a firm characteristic determined by empirically observed firm specific process intensity. Meanwhile, the parameter a is calibrated to match empirical moments, and it is a constant across firms. We refrain from using a Cobb-Douglas form for the function D, because agent's shirking action nullifies all physical investment in that specification.

given level of physical investment I_t. The parameter $a \in [0,1]$ represents the weight between physical investment and process intangibles. The CES parameter ρ measures the complementarity between I_t and θO_t. The lower ρ is, the stronger complementarity between the two components is. The factor $a^{-1/\rho}$ in front of the CES function is a normalization factor, which ensures that, without process intangibles or when the agent shirks his effort ($e_t = 0$), the production function takes the standard form $D(e_t, I_t, 0) = AI_t$.

The physical capital stock $\widehat{K}_t$ and K_t measure the physical capital in different units: efficiency units in the former and physical units in the latter. Their ratio $X_t = \widehat{K}_t / K_t$ measures the physical capital efficiency per physical unit. Introduce the physical investment rate $i_t = I_t / K_t$ and intangibility $o_t = O_t / K_t$ as the physical investment and intangible capital, both normalized by physical capital.[32] The dynamics of X_t are derived by combining (3.1) and (3.3):

$$\mathrm{d}X_t = \Big(d(e_t, i_t, \theta o_t) - i_t X_t \Big)\, \mathrm{d}t + \sigma X_t \mathrm{d}Z_t^e, \tag{3.5}$$

where

$$d(e_t, i_t, \theta o_t) = \frac{A}{a^{1/\rho}} \big(a\, i_t^\rho + e(1-a)(\theta o_t)^\rho \big)^{1/\rho}. \tag{3.6}$$

When the agent exerts full effort ($e = 1$), three properties of the (normalized) production function, d, are important for our results: (i) $d(1, i, \theta o)$ increases with the process intangibility θo; (ii) When $\rho < 1$, $\frac{\partial d}{\partial(\theta o)}$ increases with i; (iii) When $\rho_1 < \rho_2$, d_{ρ_1} and d_{ρ_2} represent the (normalized) production with complementarity coefficient ρ_1 and ρ_2 respectively, we have

$$\frac{\partial d_{\rho_1}}{\partial(\theta o)} > \frac{\partial d_{\rho_2}}{\partial(\theta o)} > 0, \tag{3.7}$$

for fixed i and o. Property (i) shows that process intangibility fosters capital efficiency growth. Property (ii) implies that the physical investment and process intangibility complement, so that the marginal value of process intangibility increases when the firm invests more in its physical capital. Meanwhile, (3.7) indicates that this complementarity increases as ρ decreases.

The function d increases with i_t. The capital stock K_t also grows with i_t, introducing $-i_t X_t$ in the expected growth of X_t. Therefore, equation (3.5) shows that X_t exhibits mean-reverting dynamics: the expected growth of X_t is positive when X_t is small so that

[32] In our empirical analysis, we use ratio of intangible capital to total capital as our measure of intangibility. This measure is a monotonic transformation of our model definition of intangibility and does not explode when K_t is small but O_t is big.

$d(e_t, i_t, \theta o_t) > i_t X_t$; otherwise, the expected growth of X_t is negative when X_t is large.

We choose to model the efficiency-adjusted physical capital directly in our model for tractability (see e.g., Brunnermeier and Sannikov (2014)).[33] The model will be solved using $\widehat{K}$ and O. However, because $\widehat{K}$ is not directly observable empirically, we will use the model derived dynamics of K and O, when we match the model to data.

Cash flow Firm produces output by aggregating physical and total intangible capital (including both process and product intangibles) via the second CES production function $\mu\left[(1-\phi)\widehat{K}_t^{\psi} + \phi O_t^{\psi}\right]^{1/\psi}$ with productivity rate μ. Physical and intangible investments are subject to convex adjustment costs $C_K(I_t)$ and $C_O(S_t)$, respectively. The instantaneous cash flow produced by the firm at time t is

$$Y_t = \mu\left[(1-\phi)\widehat{K}_t^{\psi} + \phi O_t^{\psi}\right]^{1/\psi} - I_t - S_t - C_K(I_t) - C_O(S_t), \tag{3.8}$$

which is output net of investment and adjustment costs but before payments to the agent. The adjustment costs for physical and intangible investment are assumed to be homogenous of degree one and quadratic with respect to their respective deprecation rates, so that expanding and shrinking physical and intangible capital with respect to their steady states incur adjustment costs:

$$C_K(I_t) = \tfrac{Q_K}{2}\left(I_t/\widehat{K}_t - \delta_K\right)^2 \widehat{K}_t \quad \text{and} \quad C_O(I_t) = \tfrac{Q_O}{2}\left(S_t/O_t - \delta_O\right)^2 O_t, \tag{3.9}$$

where constants Q_K and Q_O measuring the magnitude of the adjustment cost.

Agency Agent's effort e_t impacts the capital efficiency growth. When $e_t = 1$, the agent exerts full effort and the physical capital efficiency increases at a rate of $d(1, i_t, \theta o_t)$. When $e_t = 0$, the agent shirks his effort and the (normalized) production function d is reduced to $d(0, i_t, \theta o_t) = A i_t$, which is independent of process intangibles. The dependence of d on the agent's effort models the agency friction on process innovations. Define

$$\Lambda(I_t, \theta O_t) = K_t\big(d(1, i_t, \theta o_t) - d(0, i_t, \theta o_t)\big) = D(1, I_t, O_t) - D(0, I_t, O_t). \tag{3.10}$$

The function Λ measures the increment of the firm's efficiency-adjusted physical capital accumulation due to the agent's effort, conditional on the physical capital investment and process intangibles. Therefore, Λ measures how much stake the agent controls in

[33] If we were to model the capital efficiency X directly, the model becomes three dimensional.

the process innovations.

The three properties of the production function d directly translate to the same properties of the function Λ. (i) $\Lambda(I_t, \theta O_t)$ increases with θO_t, indicating a more important role for agent effort in the physical capital accumulation, when a firm possesses more process intangibles. Complementarity between physical capital investment and process intangibles in (ii) and (iii) introduces a "hold up" problem for the physical investment from the agent's effort, and this problem is more severe when the firm invests more or the complementarity is stronger.

When the agent shirks, he enjoys a flow of private benefits, which is assumed to be $\lambda_t \Lambda_t$ with $\Lambda_t = \Lambda(I_t, \theta O_t)$.[34] Conditional on the agent's impact on the physical capital accumulation, λ_t measures the magnitude of agent's private benefit from shirking. We will show later that λ_t is also the ratio between compensation volatility and physical efficiency volatility at time t, and this ratio increases in the firm's level of intangibility in our data. Therefore, we will assume later that λ_t increases with firm's intangibility.

We assume that the firm's owner (principal) only observes the dynamics of O_t, K_t, and $\widehat{K}_t$, but cannot observe the agent's effort e_t due to the random shocks in Z^e. This introduces agency frictions in process innovation.

3.2 Discussion of model assumptions

Of the total intangible capital O_t, θO_t is used in process innovation. Therefore, we call θ the firm's process intensity. Our key modeling assumption is the complementarity between physical investment and process intangibles in (3.4). We will present empirical evidence for this assumption in Section 5. This complementarity between physical investment and process intangibles is also consistent with the literature. Parisi, Schiantarelli and Sembenelli (2006) find that process innovations are more strongly associated with capital investment than product innovations. Bena, Ortiz-Molina and Simintzi (2022) find that when firms face an increase in labor dismissal cost, firms increases their process innovation to facilitate physical capital investment and adjust toward higher capital-labor ratios. They also find this adjustment is stronger among firms with high innovation ability than those with low innovation ability.

We do not include the product focused intangibles in the physical capital accumulation. This is because Section 5 documents that the association between product in-

[34] He (2009) also assumes that the agent's private benefit depends on the investment rate.

tangibles and physical investment is weaker.[35] We do not model an agency friction in intangible capital accumulation. This is because we aim to focus on process intangibles and their impact on the physical capital accumulation.[36]

Next, we present a contracting problem between the owner of our model firm (principal) and an executive or a skilled employee (agent) with expertise in process innovation.

3.3 Contracting problem

The principal offers a contract with a cumulative compensation of $C = (C_t)_{t \geqslant 0}$ to the agent. The cumulative compensation C is a non-decreasing process, because the agent does not subsidize the firm by accepting negative compensation. For a given compensation plan C, the agent's continuation utility U is

$$U_t = \max_{e \in \{0,1\}} \mathbb{E}_t^e \Big[\int_t^\tau e^{-\gamma(s-t)} \big[dC_s + (1 - e_s)\lambda_s \Lambda_s \mathrm{d}s \big] \Big]. \tag{3.11}$$

The expectation is taken with respect to a probability $\mathbb{P}^e$, induced by the agent's effort e. The Brownian motion Z^e in (3.3) is under the measure $\mathbb{P}^e$. The agent is assumed to be risk neutral, discounting future compensation and potential private shirking benefits using a subjective discount rate of γ. When the agent's continuation utility decreases to the outside value, normalized to zero, at an endogenously determined stopping time τ, the contract is terminated. Then, the agent leaves the firm, production continues, but the efficiency-adjusted physical capital accumulation decrease, with its expected value reduced to $D(0, I_t, O_t) = AI_t$, i.e., complementarity between physical investment and process intangibles is lost, because the agent with know-how has left the firm.

The principal of the firm chooses a contract to maximize the expected future cash

[35]Our decision to only include agency frictions on the use of process intangibles can be motivated by Holmstrom and Milgrom (1991). If both product and process intangibles require agent effort, but product intangibles are easier to monitor and observe, an assumption we maintain, then absent proper incentives, the agent will focus on the product intangibles. Consequently, the agency friction would be stronger on process intangibles. We take the stark case here and assume all the agency frictions are on process intangibles.

[36]We also solve an extension of our baseline model where intangible capital accumulation is subject to the agency friction of Ward (2022). Our main results in Section 4 hold qualitatively. Results available upon request. We can also consider the case where the intangible capital stock O is measured in the efficiency unit, hence the dynamics of O are subject to random shocks, for example, $\mathrm{d}O_t = (S_t - \delta_O O_t)\mathrm{d}t + \sigma_O O_t \mathrm{d}W_t$ for another Brownian motion W independent of Z^e. However, contracting on O does not provide an incentive to the agent in our model and makes the agent's continuation utility more volatile. We will show later that the principal's value function is concave in the agent's continuation utility. Therefore, the principal is implicitly risk-averse in the agent's continuation utility, and hence does not load on the intangibles in the optimal contract.

flow net compensation discounted by the interest rate r, which is assumed to be strictly less than γ. The principal chooses among the contracts incentivizing the agent's full effort $e_t = 1$ for any $t \geqslant 0$. Therefore, the principal's optimization problem at time zero is

$$\max_{I,S,C} \mathbb{E}^{e^*}\Big[\int_0^\tau e^{-rs}\left[Y_s \mathrm{d}s - \mathrm{d}C_s\right] + e^{-r\tau}\ell V_\tau^T\Big], \tag{3.12}$$

subject to the agent's incentive compatibility constraint that the agent chooses the full effort optimally, i.e., $e_t^* = 1$ for any $t \geqslant 0$, and the agent's participation constraint $U_0 \geqslant 0$. The contract termination time in (3.12) is

$$\tau = \inf\{t \geqslant 0 \,:\, U_t = 0\},$$

when the agent's continuation value from the contract reaches his outside value (normalized to zero). The contract is terminated at τ to protect the agent's limited liability with respect to his outside value. After the agent leaves the firm at time τ, the efficacy of the firm's physical capital accumulation is reduced:

$$\mathrm{d}\widehat{K}_t = AI_t\,\mathrm{d}t + \sigma\widehat{K}_t\,\mathrm{d}Z_t. \tag{3.13}$$

Hence, the firm's value after the contract's termination is

$$V_\tau^T = \max_{I,S} \mathbb{E}_\tau\Big[\int_\tau^\infty e^{-r(s-\tau)}Y_s \mathrm{d}s\Big], \tag{3.14}$$

subject to (3.2) and (3.13). The expectation is taken under $\mathbb{P}^e$ with $e = 0$. The parameter $\ell \in [0,1]$ represents the deadweight loss due to agent's departure. Contract termination is inefficient due to the deadweight loss and losing the complementarity between physical capital investment and process intangibles. Therefore, the principal aims to use the optimal contract to mitigate this inefficient termination.

4 Optimal contract and implications

4.1 Optimal contract

In order to incentivize the agent's full effort, the principal exposes the agent's continuation utility to variations in $\widehat{K}_t$. Introducing a pay-performance sensitivity φ_t to $\mathrm{d}\widehat{K}_t$ yields the benefit of working $\varphi_t\Lambda_t$ for the agent. Comparing to the cost of working (losing the shirking benefits) $\lambda_t\Lambda_t$, the principal needs to choose $\varphi_t \geqslant \lambda_t$ to incentivize the agent's full effort. Motivated by empirical evidence (Table IA.3 in Internet Appendix IA.1), we assume that λ_t depends on firm's intangibility. The following result summarizes the agent's optimal effort choice and dynamics of the continuation utility.

Lemma 4.1 *For a given cumulative compensation $C = (C_t)_{t \geqslant 0}$, there exists a process $\varphi = (\varphi_t)_{t \geqslant 0}$ such that the agent's continuation utility follows*

$$dU_t = \gamma U_t dt + \varphi_t \widehat{K}_t \sigma dZ_t^{e^*} - dC_t, \tag{4.1}$$

where the agent's optimal effort is

$$e_t^* = \begin{cases} 1, & \varphi_t \geqslant \lambda_t, \\ 0, & \textit{otherwise}. \end{cases} \tag{4.2}$$

Therefore, in order to incentivize full effort, the incentive compatibility constraint for the principal is

$$\varphi_t \geqslant \lambda_t. \tag{4.3}$$

We now turn to the principal's problem (3.12). Introduce the principal's value function as

$$V(\widehat{K}_t, U_t, O_t) = \max_{I,S,C} \mathbb{E}_t^{e^*} \Big[\int_t^\tau e^{-r(s-t)} \big(Y_s \mathrm{d}s - \mathrm{d}C_s \big) + e^{-r(\tau - t)} \ell V_\tau^T \Big]. \tag{4.4}$$

The homogeneity in $\widehat{K}_t$ allows us to introduce a function v via

$$V(\widehat{K}_t, U_t, O_t) = \widehat{K}_t \, v(\widehat{u}_t, \widehat{o}_t), \tag{4.5}$$

where

$$\widehat{u}_t = U_t / \widehat{K}_t \quad \text{and} \quad \widehat{o}_t = O_t / \widehat{K}_t,$$

are the continuation utility and the intangible capital normalized by efficiency-adjusted physical capital, respectively.[37] Using $\widehat{u}$ and $\widehat{o}$ as two state variables for the principal's problem, the optimal contract and the optimal investment strategies are characterized by the following result.

Proposition 4.1 *The function v, the optimal contract, and the optimal investment are described as follows:*

[37]We choose to normalize by physical capital to better map to the existing literature, see e.g. Eisfeldt and Papanikolaou (2013).

(i) The function v satisfies the HJB equation

$$
\begin{aligned}
0 = \max\Big\{ &-(r+\delta_K)v + \max_{\widehat{i}\geqslant 0, \widehat{s}\geqslant 0, \varphi\geqslant\lambda} \Big\{ \big(v - \widehat{o}\,\partial_{\widehat{o}}v - \widehat{u}\,\partial_{\widehat{u}}v\big) d(\widehat{i}, \theta\widehat{o}) \\
&+ \big(\widehat{s} - (\delta_O - \delta_K)\widehat{o}\big)\partial_{\widehat{o}}v + (\gamma+\delta_K)\widehat{u}\,\partial_{\widehat{u}}v \\
&+ \frac{1}{2}\widehat{o}^2\sigma^2\partial^2_{\widehat{o}\widehat{o}}v + \frac{1}{2}(\varphi-\widehat{u})^2\sigma^2\partial^2_{\widehat{u}\widehat{u}}v - \widehat{o}(\varphi-\widehat{u})\sigma^2\partial^2_{\widehat{o}\widehat{u}}v \\
&+ \mu\big[1-\phi+\phi\widehat{o}^{\psi}\big]^{1/\psi} - \widehat{i} - \widehat{s} - \tfrac{Q_K}{2}(\widehat{i}-\delta_K)^2 - \tfrac{Q_O}{2}(\widehat{s}/\widehat{o}-\delta_O)^2\widehat{o}\Big\}, \\
&- \partial_{\widehat{u}}v - 1\Big\},
\end{aligned} \tag{4.6}
$$

where $\widehat{i} = I/\widehat{K}$ and $\widehat{s} = S/\widehat{K}$ are the physical and intangible investment normalized by efficiency-adjusted physical capital, respectively.

(ii) Define $\overline{u}(\widehat{o}) = \inf\{\widehat{u} : \partial_{\widehat{u}}v(\widehat{u},\widehat{o}) = -1\}$ for any $\widehat{o} > 0$. The optimal compensation is a reflection type. Whenever $\widehat{u}_t < \overline{u}(\widehat{o}_t)$, no compensation is paid, i.e., $dC^*_t = 0$. Only when $\widehat{u}_t = \overline{u}(\widehat{o}_t)$, compensation is paid to keep the state process $(\widehat{u}_t, \widehat{o}_t)$ below the payment boundary $\overline{u}$.

(iii) When

$$
\partial^2_{\widehat{u}\widehat{u}}v < 0 \quad \textit{and} \quad \lambda > \widehat{u} + \frac{\widehat{o}\partial^2_{\widehat{o}\widehat{u}}v}{\partial^2_{\widehat{u}\widehat{u}}v}, \tag{4.7}
$$

the optimal contract sensitivity φ^*_t is λ_t.

(iv) The optimal physical investment rate $\widehat{i}^*$ satisfies the first order condition

$$
\big(v - \widehat{o}\,\partial_{\widehat{o}}v - \widehat{u}\,\partial_{\widehat{u}}v\big)\partial_{\widehat{i}}d(\widehat{i}^*, \theta\widehat{o}) = 1 + Q_K(\widehat{i}^* - \delta_K); \tag{4.8}
$$

The optimal intangible investment rate $\widehat{s}^*$ is given by

$$
\widehat{s}^* = \widehat{o}\Big(\frac{\partial_{\widehat{o}}v - 1}{Q_O} + \delta_O\Big). \tag{4.9}
$$

In (4.8) and (4.9), Q_K and Q_O are from (3.9).

To understand the HJB equation (4.6), we first use (3.2), (3.3), and (4.1) to derive the dynamics of $\widehat{u}_t$ and $\widehat{o}_t$:

$$
d\widehat{o}_t = \big[\widehat{s}_t - (\delta_O - \delta_K)\widehat{o}_t - \widehat{o}_t\,d(\widehat{i}_t, \theta\widehat{o}_t) + \widehat{o}_t\,\sigma^2\big]dt - \widehat{o}_t\,\sigma dZ_t, \tag{4.10}
$$

$$
d\widehat{u}_t = \big[(\gamma+\delta_K)\widehat{u}_t - \widehat{u}_t\,d(\widehat{i}_t, \theta\widehat{o}_t) + \sigma^2(\widehat{u}_t - \varphi_t)\big]dt + \sigma(\varphi_t - \widehat{u}_t)dZ_t - \frac{1}{\widehat{K}_t}dC_t, \tag{4.11}
$$

where $\widehat{i}_t = I_t/\widehat{K}_t$, $\widehat{s}_t = S_t/\widehat{K}_t$, and the superscript 1 is suppressed on Z^1 to simplify notation. Equation (4.6) divides the state space into two regions: (i) continuation region where

$$\underbrace{r\,\widehat{K}v}_{\text{Expected change}} = \underbrace{\mathbb{E}\big[\mathrm{d}(\widehat{K}v)\big]}_{\text{Expected change in } V} + \underbrace{Y}_{\text{Net cash flow}}$$
$$= \widehat{K}\,\mathbb{E}[\mathrm{d}v] + v\,\mathbb{E}[\mathrm{d}\widehat{K}] + \mathbb{E}[\mathrm{d}\widehat{K}\,\mathrm{d}v] + Y;$$

(ii) compensation region, where the marginal benefit of compensation $-\partial_{\widehat{u}}v$ equals the unit marginal cost. The right-hand side of (4.6) compares two groups of terms corresponding to continuation and compensation, respectively. Only one group equals zero for each point in the state space. The boundary between the continuation and the compensation region is $\overline{u}$. The optimal compensation satisfies $dC_t^* = 0$ when $\widehat{u}_t < \overline{u}(\widehat{o}_t)$ and $dC_t^* > 0$ when $\widehat{u}_t = \overline{u}(\widehat{o}_t)$. This compensation maintains the state process to be lower than the compensation boundary $\overline{u}$ and reflects the state process whenever the compensation boundary is reached.

The optimal pay-performance sensitivity is determined by the constrained optimization problem

$$\max_{\varphi \geqslant \lambda_t} \left\{ \frac{1}{2}(\varphi - \widehat{u})^2 \sigma^2 \partial^2_{\widehat{u}\widehat{u}} v - \widehat{o}(\varphi - \widehat{u})\sigma^2 \partial^2_{\widehat{o}\widehat{u}} v \right\},$$

where the pay-performance sensitivity φ is subject to the incentive compatibility constraint $\varphi \geqslant \lambda_t$. When the conditions (4.7) are satisfied, the incentive compatibility constraint is binding, i.e., $\varphi_t^* = \lambda_t$. Conditions (4.7) will be verified numerically in our experiments.

The optimal investments are determined by their first-order conditions. The optimal physical investment rate $\widehat{i}^*$ satisfies the first order condition (4.8), where the right-hand side is the marginal cost of physical investment. The left-hand side of (4.8) consists of two components. First, the marginal impact of physical investment on the growth rate of the physical capital is $\partial_{\widehat{i}} d(\widehat{i}^*, \widehat{o})$. Therefore, the marginal benefit on the value function, due to the change of physical capital accumulation, is $v\,\partial_{\widehat{i}} d(\widehat{i}^*, \widehat{o})$. Second, the growth in physical capital reduces the intangible and physical capital ratio, at the rate of $\widehat{o}\,\partial_{\widehat{i}} d(\widehat{i}^*, \widehat{o})$, and also reduces the continuation utility and physical capital ratio, at the rate of $\widehat{u}\,\partial_{\widehat{i}} d(\widehat{i}^*, \widehat{o})$. Both reductions introduce the marginal cost $(\widehat{o}\,\partial_{\widehat{o}} v + \widehat{u}\,\partial_{\widehat{u}} v)\partial_{\widehat{i}} d(\widehat{i}^*, o)$. The optimal investment in the physical capital balances the net marginal benefit on the left-hand side of (4.8) and the marginal cost on the right-hand side. The optimal investment rate in the intangible capital, $\widehat{s}^*$, satisfies the following first-order condition

$$\partial_{\widehat{o}} v = 1 + Q_O\big(\widehat{s}^*/\widehat{o} - \delta_O\big),$$

where the marginal cost on the right-hand side matches the marginal benefit $\partial_{\widehat{o}} v$ on the left.

The HJB equation (4.6) is combined with several boundary conditions. When U reaches 0, the contract terminates, and the firm continues production without process intangibles. Therefore, the boundary condition at $\widehat{u} = 0$ is

$$v(0, \widehat{o}) = \ell v^T(\widehat{o}), \tag{4.12}$$

where the contract termination value v^T satisfies the HJB equation

$$\begin{aligned}(r + \delta_K) v^T = \max_{\widehat{i} \geqslant 0, \widehat{s} \geqslant 0} \Big\{ & (v^T - \widehat{o}\, \partial_{\widehat{o}} v^T) A \widehat{i} + (\widehat{s} - (\delta_o - \delta_K)\widehat{o}) \partial_{\widehat{o}} v^T + \frac{1}{2} \widehat{o}^2 \sigma^2 \partial^2_{\widehat{o}\widehat{o}} v^T \\ & + \mu \big[1 - \phi + \phi \widehat{o}^{\psi}\big]^{1/\psi} - \widehat{i} - \widehat{s} - \tfrac{Q_K}{2}(\widehat{i} - \delta_K)^2 - \tfrac{Q_O}{2}(\widehat{s}/\widehat{o} - \delta_O)^2 \widehat{o} \Big\}. \end{aligned} \tag{4.13}$$

After the equation (4.6) is solved, the compensation boundary $\overline{u}$ is determined endogenously via $\overline{u}(\widehat{o}) = \inf\{\widehat{u} : \partial_{\widehat{u}} v(\widehat{u}, \widehat{o}) = -1\}$. Several other technical boundary conditions and our numeric algorithm are discussed in Internet Appendix IA.4.

4.2 Stationary distribution

After the optimal contract and investment strategies are characterized for an individual firm in the previous section, we examine in this section the stationary distribution of the state variables (see e.g., Hopenhayn (1992)).

Because the volatility of $\widehat{u}$ in (4.11) is non-degenerate at $\widehat{u} = 0$, firm liquidation happens with positive probability under the optimal contract. In order to maintain a stationary mass of firms, we introduce firm entry. The stationary density g of the state variable $(\widehat{u}, \widehat{o})$ satisfies the stationary Fokker-Planck-Kolmogorov equation:

$$\mathcal{L}^*_{\widehat{u},\widehat{o}}\, g(\widehat{u}, \widehat{o}) + m\, \psi(\widehat{u}, \widehat{o}) = 0, \tag{4.14}$$

where $\mathcal{L}^*_{\widehat{u},\widehat{o}}$ is the adjoint operator of the infinitesimal generator for the state variable, see (IA.4) in the Internet Appendix for the form of $\mathcal{L}^*_{\widehat{u},\widehat{o}}$, $\psi(\widehat{u}, \widehat{o})$ represents an entry density integrating to one, and m is an entry rate. To ensure that the stationary density

g integrates into one, the entry rate m is chosen to match the existing mass:

$$m = -\int_0^\infty \int_0^{\overline{u}(\widehat{o})} \mathcal{L}^*_{\widehat{u},\widehat{o}} g(\widehat{u},\widehat{o})\, \mathrm{d}\widehat{u}\, \mathrm{d}\widehat{o}.$$

The stationary density g describes the behavior of the equilibrium state variables.

4.3 Quantitative model implications

We examine the quantitative implications of our model in this section. Several of the model parameters, especially those in the physical capital production function D in (3.4), are calibrated to the data. Efficiency-adjusted physical capital $\widehat{K}$ is not directly observed in data, only the capital in physical units K is. To bridge the gap between model generated quantities, such as $O/\widehat{K}$, $U/\widehat{K}$, $I/\widehat{K}$, and the empirically observed quantities, such as O/K, U/K, I/K, we use the model generated stationary distribution of $(O/\widehat{K}, U/\widehat{K})$ and simulation of X to translate $\widehat{K}$-normalized quantities to K-normalized quantities.[38]

It follows from Proposition 4.1 (iii) that the sensitivity of changes in U with respect to changes in $\widehat{K}$ is λ_t. Table IA.3 in Internet Appendix IA.1 shows that this sensitivity increases with the intangibility. Motivated by this empirical fact, we assume

$$\lambda_t = \overline{\lambda}\, \widehat{o}_t, \tag{4.15}$$

for a constant parameter $\overline{\lambda}$.

For firm entry, we assume that the principal has all bargaining power so that a new firm starts the agent's promised utility $\widehat{u}$ at $\widehat{u}^e(\widehat{o})$ which maximizes the principal's value $v(\cdot, \widehat{o})$ for a given intangibility level $\widehat{o}$. The entry density $\vartheta(\widehat{u}, \widehat{o})$ is assumed to have the decomposition $\vartheta(\widehat{u}, \widehat{o}) = \zeta(\widehat{o})\xi(\widehat{u}|\widehat{o})$, where ζ is the density of a log normal distribution with parameters μ^e and σ^e, and $\xi(\cdot|\widehat{o})$ has a unit mass at $\widehat{u}^e(\widehat{o})$.

The model parameters are calibrated to mean of the physical investment rate I/K and the compensation rate U/K conditioning on quintiles of intangibility O/K. The empirical physical investment rate is measured by physical investment (Compustat item CAPX) divided by lagged physical capital (Compustat item PPEGT) and the compensation rate is measured by the total compensation (Execucomp item TDC1) divided by lagged physical capital. Other than calibrated parameters, the process intensity parameter, θ, is estimated from average firm-level process intensity using patent data. The

[38] Recall that $X_t = \widehat{K}_t/K_t$. Hence $O_t/K_t = X_t\, O_t/\widehat{K}_t$. See Internet Appendix IA.4 for more details on the calibration procedure.

volatility parameter, σ, is estimated from the standard deviation of annual changes in the log physical capital stock. Other model parameters are studied extensively in the literature. Their values are chosen to be consistent with the literature. All model parameters are summarized in Table 2.

Table 2 Here

The data panel in Table 3 presents the mean physical investment rate and the ratio between total compensation and physical capital in different intangibility quintiles in our sample. The calibrated CES parameter ρ in the physical capital accumulation function D is 0.55, which is consistent with the value of a similar parameter in Lin (2012).

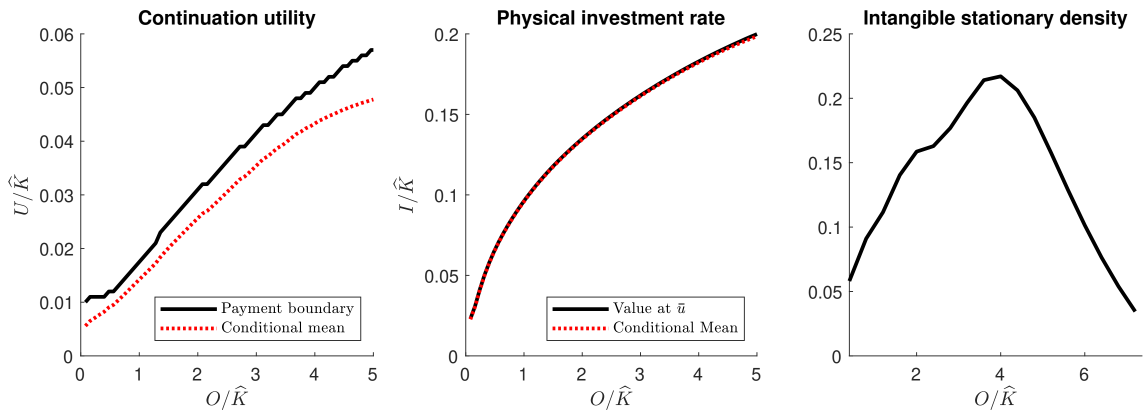
Table 3 Here

We now present several implications of our calibrated model. The left panel of Figure 3 presents the payment boundary $\overline{u}$ (black solid line) and the mean of agent's continuation utility over capital ratio conditioning on $O/\widehat{K}$ (red dotted line) under the stationary distribution of model state variables $U/\widehat{K}$ and $O/\widehat{K}$. The compensation boundary $\overline{u}$ increases with $O/\widehat{K}$. This is due to two effects. First, the production function d of the physical capital investment increases with process intangibles. For a given process intensity θ, a higher intangibility means more process intangibles, which improves the physical capital efficiency. Meanwhile, more process intangibles also elevate the importance of the agent's effort in the physical capital accumulation. Second, the private benefit rate λ_t increases with $O_t/\widehat{K}_t$ in (4.15). Both effects imply that the agency friction worsens with higher intangibility. In order to mitigate inefficient liquidation, the principal increases the compensation boundary $\overline{u}$ to build up the agent's continuation utility by deferring more compensations into the future. The conditional mean of $U/\widehat{K}$ (red dotted line in the left panel) also increases with $O/\widehat{K}$, following the same pattern of the payment boundary and indicating a positive relationship between the average deferred compensation and intangibility.

The middle panel of Figure 3 presents the physical investment rate $I/\widehat{K}$ at the payment boundary $\overline{u}$ (black solid line) and the mean physical investment rate conditioning on $O/\widehat{K}$ under the stationary distribution. Both investment rates increase with $O/\widehat{K}$. Finally, the stationary marginal distribution of $O/\widehat{K}$ is presented in the right panel of Figure 3.

Compensation and physical investment, normalized by capital in physical units K, also increase with O/K. The data panel in Table 3 show that both U/K and I/K increase

Figure 3: Optimal Contract and Physical Investment



This figure presents agent's continuation utility and physical investment for different intangibility $O/\widehat{K}$ under the optimal contract. Agent's continuation utility is $\widehat{u} = U/\widehat{K}$ and the physical investment rate is $\widehat{i}^* = I^*/\widehat{K}$. The black solid lines in the left and middle panels present the payment boundary $\overline{u}$ and the physical investment rate at $\overline{u}$. The red dotted lines plot the mean conditioning on $O/\widehat{K}$ under the stationary distribution. The right panel presents the marginal stationary density of $O/\widehat{K}$ from 1 to 99 percentile. All parameters are listed in Table 2.

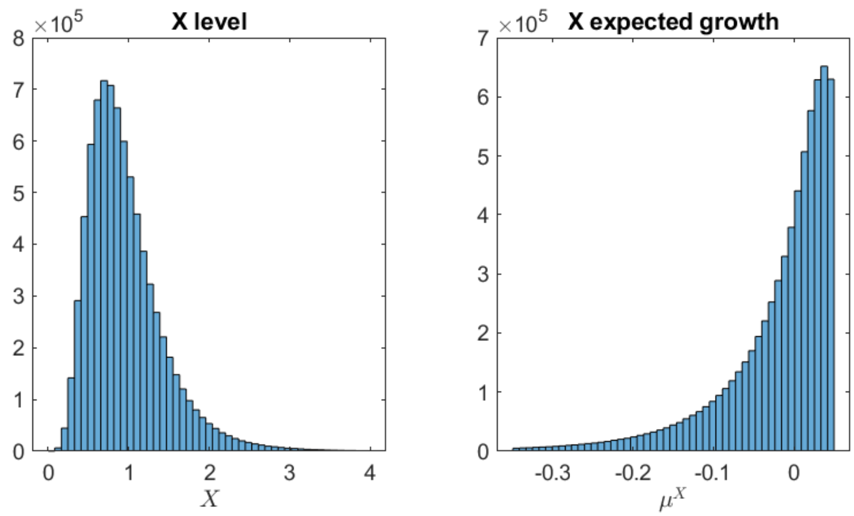
with quintiles of O/K, in the full sample, the low process intensity subsample, and the high process intensity subsample.[39] The model panel in Table 3 presents the model generated counterparts. The model is not calibrated using the moments conditioning on process intensity. We solve the model under the low process intensity ($\theta = 0.05$) and high process intensity ($\theta = 0.67$) parameters, which match the empirical mean of process intensities in the low and high process intensity subsamples, meanwhile keeping the remaining parameters unchanged from the main calibration. The models generate physical investment rates and compensations that are consistent with their empirical counterparts in Table 3.

Figure 4 presents the distributions of physical capital efficiency, X_t, and its expected growth from the calibrated model. The mean of X_t is 0.963 and its median is 0.867. Around 60% of simulated X_t display positive growth. Negative growth of X_t happens for large values of X_t due to the mean-reverting behavior in (3.5).

The impact of θ on compensation and investment is presented in Figure 5. When θ increases, more proportion of the intangible capital is used in the process innovation.

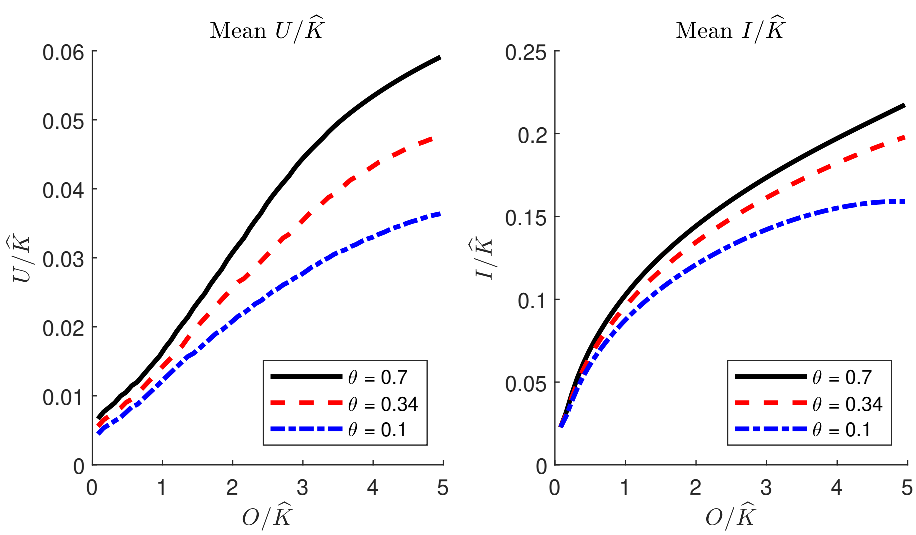
[39]The low (resp. high) process intensity subsample contains firms whose process intensities are lower (resp. higher) than 30% (resp. 70%) percentile of the full sample process intensity distribution. Firms are assigned to the low or high bin each year. All data moments are computed by first taking cross-sectional medians with each intangibility bin by process intensity bin by year. Then, we take time-series averages of these medians. These are the numbers reported in the table.

Figure 4: Physical Capital Efficiency



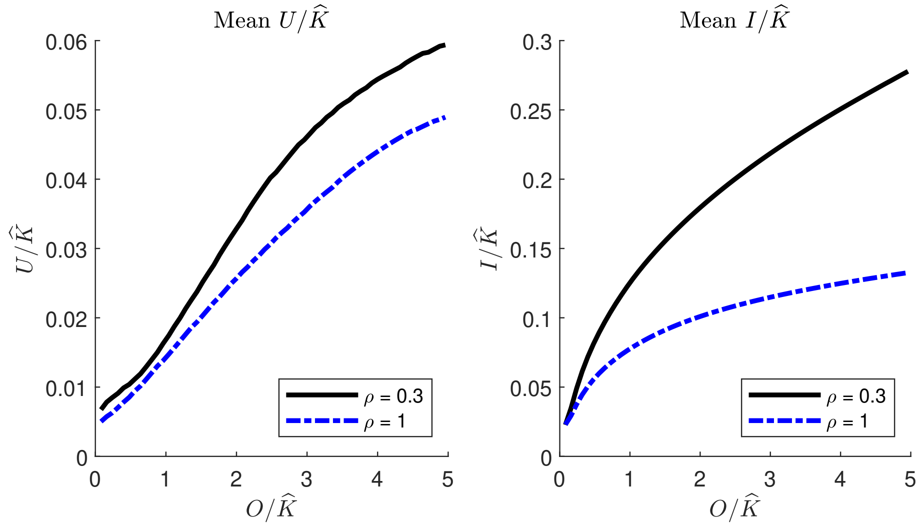
This figure presents distributions of X_t and its expected growth $\mu_t^X = d(e_t, i_t, \theta o_t) - i_t X_t$ from the calibrated model. All parameters are listed in Table 2. These histograms are generated by 10^5 simulations. Each simulation lasts 10 years with the first 2 years burned out.

Figure 5: Compensation and physical investment: varying θ



This figure presents the conditional mean of agent's continuation utility $U/\widehat{K}$ and the physical investment rate $I/\widehat{K}$ for different process intensity θ. Other parameters are listed in Table 2.

Figure 6: Compensation and physical investment: varying ρ



This figure presents the conditional mean of agent's continuation utility $U/\widehat{K}$ and the physical investment rate $I/\widehat{K}$ for different CES complementarity parameter ρ. Other parameters are listed in Table 2.

Given $I/\widehat{K}$ and $O/\widehat{K}$, the physical capital accumulation function d increases in θ when the agent exerts full effort. As a result, the physical capital efficiency depends more on the agent's effort and the agency friction is more severe as θ increases. The left panel of Figure 5 shows that the conditional mean of agent's continuation utility increases with θ, implying that the principal defers more compensation into the future when more proportion of the intangible capital is used for process innovations. The right panel of Figure 5 shows the same pattern as physical capital efficiency improves with the process intensity. Conditioning on intangibility quintiles, and comparing the physical investment rate and compensation over capital ratio in each column of Table 3, we see that they also increase with process intensity in both data and model generated moments.

The impact of changing the CES parameter ρ is presented in Figure 6. When ρ increases, complementarity between physical capital investment and process intangibles weakens. Process intangibles play an diminishing role in physical capital accumulation and efficiency of physical capital reduces. Therefore, the physical investment rate decreases with ρ in the right panel of Figure 6. In particular, when $\rho = 1$, physical capital investment and process intangibles perfectly substitute each. The physical investment rate is significantly reduced. This demonstrates the importance of complementarity in our calibrated model: without the complementarity (increasing ρ from 0.3 to 1), physical investment rate drops from 20.9% to 12.1% among firms with median intangibility $O/\hat{K}$. As ρ increases, the agent's effort becomes less important in the physical capital accumulation and the agency friction subsides. The left panel of Figure 6 shows that the conditional mean of agent's continuation utility decrease with ρ.

These model predictions on the intensity of process innovation and the complementarity between the physical capital investment and the process intangibles will be tested in our empirical analysis next.

5 Process Intensity, Investment, and Capital Efficiency

This section tests two underlying assumptions of our model. These assumptions are the key features of the efficiency-adjusted capital accumulation in our model. Here, we are interested in the associations between process intensity and physical investment. First, we show that higher process intensity or intangibility is associated with more future investment. Second, we show that process intensity and intangibility interact with the investment rate in a complementary way when it comes to the improvement in the firm's efficiency. These two facts combined gives us the properties of the capital accumulation equation, (3.3). As we saw in the model section, it is equation (3.3), with the agency friction, that deliver our main results about compensation. We defer the empirical results on compensation to the Section 7.

5.1 Physical Investment

In this subsection, we show that higher process intensity and intangibility are associated with higher physical investment rates. Timing is important here (and in all our regressions). Investment is a flow variable, and a control in the model, while intangibility is a stock variable. The timing conventions in Compustat are such that flows are measured over a period (e.g., investment over the year 2000), while stocks are measured at the end of the period (e.g., capital at the end of 2000). Consequently, here and throughout the paper, dependent flow variables are always leaded at least one period ahead. The interpretation is that the firm enters the period with some capital stock and then chooses investment.[40] Therefore, our regression specification is:

$$PhyInv_{f,t+1} = \alpha_j + \alpha_t + \beta_1 \text{Process Measure}_{ft} + \beta_2 \text{Non-Process Measure}_{ft} + \boldsymbol{\beta X}_{ft} + \varepsilon_{f,t+1}. \tag{5.1}$$

[40]See Internet Appendix IA.1 for explicit definitions.

The dependent variable is physical investment in year $t+1$ (Compustat code: CAPX) divided by physical capital in year t (PPEGT). The first two terms on the right-hand side, α_j and α_t, are industry and date fixed effects, respectively. Industry is defined as 2 digits SIC code prior to 2002 and 2 digit NAICS code after (Belo et al. (2017)). For Process Measure$_{ft}$ and Non-Process Measure$_{ft}$, we consider two specifications. First, Process Measure$_{ft}$ is the ratio between process intangibles and total capital in year t, i.e., process intangibility; Non-Process Measure$_{ft}$ is the rest of intangibles over total capital, i.e., non-process intangibility. Second, we consider the effect of fixing intangiblity and varying only the process intensity. Hence, Process Measure$_{ft}$ is process intensity and Non-Process Measure$_{ft}$ is (total) intangibility in the second specification. We find both measures informative. Finally, $\boldsymbol{X}_{ft}$ is a vector of controls: log market cap, the iB/M ratio (book equity plus intangibles divided by market value; Park (2019) and Kazemi (2022)), and the sales to capital ratio.

Table 4 displays the results. The dependent variable is multiplied by 100 so that all coefficients can be interpreted as the "percent of physical capital." The first two columns use process intangibility and non-process intangibility as the key measures, while the last two use process intensity and total intangibility. Standard errors are clustered at the firm level. Looking at the first two columns, we see that both process intangibility and non-process intangibility forecast more physical investment in the future. This is consistent with our model. Process intangibles, as we will explore below, forecast more investment because they increase the marginal product of investment itself. Non-process intangibles forecast higher investment because they increase the marginal value of capital. Note, however, that the magnitude of process intangibility is larger. For example, moving from the 25th (0.027) to the 75th (0.32) percentile in process intangibility is associated with around an 8% increase in investment per physical capital. at the mean level of physical capital ($1.55 billion), this corresponds to an increase of $119 million in physical investment. The same increase for non-process intangibility is associated with about a $10 million smaller increase in investment. Looking at the final two columns, we see that fixing the intangibility of the firm, increasing process intensity, θ, has a significant association with future investment. Going from 0 to 1 process intensity at the mean level of capital is associated with a $31 million increase in investment. Meanwhile, a one standard deviation (0.287) increase in process intensity is associated with about a $9 million increase in investment at the mean capital level.

Table 4 Here

5.2 Efficiency

In this subsection, we wish to test the implications of equation (3.5) in the data. In particular, we wish to show that process intangibility and intensity increase capital efficiency, and that this effect is strong if the firm undertakes more physical investment. That is, we expect to see positive complementarity between investment and our process measures. Before we can define our regression specification, we need to define efficiency itself, as it is not a directly observable variable. Following Liu, Sojli and Tham (2022), we use the total factor productivity (TFP) measures from İmrohoroğlu and Tüzel (2014).[41] İmrohoroğlu and Tüzel (2014) create these TFP measures using the methods of Olley and Pakes (1996). Essentially, their estimates of TFP are a regression residual, where the regression in question corresponds to an (adjusted) production function. Because their method does not take into account intangible capital, we also use a residualized version (TFPR) that is created by regressing the TFP measure on three lags of intangible capital and then calling the regression residual our new efficiency measure.

The dynamics of X_t in (3.5) imply that

$$\mathbb{E}\left[\mathrm{d}X_t\right] + i_t X_t \mathrm{d}t = d(e_t, i_t, \theta o_t)\mathrm{d}t. \tag{5.2}$$

What this last equation says is that efficiency growth (adjusted by investment) is a function of investment and process intangibility. Also, given our choice of $d(\cdot,\cdot,\cdot)$, we expect some degree of interaction between the investment rate and process intangibility. This interaction is the complementarity effect we have discussed earlier. We now define our adjusted efficiency growth measure in discrete time so we can map to the data:

$$\text{Adjusted Efficiency Growth}_{f,t\to t+1} = X_{f,t+1} - X_{ft} + i_{f,t+1}X_{ft}. \tag{5.3}$$

Note well the timing convention again. We treat efficiency as a stock variable. So, for example, the firm enters the year 2000 with some efficiency and capital. Over that year, using those inputs, it invests and makes profits. At the end of 2000, shocks happen, and combined with choices the firm made in 2000, a new efficiency level is realized for 2001.

In order to capture the complementarity between investment and process intangibility, we consider not just the direct effect of process intangibility on adjusted efficiency/TFP growth, but also how this effect changes as we move from low to high investment firms. Every year, we assign firms to one of two bins, depending on whether the firms were above or below the median investment rate for that year. Let BH_{ft} be a

[41] Available here: https://sites.google.com/usc.edu/selale-tuzel/home?authuser=2.

dummy variable equal to one if the firm is in the above median bin in year t. Thus, our regression specification is:

$$AdjEffGr_{f,t\to t+1} = \alpha_j + \alpha_t + \alpha_B + \beta_1 \text{Process Measure}_{ft} + \beta_2 \text{Non-Process Measure}_{ft} \\ + \beta_3 \text{Process Measure}_{ft} \times BH_{f,t+1} + \beta_4 \text{Non-Process Measure}_{ft} \times BH_{f,t+1} + \boldsymbol{\beta X}_{ft} + \varepsilon_{f,t+1}, \tag{5.4}$$

where α_B is a bin fixed effect. We are mainly interested in $\beta_1, ..., \beta_4$. The first two coefficients tell us the direct effect of process vs non-process intangibility on adjusted efficiency growth. The last two coefficients tell us how much stronger the association is for high investment firms on top of the baseline effect. When our process measure is process intangibility, we expect β_1 and β_3 to be positive and larger than β_2 and β_4, respectively. When we use process intensity, we expect β_1 and β_3 to be positive.

Tables 5 and 6 display the results for adjusted efficiency growth and adjusted efficiency growth residualized with respect to intangible capital, respectively. Control variables have been suppressed for compactness of presentation. Looking at the first two columns of each table, we see our expectations about the coefficients are met. Moving from the 25th to the 75th percentile of process intangibility is associated with a 0.065 standard deviation increase in adjusted efficiency growth. This is change would move a firm from the 25th percentile of growth to the 43rd. For residualized growth, the same move in process intangibility is associated with a move from the 25th to the 49th percentile of residualized growth. These numbers are for the baseline effect. When we look at a high investment firms, a 25-75 move in process intangibility is associated with a 0.32 standard deviation increase in adjusted growth and a similar increase for residualized growth. These moves would take a firm from the 25th to 85th and 83rd percentiles of growth and residualized growth, respectively. Looking at the last two columns, using process intensity, we see similar effects. Though the direct effect coefficient is not significant at conventional levels, it is positive. Importantly, the interaction of process intensity with high investment is significant and positive, indicating that, all else equal, higher process focused firms experience complementarity between intangibility and investment in efficiency growth.

Table 5 Here

Table 6 Here

6 Burning Glass Technologies

We introduce in this section a dataset on skilled labor. We leave the rest of our data discussion to Internet Appendix IA.1. There, we discuss more traditional datasets that we use (including CRSP/Compustat and Execucomp), as well as provide more details on the process intensity measure of Bena and Simintzi (2019).[42]

We are not only interested in the payments to top executives but also the payments to specialists/skilled labor. Though executives are unlikely to be directly involved in innovation activities, they are arguably the best positioned to extract rents from the firm. Indeed, many papers studying agency conflicts use data from Execucomp to test their model predictions. Meanwhile, it is plausible that the skilled labor directly involved with innovation has the most information about the technology in question. Therefore, these workers are also well positioned to extract knowledge-based rents.

Burning Glass Technologies (BGT) is a labor market data firm that collects vacancy and resume data from the Internet using machine learning techniques. The data set we use is collected by an "electronic spider" that scrapes job posting sites like Indeed.com and Monster.com for information about the vacancies posted there.

BGT collects the unstructured data on the websites and arranges them in a database with standardized variables. This allows cross-firm and intertemporal comparisons. Most importantly for us, BGT standardizes the set of skills that firms are looking for. For example, one firm may want to hire someone "proficient at Microsoft Word". Another firm might simply state that "the job will require a good deal of writing, so facility with word processors like Microsoft Word is a must". BGT would assign "Microsoft Word" as a skill for both firms. For each job posting, BGT assigns a number of employee skills.[43]

There are three lists of skills, and the difference between these lists is the level of granularity. For example, the least granular list has 29 different levels, such as Administration, Design, Business, and Health Care. We use the middle list (in terms of granularity) that has 677 levels. Examples of skills here include Litigation, Water Testing and Treatment, and Technical Support.

We classify certain skills as being innovation intensive (II) versus not. We call a job posting an innovation intensive job (II job) posting if it has one of these skills assigned

[42] We refer the reader to their paper for full details.

[43] 7% of jobs have no assigned skills or skills are assigned to a non-existent job posting. We drop these cases from the dataset.

to it.[44] Our selection of skills for this categorization is subjective. We ask ourselves "What skills are associated with the creation of new ideas and processes?" Note that this is related to, but different from, "high skill". For example, medical doctors are highly skilled and educated, but we do not consider them to typically be involved in the creation of new ideas. Consequently, medical doctors are not "innovation" job holders.

Around 5% of all BGT job postings have an associated salary. These are the salaries the employer is offering for the position. For each firm-year, we compute the average II job salary. Similarly, we compute the average II job salary within an industry year.

There are two drawbacks to the BGT data. First, BGT data only go back to 2010. Second, as alluded to above, even though the II workers in BGT are the ones actually undertaking the innovative work, it is not clear how much power these workers have to extract rents from shirking. This second drawback is not so problematic, since we examine Execucomp data, as well. Our results are consistent with either the executives or the II workers, or both, being subject to agency conflicts.

7 Compensation and Process Intensity

This section displays our main empirical results linking compensation and process intensity. First, we consider the direct effect: The level effect of process intensity on compensation. Second, we examine the indirect effect: The changing slope of the compensation-process intensity relationship. The direct effect says that an increase in process intangibles increases the benefit from shirking, and, therefore, increases optimal compensation. The indirect effect says that the strength of that relationship varies depending on the amount of physical investment. The intuition for this strengthening relationship is a hold-up problem: As the firm increases investment, the effect of the agent's effort on investment's marginal product increases. Therefore, he can extract more and more rents from ownership.

[44] We require only one skill to be II because it is not true that more non-II skills reduce the innovativeness of the job, so to speak. For example, one company may want someone who understands artificial intelligence, while another company wants this same role to also manage people and write reports. The second company's posting would have a smaller fraction of skills classified as II, but that role just described is no less innovative.

7.1 The Direct Effect

We show that higher process intensity is associated with higher total and deferred compensation, as well as higher salaries for II job employees relative to their industry peers. These results correspond to what we called the direct effect of agency frictions on the process intensity-compensation association.

We estimate specifications of the form:

$$\begin{aligned}&\text{Compensation Measure}_{f,t+1} = \alpha_j + \alpha_t + \beta_1 \text{Process Measure}_{ft} \\ &+ \beta_2 \text{Non-Process Measure}_{ft} + \boldsymbol{\beta X}_{ft} + \varepsilon_{f,t+1}. \end{aligned} \tag{7.1}$$

Our compensation measure is one of four possibilities: Total executive compensation (Execucomp: TDC1) over total capital, deferred executive compensation (Execucomp: STOCK_AWARDS + OPTION_AWARDS) over total capital, the skilled wage (relative to the yearly average skilled wage), and the fraction of executive compensation deferred. We take these in order.

Table 7 displays the results for total executive compensation. The first two columns, using process and non-process intangibility, show that process intangibility has a stronger association with executive compensation than non-process intangibility. Similarly, the final two columns show that, holding total intangibility fixed, an increase in process intensity is associated with higher executive compensation. A 25th percentile to 75th percentile ("25-75 change") in process intangibility is associated with a move in executive compensation per capital starting in the 25th percentile to the 66th percentile. For a firm with the 25th percentile of total capital, this amounts to a $1.1 million increase in compensation. For a firm with the mean amount of total capital, it is $18 million increase.

Table 7 Here

Table 8 shows the results when our compensation measure is deferred executive compensation over total capital. Deferred compensation is the value of stocks and options granted to the firm's executives. Our preferred measure of compensation, the one that maps most closely to the model object, "promised utility," is the fraction of compensation deferred. However, we find it informative to look at all components (total and deferred) since there is no exact way to map to the model object.

Once again, we see that increases in process intangibility (process intensity) are associated with higher deferred compensation. This effect is on top of the effects of intangibility alone, as the last two columns show. A 25-75 change in process intangibility is associated with a 25-68 change in deferred compensation per unit capital. In dollar terms, assuming a firm with 25th percentile total capital, the association implies a $362 thousand increase. For a firm with the mean total capital, the amount is $5.7 million.

Table 8 Here

Table 9 shows the results when our compensation measure is the fraction of executive compensation deferred (the ratio of deferred compensation to total compensation). Increases in process intangibility (process intensity) are associated with a larger fraction of compensation being deferred. This effect is on top of the effects of intangibility alone, as the last two columns show. A 25-75 change in process intangibility is associated with a 25-54 change. The 25-75 change in process intangibility is associated with a 4% increase in the fraction of compensation deferred (e.g., deferred compensation going from 5% to 9% of total compensation).

Table 9 Here

When it comes to the other controls, we see that firm size is associated with decreases in total compensation and deferred compensation. This is consistent with results in Hall and Liebman (1998) and Murphy (1999) which document that pay-for-performance sensitivity decreases with firm size. Higher iB/M ratios are also associated with lower compensation. Book-to-market ratios can be used as measures of performance. For example, a low iB/M ratio implies that the market values the firm much more than its balance sheet shows. This higher valuation could be associated with better management and, therefore, higher pay for executives. Higher sales are associated with higher total and deferred compensation.

Up to this point, we have examined executive pay, but now we turn to the pay of high-skilled II job workers. Executives are unlikely to be directly involved in the innovation or investment process. At the same time, executives probably have the most scope for extracting rents from their firms. II job employees, though less powerful than c-suite executives, are directly involved in implementing and developing new processes. It is their efforts that determine success or failure. Because internal process innovations and improvements are inherently opaque (especially to outsiders), it is difficult to assess the efficacy of the hours worked by II employees even if their managers can see that the quantity of working hours is high. In the next table, we will show that II employee wages and salaries are also increasing in process intangibility (intensity), lending credence to the hypothesis that non-executives can extract rents, too. To the best of our knowledge, we are one of the first papers to test the consequences of dynamic agency theory in compensation of non-executives.

Table 10 shows our estimation results when we use the relative II job salary as the dependent variable in equation (7.1). These are posted salaries, not total wage bills. Therefore, we cannot scale by firm size or capital. Instead, we scale by the annual mean of the II job salary. The coefficients can be interpreted as "how much more does a firm pay for a given set of skills?"

The firs two columns show that higher process intangibility is associated with higher relative skilled salaries. This effect is larger than that of non-process intangibility. Similarly, the last two columns show that process intensity has a significant positive association with relative skilled salaries above and beyond that of total intangibility. A 25-75 move in process intangibility is associated with a 25-39 move in relative skilled salaries. For a year when the average skilled salary is its 25th percentile, this association amounts to a $10,000 increase salary. When the the aggregate skilled salary is at its time-series mean, the association amounts to an $11,000 increase.[45]

Table 10 Here

7.2 The Indirect Effect

This section verifies what we have called the indirect effect of agency frictions on the process intangibility-compensation association. In Section 5, we showed that, empirically, process intangibles and physical investment are complements. This complementarity implies that the benefits of shirking are more sensitive to process intensity for agents employed at firms undertaking more physical capital investment, everything else equal.

We test this implication in two steps. First, we divide firms into high and low physical investment portfolios. Each year, we assign a firm to a portfolio depending on whether that firm is above or below the median physical capital investment rate that year. Second, We re-estimate our compensation equations, allowing the coefficients to vary across bin

[45]The range is small because the BGT data starts in 2010, so the mean skilled wage is only $7,000 higher than the 25th percentile. Similarly, we see that the number of observations in Table 10 is much smaller than in our previous tables.

assignments:

$$\text{Compensation Measure}_{f,t+1} = \alpha_j + \alpha_t + \alpha_B + \beta_1 \text{Process Measure}_{ft} + \beta_2 \text{Non-Process Measure}_{ft} + \beta_3 \text{Process Measure}_{ft} \times BH_{f,t+1} + \beta_4 \text{Non-Process Measure}_{ft} \times BH_{f,t+1} + \boldsymbol{\beta X}_{ft} + \varepsilon_{f,t+1}. \tag{7.2}$$

Just as in regression (5.4), BH is a dummy variable equal to one when a firm undertakes above median investment in a given year. Our hypothesis is that $\beta_3 > 0$, the compensation-process intangibility sensitivity is increasing in investment.

Tables 11 to 14 displays the results. For brevity, we retain only the coefficients on process/non-process intangibility, process intensity, and total intangibility in the table. (Inv. = H) refers to firms in the high physical investment bin, so that the numbers in this row correspond to the additional compensation-process intangibility sensitivity coming from higher physical investment (e.g., because of the agent's hold up power). First, we see that across all columns, the second row coefficient is positive. Second, for the first two columns, $\beta_3 > \beta_4$, that is, process intangibility is associated with a much stronger indirect effect than non-process intangibility. In fact, for our preferred measure, the fraction of compensation deferred, only the process intangibility interaction coefficient is significant. Third, for the skilled labor regressions, due to a much smaller sample size, we do not have significance for the indirect effect. However, note that the sign on process intangibility interacted with high investment (and process intensity interacted with high investment) is positive, while the sign of the non-process intangibility indirect effect is negative.

Tables 11 - 14 Here

This section has established our main results: Compensation and process intensity are tightly linked. This statement applies to total and deferred executive pay. It also applies to the salaries of highly skilled, innovation-based workers. Finally, these results interact with physical capital investment. The more important process intensity is to the firm's capital growth process, the more rents the agent can extract, ceteris paribus.

8 Policy Implications

In a world obsessed with innovation and growth, the way firms compensate their executives and skilled labor is of utmost importance. Could compensation restrictions be inadvertently harming firms by stifling innovation and reducing their value, especially among high intangibility firms?[46] Addressing this question is key to promoting sustainable growth and encouraging innovation in a time when it's needed most.

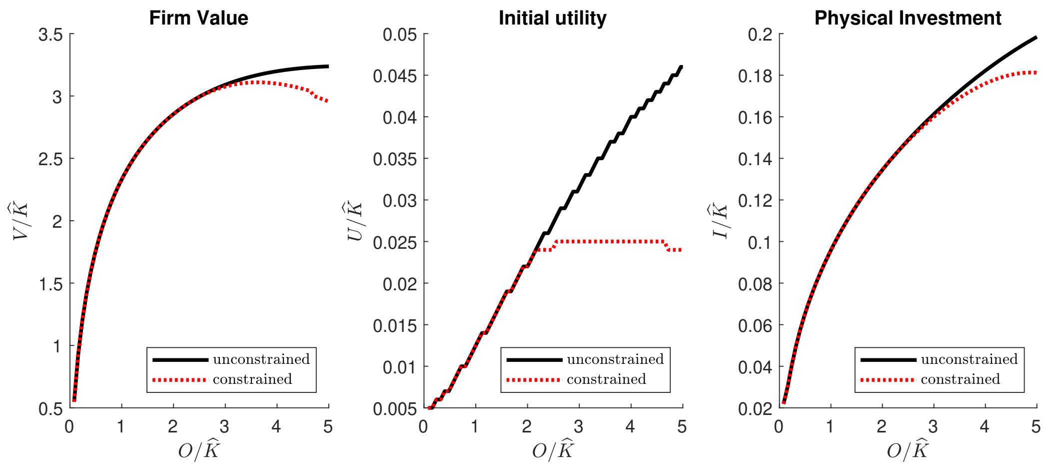
Our model implies that firms with high intangibility optimally compensate their executives and skilled labor more to incentivize their effort. In particular, the agent's initial utility U_0, which measures the present value of future deferred compensation, increases with the firm's intangibility $O_0/\widehat{K}_0$ at time zero. Higher values of U_0 mean the agent is farther from his outside value (normalized to be zero). This helps mitigate agency frictions in firms with high intangibility, where the effectiveness of their large stock of process intangibles is lost if the agent shirks or leaves the firm. We consider a policy experiment where the policy maker restricts executive compensation by setting an upper bound on the ratio between the agent's initial utility U_0 and the firm's initial value $V(U_0, \widehat{K}_0, O_0)$.

Figure 7 presents the impact on firm values and physical investment from this policy. Without the compensation constraint, U_0 is chosen by the principal to maximize the initial firm value $V(U_0, \widehat{K}_0, O_0)$, assuming that the principal has all the bargaining power. The black solid line in the middle panel of Figure 7 shows that U_0 increases with the firm intangibility $O_0/\widehat{K}_0$ at time zero, and reaches 1.6% of firm's initial value at $O_0/\widehat{K}_0 = 6$. When the compensation constraint is in place, the principal chooses U_0 to maximize the firm's initial value $V(U_0, \widehat{K}_0, O_0)$, subject to the constraint that U_0 is less than an upper bound (chosen as 0.8% in the experiment) of the firm's initial value. The right panel of Figure 7 shows that the physical investment remains the same among low intangibility firms, but is depressed significantly among high intangibility firms. For these high intangibility firms, restricting the agent's initial utility U_0 weakens their ability to mitigate the agency friction by choosing a high U_0. The constraint, when binding, reduces the distance to the inefficient termination. Hence, the probability of agent leaving and firm losing complementarity is higher and the physical investment is depressed as a result. This effect is more pronounced when the firm's outside value is less efficient. The depressed physical investment reduces the firm's value, as the left panel of Figure 7 shows.

This policy experiment shows that restricting compensation for executives and skilled

[46] There are several compensation restriction policies, such as the "Say on Pay Legislation", which is implemented in several countries and gives shareholders a non-binding vote on executive compensation. Another is the "Pay Ratio Disclosure Rule" in the United States, which requires publicly traded companies to disclose the CEO's compensation to the median compensation of its employees. And, finally, the "FASB Stock Option Expensing" required U.S. firms, starting in 2005, to expense stock options, which led to a significant decrease in the use of stock options as a form of compensation.

Figure 7: Constrained Compensation



This figure presents the firm value, the agent's initial utility at time zero, and the conditional mean of physical investment under the stationary distribution when the firm chooses agent's initial utility optimally (unconstrained case) and when the firm can only promise agent a compensation package up to 0.8% of firm's initial value (constrained case). Contract termination is inefficient. Firm's outside value (V_τ in (3.12)) is assumed to be zero at contract termination. All other parameters are listed in Table 2.

labor has unintended consequences: it could depress physical investment and reduce firm value, particularly among high intangibility firms. Therefore, a blanket industry-wide compensation constraint is suboptimal. The optimal policy should differentiate sources of high compensation and leave room for firms to incentivize key employees in process innovation.

9 Conclusion

We presented and studied a new empirical fact: Higher process intensity is associated with higher pay for executives and skilled employees. To rationalize this fact, we developed a dynamic principal-agent model in which agent effort determined the efficacy of process intangibles on the efficiency-adjusted physical capital growth. The key model assumption is that process intangibles and physical capital investment are complements in increasing efficiency-adjusted capital growth. We estimated a proxy for this unobservable efficiency and showed that our assumptions are true in the data. The model delivered two key channels: A direct effect and indirect effect of process intensity on compensation. The direct effect states that higher process intensity increases the benefits of shirking, so the agent must be further compensated to ensure full effort. The indirect effect states that for a given level of process intensity, higher physical capital investment increases the hold up power the agent has over the firm. This leads to a larger effect of process intensity on compensation for all levels of process intensity.

We verified these two main effects in the data using measures of executive and skilled labor pay. Our main specifications showed that a one standard deviation increase in process intangibility is associated with an 3% increase in the fraction of executive pay deferred and a 10% increase in skilled labor pay. When physical investment is high (i.e., the hold up problem is serious), these numbers increase.

We focused on the cross-sectional implications of process focused intangibles in this paper. There is also an intriguing aggregate pattern. Bena, Ortiz-Molina and Simintzi (2022) document a substantial increase in the process patent claims from 1975 to 1997. Executive equity pay also increases in recent decades (see eg. Eisfeldt et al. (2023)). Meanwhile, aggregate investment has declined around 3% in the last two decades (see Gutierrez and Philippon (2017) and Crouzet and Eberly (2018)). We leave more in depth study in matching the aforementioned aggregate patterns to the future.

We have taken the level of process intensity as given. However, even if changing this ratio is costly, over the medium to long-term we expect it to be endogenous. Studying this choice is left for future work. We have also not considered the asset pricing implications of process intensity and agency. Adding a stochastic discount factor as in Kogan and Papanikolaou (2014) to the model would provide further interesting testable implications.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. 2022. "Income and wealth distribution in macroeconomics: a continuous-time approach."

Ai, Hengjie, Dana Kiku, and Rui Li. 2023. "A Quantitative Model of Dynamic Moral Hazard." *Review of Financial Studies*, 36(4): 14081463.

Angenendt, David T. 2018. "Easy to Keep, But Hard to Find: How Patentable Inventions are Being Kept Secret." *Unpublished manuscript, University of Bologna*.

Bai, Hang, Erica XN Li, Chen Xue, and Lu Zhang. 2022. "Asymmetric investment rates." National Bureau of Economic Research.

Belo, Frederico, Jun Li, Xiaoji Lin, and Xiaofei Zhao. 2017. "Labor-force heterogeneity and asset prices: The importance of skilled labor." *Review of Financial Studies*, 30(10): 3669–3709.

Belo, Frederico, Vito D Gala, Juliana Salomao, and Maria Ana Vitorino. 2022. "Decomposing firm value." *Journal of Financial Economics*, 143(2): 619–639.

Bena, Jan, and Elena Simintzi. 2019. "Machines could not compete with Chinese labor: Evidence from US firms innovation." *Available at SSRN 2613248*.

Bena, Jan, Hernan Ortiz-Molina, and Elena Simintzi. 2022. "Shielding firm value: Employment protection and process innovation." *Journal of Financial Economics*, 146(2): 637–664.

Bhandari, Anmol, and Ellen R McGrattan. 2021. "Sweat equity in US private business." *Quarterly Journal of Economics*, 136(2): 727–781.

Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet. 2007. "Dynamic security design: Convergence to continuous time and asset pricing implications." *Review of Economic Studies*, 74(2): 345–390.

Brunnermeier, Markus K., and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *American Economic Review*, 104(2): 379–421.

Chen, Yuchen, Xuelin Li, Richard T Thakor, and Colin Ward. 2023. "Appropriated growth." *Available at SSRN 4244951*.

Crouzet, Nicolas, and Janice Eberly. 2018. "Intangibles, investment, and efficiency." *AEA Papers and Proceedings*, 108: 426–31.

Crouzet, Nicolas, Janice C Eberly, Andrea L Eisfeldt, and Dimitris Papanikolaou. 2022. "The economics of intangible capital." *Journal of Economic Perspectives*, 36(3): 29–52.

DeMarzo, Peter M., and Michael J. Fishman. 2007*a*. "Agency and optimal investment dynamics." *Review of Financial Studies*, 20(1): 151–188.

DeMarzo, Peter M., and Michael J. Fishman. 2007*b*. "Optimal long-term financial contracting." *Review of Financial Studies*, 20(6): 2079–2128.

DeMarzo, Peter M., and Yuliy Sannikov. 2006. "Optimal security design and dynamic capital structure in a continuous-time agency model." *Journal of Finance*, 61(6): 2681–2724.

DeMarzo, Peter M., Michael J. Fishman, Zhiguo He, and Neng Wang. 2012. “Dynamic agency and the q theory of investment.” *Journal of Finance*, 67(6): 2295–2340.

Edmans, Alex, Xavier Gabaix, and Dirk Jenter. 2017. “Executive compensation: A survey of theory and evidence.” *The handbook of the economics of corporate governance*, 1: 383–539.

Eisfeldt, Andrea, Antonio Falato, Dongryeol Lee, and Mindy Z. Xiaolan. 2023. “Equity pay inequality.”

Eisfeldt, Andrea L., and Dimitris Papanikolaou. 2013. “Organization capital and the cross-section of expected returns.” *Journal of Finance*, 68(4): 1365–1406.

Eisfeldt, Andrea L, Antonio Falato, and Mindy Z Xiaolan. 2021. “Human capitalists.” National Bureau of Economic Research.

Eisfeldt, Andrea L., Edward Kim, and Dimitris Papanikolaou. 2020. “Intangible value.” National Bureau of Economic Research.

Ewens, Michael, Ryan H. Peters, and Sean Wang. 2019. “Measuring intangible capital with market prices.” National Bureau of Economic Research.

Fama, Eugene F, and Kenneth R French. 2015. “A five-factor asset pricing model.” *Journal of Financial Economics*, 116(1): 1–22.

Frésard, Laurent, Loriano Mancini, Enrique J Schroth, and Davide Sinno. 2023. “How Do Firms Choose Between Growth and Efficiency?” *Available at SSRN 4383725*.

Ganglmair, Bernhard, W Keith Robinson, and Michael Seeligson. 2022. “The rise of process claims: Evidence from a century of US patents.” *ZEW-Centre for European Economic Research Discussion Paper*, , (22-011).

Grabner, Isabella. 2014. “Incentive system design in creativity-dependent firms.” *The Accounting Review*, 89(5): 1729–1750.

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell. 1997. “Long-run implications of investment-specific technological change.” *The American economic review*, 342–362.

Gutierrez, German, and Thomas Philippon. 2017. “Investment-less growth: An Empirical Investigation.” *Brookings Papers on Economic Analysis*, 89–189.

Hall, Brian J., and Jeffrey B. Liebman. 1998. “Are CEOs really paid like bureaucrats?” *The Quarterly Journal of Economics*, 113(3): 653–691.

He, Zhiguo. 2009. "Optimal executive compensation when firm size follows geometric brownian motion." *The Review of Financial Studies*, 22(2): 859–892.

Holmstrom, Bengt, and Paul Milgrom. 1991. "Multitask principal–agent analyses: Incentive contracts, asset ownership, and job design." *The Journal of Law, Economics, and Organization*, 7(special_issue): 24–52.

Hopenhayn, Hugo A. 1992. "Entry, exit, and firm dynamics in long run equilibrium." *Econometrica: Journal of the Econometric Society*, 1127–1150.

İmrohoroğlu, Ayşe, and Şelale Tüzel. 2014. "Firm-level productivity, risk, and return." *Management Science*, 60(8): 2073–2090.

Jorgenson, Dale W. 2005. "Accounting for growth in the information age." *Handbook of economic growth*, 1: 743–815.

Kazemi, Maziar M. 2022. "Intangible Investment, Displacement Risk, and the Value Discount."

Kline, Patrick, Neviana Petkova, Heidi Williams, and Owen Zidar. 2019. "Who profits from patents? rent-sharing at innovative firms." *Quarterly Journal of Economics*, 134(3): 1343–1404.

Kogan, Leonid, and Dimitris Papanikolaou. 2014. "Growth opportunities, technology shocks, and asset prices." *Journal of Finance*, 69(2): 675–718.

Kogan, Leonid, Dimitris Papanikolaou, Lawrence DW Schmidt, and Jae Song. 2020. "Technological innovation and labor income risk." National Bureau of Economic Research.

Kung, Howard, and Lukas Schmid. 2015. "Innovation, growth, and asset prices." *Journal of Finance*, 70(3): 1001–1037.

Kushner, Harold J., and Paul Dupuis. 2001. *Numerical methods for stochastic control problems in continuous time.* Vol. 24 of *Applications of Mathematics (New York)*. Second ed., Springer-Verlag, New York. Stochastic Modelling and Applied Probability.

Lerner, Josh, and Julie Wulf. 2007. "Innovation and incentives: Evidence from corporate R&D." *Review of Economics and Statistics*, 89(4): 634–644.

Lev, Baruch. 2000. *Intangibles: Management, measurement, and reporting*. Brookings institution press.

Lev, Baruch, and Suresh Radhakrishnan. 2005. "The valuation of organization capital." In *Measuring capital in the new economy*. 73–110. University of Chicago Press.

Lin, Xiaoji. 2012. "Endogenous technological progress and the cross-section of stock returns." *Journal of Financial Economics*, 103(2): 411–427.

Liu, Leo, Elvira Sojli, and Wing Wah Tham. 2022. "Advanced Manufacturing, Product Innovations, Productivity and Growth."

Lustig, Hanno, Chad Syverson, and Stijn Van Nieuwerburgh. 2011. "Technological change and the growing inequality in managerial compensation." *Journal of Financial Economics*, 99(3): 601–627.

Mohnen, Pierre, and Bronwyn H. Hall. 2013. "Innovation and productivity: An update." *Eurasian Business Review*, 3(1): 47–65.

Murphy, Kevin. 1999. "Executive compensation." In *Handbook of Labor Economics*. , ed. O. Ashenfelter and D. Card, Chapter 38, 2485–2563. Elsevier Science North, Holland.

OECD. 2005. "Oslo manual." *Paris and Luxembourg: OECD/Euro-stat*.

Olley, G Steven, and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica*, 64(6): 1263–1297.

Pan, Xiaojun, and Shoude Li. 2016. "Dynamic optimal control of process–product innovation with learning by doing." *European Journal of Operational Research*, 248(1): 136–145.

Parisi, Maria Laura, Fabio Schiantarelli, and Alessandro Sembenelli. 2006. "Productivity, innovation and R&D: Micro evidence for Italy." *European Economic Review*, 50(8): 2037–2061.

Park, Hyuna. 2019. "An intangible-adjusted book-to-market ratio still predicts stock returns." *Critical Finance Review*, 25(1): 207–236.

Peters, Ryan H, and Lucian A Taylor. 2017. "Intangible capital and the investment-q relation." *Journal of Financial Economics*, 123(2): 251–272.

Sannikov, Yuliy. 2008. "A continuous-time version of the principal-agent problem." *Review of Economic Studies*, 75(3): 957–984.

Song, Jae, David J Price, Fatih Guvenen, Nicholas Bloom, and Till Von Wachter. 2019. "Firming up inequality." *Quarterly journal of Economics*, 134(1): 1–50.

Sun, Qi, and Mindy Z Xiaolan. 2019. "Financing intangible capital." *Journal of Financial Economics*, 133(3): 564–588.

Ward, Colin. 2022. "Agency in intangibles." *Available at SSRN 3242478*.

Appendix

A Tables

Table 2: Model Parameters

Symbol	Variable	Value	Reference
a	Weight of the physical investment in the physical capital accumulation	0.85	Calibrated from data
A	Scale parameter in the physical capital accumulation	0.5	Calibrated from data
θ	Percentage of intangibles used in the process innovation	0.33	Estimated from the average firm-level process intensity
ρ	CES parameter in the physical capital accumulation	0.55	Calibrated from data
ϕ	Weight of the intangible capital used in the production	0.4	Calibrated from data
ψ	CES parameter in the production	-0.5	Eisfeldt, Falato and Xiaolan (2021)
μ	Productivity rate	0.45	Ward (2022)
σ	Volatility of $\log K$	0.29	Estimated from standard deviation of annual changes in the log physical capital stock
δ_K	Physical capital deprecation rate	0.1	Ward (2022)
δ_O	Intangible capital deprecation rate	0.15	Ward (2022)
Q_K	Scale parameter of the physical investment adjustment cost	13	Belo et al. (2022)
Q_O	Scale parameter of the intangible investment adjustment cost	22	Belo et al. (2022)
γ	Agent impatient parameter	0.08	Ward (2022)
$\overline{\lambda}$	Agent shirking benefit parameter	0.007	Calibrated from data
μ^e	Mean parameter for the entry log-normal distribution	4	Calibrated from data
σ^e	Dispersion parameter for the entry log-normal distribution	1.5	Calibrated from data
r	Interest rate	0.06	
ℓ	Physical capital recovery rate	0.8	

This table shows the parameters used in our simulations. Citations are given for the parameters which are consistent with the existing literature.

Table 3: Quintile Statistics

			Intangible Quintiles				
		Variable	1	2	3	4	5
Data	Process Intensity	Intangibility O/K	0.517	1.252	1.952	3.054	4.076
	Full Sample	Physical Investment I/K	0.106	0.117	0.122	0.142	0.129
	Mean: 0.33, Std: 0.28	Compensation U/K	0.008	0.023	0.021	0.033	0.038
	Low Process Intensity	Intangibility O/K	0.537	1.042	1.403	2.402	2.952
	Bottom 30% Process Intensity	Physical Investment I/K	0.094	0.098	0.102	0.124	0.118
	Mean: 0.05, Std: 0.06	Compensation U/K	0.010	0.014	0.015	0.032	0.024
	High Process Intensity	Intangibility O/K	0.359	1.447	2.327	4.003	5.576
	Top 30% Process Intensity	Physical Investment I/K	0.102	0.121	0.132	0.144	0.139
	Mean: 0.67, Std: 0.20	Compensation U/K	0.007	0.027	0.035	0.053	0.043
Model	Process Intensity	Intangibility O/K	1.058	1.650	2.027	2.448	3.313
	$\theta = 0.33$	Physical Investment I/K	0.087	0.115	0.126	0.136	0.157
		Compensation U/K	0.017	0.024	0.028	0.032	0.039
	Low Process Intensity	Intangibility O/K	0.973	1.591	2.072	2.632	3.745
	$\theta = 0.05$	Physical Investment I/K	0.064	0.083	0.092	0.099	0.110
		Compensation U/K	0.012	0.018	0.021	0.025	0.031
	High Process Intensity	Intangibility O/K	1.106	1.630	1.940	2.291	3.049
	$\theta = 0.67$	Physical Investment I/K	0.102	0.134	0.143	0.153	0.176
		Compensation U/K	0.023	0.031	0.036	0.041	0.049

This table presents the physical investment and compensation for intangible quintiles in both data and the calibrated model. Intangibility quintiles are formed by O/K. Physical investment is defined as capital expenditures (Compustat item CAPX) divided by lagged physical capital (PPEGT). Compensation is total compensation (Execucomp item TDC1) divided by lagged physical capital. In the calibrated model, quintiles of O/K are formed using the stationary distribution of $O/\widehat{K}$ together with simulations in capital efficiency X. The physical investment is the conditional mean of I/K and the compensation is the conditional mean of U/K. Parameters are listed in Table 2.

Table 4: Investment and Process Intangibles

	Dependent variable: 100 × Physical Investment / Physical Capital			
	(1)	(2)	(3)	(4)
Process Intangibility	30.094***	26.224***		
	(2.659)	(1.926)		
Non-Process Intangibility	24.037***	22.884***		
	(2.286)	(2.226)		
Size		−0.062		−0.072
		(0.188)		(0.188)
iB/M Ratio		−2.359***		−2.365***
		(0.209)		(0.208)
Sales / Total Capital		0.967***		0.967***
		(0.223)		(0.222)
Process Intensity			3.175**	2.071**
			(1.238)	(0.920)
Total Intangibility			26.389***	24.139***
			(2.036)	(1.840)
Fixed effects	Industry + Date	Industry + Date	Industry + Date	Industry + Date
Observations	22,552	19,437	22,552	19,437
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between next period physical investment (scaled by physical capital) and process intangibility. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 5: Adjusted Efficiency Growth and Process Intangibles

	Dependent variable:			
	Adjusted TFP Growth (SD Units)			
	(1)	(2)	(3)	(4)
Process Intangibility	0.224***	0.374***		
	(0.073)	(0.080)		
Non-Process Intangibility	0.132***	0.325***		
	(0.050)	(0.060)		
Process Intangibility (Inv. = H)	0.909***	0.743***		
	(0.129)	(0.118)		
Non-Process Intangibility (Inv. = H)	0.671***	0.479***		
	(0.094)	(0.089)		
Process Intensity			0.034	0.011
			(0.033)	(0.033)
Total Intangibility			0.165***	0.343***
			(0.047)	(0.058)
Process Intensity (Inv. = H)			0.165***	0.167***
			(0.057)	(0.053)
Total Intangibility (Inv. = H)			0.756***	0.577***
			(0.092)	(0.086)
Fixed effects	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin
Controls	N	Y	N	Y
Observations	17,640	15,711	17,640	15,711
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between adjusted efficiency growth and process intangibility. The first two columns use process intangibility and non-process intangibility as the key variables. The term (Inv. = H) refers to a dummy variable equal to one when a firm has an above median investment rate in a given year. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 6: Adjusted Efficiency Growth (Residualized) and Process Intangibles

	Dependent variable:			
	Adjusted TFP Growth (Residualized, SD Units)			
	(1)	(2)	(3)	(4)
Process Intangibility	0.356***	0.463***		
	(0.091)	(0.095)		
Non-Process Intangibility	0.169***	0.314***		
	(0.056)	(0.062)		
Process Intangibility (Inv. = H)	0.679***	0.656***		
	(0.143)	(0.142)		
Non-Process Intangibility (Inv. = H)	0.618***	0.568***		
	(0.106)	(0.105)		
Process Intensity			0.068*	0.053
			(0.039)	(0.040)
Total Intangibility			0.240***	0.371***
			(0.054)	(0.061)
Process Intensity (Inv. = H)			0.089	0.098
			(0.064)	(0.063)
Total Intangibility (Inv. = H)			0.635***	0.596***
			(0.102)	(0.101)
Fixed effects	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin
Controls	N	Y	N	Y
Observations	14,366	14,179	14,366	14,179
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between adjusted efficiency growth (residualized with respect to intangible capital) and process intangibility. The first two columns use process intangibility and non-process intangibility as the key variables. The term (Inv. = H) refers to a dummy variable equal to one when a firm has an above median investment rate in a given year. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 7: Total Executive Compensation and Process Intangibility

	Dependent variable:			
	100 × Total Compensation / Total Capital			
	(1)	(2)	(3)	(4)
Process Intangibility	2.198***	2.135***		
	(0.208)	(0.178)		
Non-Process Intangibility	1.628***	1.555***		
	(0.167)	(0.151)		
Size		−0.365***		−0.366***
		(0.026)		(0.027)
iB/M Ratio		−0.440***		−0.441***
		(0.044)		(0.044)
Sales / Total Capital		0.100***		0.100***
		(0.021)		(0.021)
Process Intensity			0.245**	0.306***
			(0.116)	(0.110)
Total Intangibility			1.857***	1.782***
			(0.144)	(0.125)
Fixed effects	Industry + Date	Industry + Date	Industry + Date	Industry + Date
Observations	11,995	11,074	11,995	11,074
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between total executive compensation divided by total capital and process intangibility. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 8: Deferred Executive Compensation and Process Intangibility

	Dependent variable:			
	100 × Deferred Compensation / Total Capital			
	(1)	(2)	(3)	(4)
Process Intangibility	0.697***	0.674***		
	(0.096)	(0.088)		
Non-Process Intangibility	0.497***	0.509***		
	(0.067)	(0.069)		
Size		−0.088***		−0.088***
		(0.008)		(0.008)
iB/M Ratio		−0.094***		−0.095***
		(0.010)		(0.010)
Sales / Total Capital		0.019***		0.019***
		(0.005)		(0.005)
Process Intensity			0.088**	0.086**
			(0.037)	(0.034)
Total Intangibility			0.577***	0.573***
			(0.065)	(0.064)
Fixed effects	Industry + Date	Industry + Date	Industry + Date	Industry + Date
Observations	11,995	11,074	11,995	11,074
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between the deferred executive compensation divided by total capital and process intangibility. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 9: Fraction of Executive Compensation Deferred and Process Intangibility

	Dependent variable:			
	100 × Fraction Compensation Deferred			
	(1)	(2)	(3)	(4)
Process Intangibility	10.882***	13.829***		
	(1.840)	(1.855)		
Non-Process Intangibility	4.666***	8.339***		
	(1.714)	(1.729)		
Size		2.969***		2.957***
		(0.269)		(0.268)
iB/M Ratio		0.235		0.222
		(0.225)		(0.225)
Sales / Total Capital		−0.265		−0.266
		(0.224)		(0.224)
Process Intensity			4.093***	3.261***
			(0.985)	(0.955)
Total Intangibility			7.039***	10.457***
			(1.514)	(1.519)
Fixed effects	Industry + Date	Industry + Date	Industry + Date	Industry + Date
Observations	11,870	10,972	11,870	10,972
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between the fraction executive compensation deferred and process intangibility. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 10: Skilled Labor Salaries and Process Intangibility

	Dependent variable:			
	100 × Relative Skilled Wage			
	(1)	(2)	(3)	(4)
Process Intangibility	49.741***	47.509***		
	(10.355)	(10.887)		
Non-Process Intangibility	33.749***	27.000***		
	(9.847)	(10.356)		
Size		3.501***		3.401***
		(1.094)		(1.107)
iB/M Ratio		4.672		4.542
		(2.887)		(2.879)
Sales / Total Capital		−2.083***		−2.106***
		(0.678)		(0.679)
Process Intensity			11.425*	13.462*
			(6.873)	(7.336)
Total Intangibility			39.779***	34.814***
			(8.096)	(8.365)
Fixed effects	Industry + Date	Industry + Date	Industry + Date	Industry + Date
Observations	1,908	1,784	1,908	1,784
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between innovative skilled labor's salaries and process intangibility. Skilled labor salaries are measured relative to the annual average skilled wage. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 11: Total Executive Compensation, Investment, and Process Intangibility

	Dependent variable:			
	100 × Total Compensation / Total Capital			
	(1)	(2)	(3)	(4)
Process Intangibility	1.224***	1.117***		
	(0.211)	(0.146)		
Non-Process Intangibility	0.958***	0.980***		
	(0.148)	(0.138)		
Process Intangibility (Inv. = H)	1.309***	1.396***		
	(0.257)	(0.259)		
Non-Process Intangibility (Inv. = H)	0.850***	0.675***		
	(0.220)	(0.205)		
Process Intensity			0.045	0.012
			(0.099)	(0.088)
Total Intangibility			1.070***	1.364***
			(0.128)	(0.130)
Process Intensity (Inv. = H)			0.333**	0.386**
			(0.155)	(0.169)
Total Intangibility (Inv. = H)			1.024***	0.930***
			(0.181)	(0.186)
Fixed effects	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin
Controls	N	Y	N	Y
Observations	11,934	11,019	11,934	11,019
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between total executive compensation divided by total capital and process intangibility. The term (Inv. = H) indicates a dummy variable equal to one when the firm's investment rate is above the annual median. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 12: Deferred Executive Compensation, Investment, and Process Intangibility

	Dependent variable:			
	100 × Deferred Compensation / Total Capital			
	(1)	(2)	(3)	(4)
Process Intangibility	0.326***	0.238***		
	(0.101)	(0.072)		
Non-Process Intangibility	0.298***	0.303***		
	(0.057)	(0.060)		
Process Intangibility (Inv. = H)	0.583***	0.701***		
	(0.111)	(0.123)		
Non-Process Intangibility (Inv. = H)	0.298***	0.306***		
	(0.084)	(0.089)		
Process Intensity			−0.00003	−0.014
			(0.036)	(0.027)
Total Intangibility			0.310***	0.275***
			(0.057)	(0.057)
Process Intensity (Inv. = H)			0.161***	0.189***
			(0.053)	(0.061)
Total Intangibility (Inv. = H)			0.411***	0.464***
			(0.075)	(0.078)
Fixed effects	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin
Controls	N	Y	N	Y
Observations	11,934	11,019	11,934	11,019
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between deferred executive compensation divided by total capital and process intangibility. The term (Inv. = H) indicates a dummy variable equal to one when the firm's investment rate is above the annual median. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 13: Fraction of Executive Compensation Deferred, Investment, and Process Intangibility

	Dependent variable:			
	100 × Fraction Compensation Deferred			
	(1)	(2)	(3)	(4)
Process Intangibility	4.904**	8.700***		
	(2.457)	(2.490)		
Non-Process Intangibility	2.178	6.026***		
	(2.255)	(2.252)		
Process Intangibility (Inv. = H)	9.174***	8.741***		
	(2.931)	(3.044)		
Non-Process Intangibility (Inv. = H)	3.461	3.890		
	(2.612)	(2.657)		
Process Intensity			2.055*	1.755
			(1.183)	(1.150)
Total Intangibility			3.232	7.082***
			(2.011)	(2.024)
Process Intensity (Inv. = H)			3.823**	2.926*
			(1.610)	(1.631)
Total Intangibility (Inv. = H)			5.571**	5.688**
			(2.322)	(2.363)
Fixed effects	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin
Controls	N	Y	N	Y
Observations	11,811	10,919	11,811	10,919
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between the fraction of executive compensation deferred and process intangibility. The term (Inv. = H) indicates a dummy variable equal to one when the firm's investment rate is above the annual median. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Table 14: Relative Skilled Labor Salaries, Investment, and Process Intangibility

	Dependent variable:			
	100 × Relative Skilled Wage			
	(1)	(2)	(3)	(4)
Process Intangibility	42.571***	38.764**		
	(15.549)	(15.509)		
Non-Process Intangibility	36.759***	29.797**		
	(13.795)	(14.603)		
Process Intangibility (Inv. = H)	12.075	14.771		
	(18.408)	(19.213)		
Non-Process Intangibility (Inv. = H)	−5.127	−5.131		
	(16.478)	(16.999)		
Process Intensity			10.089	11.915
			(10.176)	(10.786)
Total Intangibility			38.507***	32.818***
			(10.430)	(10.493)
Process Intensity (Inv. = H)			2.859	3.151
			(12.596)	(13.250)
Total Intangibility (Inv. = H)			2.525	3.529
			(12.667)	(12.735)
Fixed effects	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin	Industry + Date + Bin
Controls	N	Y	N	Y
Observations	1,907	1,783	1,907	1,783
Note:				*p<0.1; **p<0.05; ***p<0.01

This table shows the relationship innovative skilled labor salaries and process intangibility. The term (Inv. = H) indicates a dummy variable equal to one when the firm's investment rate is above the annual median. Skilled labor salaries a normalized by the annual average skilled salary. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

Internet Appendix
"Process Intangibles and Agency Conflicts"

This internet appendix provides supplemental materials for the paper. Section IA.1 provides details of our data set and variable constructions. Section IA.2 presents a robustness result to show that our compensation result is not driven by executives self-select into process intense firms to receive higher compensation. Section IA.3 presents all proofs for the paper. Section IA.4 provides details of numeric algorithm and model calibration procedure. Section IA.5 examines the first best benchmark. Finally Section IA.6 presents three example of process focused patents which enhance the operational or production efficiency in firms.

IA.1 Data

This section describes our data sources and how we construct our final data set. We discuss CRSP/Compustat, Execucomp, and the Bena and Simintzi (2019) data.

IA.1.1 CRSP and Compustat

We begin by describing our data preparation procedure for CRSP/Compustat. These data sets give information on the firm balance sheet and income statement variables. The key variables we will construct from CRSP/Compustat are the physical investment rate and the two capital stocks.[47] We will also construct a number of variables commonly used in the finance literature as controls in our regressions.

We employ a number of standard filters on our data. First, we only retain firms traded on AMEX, NASDAQ, or NYSE stock exchanges. Second, following Fama and French (2015), we drop the first two years a firm appears in the data.[48] Third, we drop firms in the Transportation, Finance, and Public industries. Fourth, we drop micro-cap firms as defined by Fama and French (2015).[49]

[47] The intangible investment rate is necessary for constructing intangible capital. However, we do not focus on intangible investment in this paper.

[48] We drop entirely firms that do not have beyond two full years of data.

[49] Micro-caps are defined as firms whose market capitalization is less than the market capitalization of the 20th percentile NYSE firm's size.

Following Belo et al. (2017) we define industries by 2-digit SIC codes before 2002 and NAICS code after.

We describe the construction of our intangible capital stock and investment variables in the next subsection. We will describe the other CRSP/Compustat variables as we use them, since they are more standard.

IA.1.2 Definition of Intangible Capital

Internally generated intangible capital stocks and their associated investment rates are not reported on firm balance sheets, so we must construct these variables ourselves. To do so, we follow Peters and Taylor (2017). First, if any of the following Compustat variables are NAs, we set the values to 0: xrd (R&D), xsga (Selling, General, and Administrative), rdip (R&D in progress), cogs (Costs of Goods Sold). Second, we construct a variable called SGA.

SGA is defined as follows. If R&D is greater than Selling, General, and Administrative expenses and R&D is less than Costs of Goods Sold, then we set SGA equal to Selling, General, and Administrative expenses. Otherwise, we set SGA equal to Selling, General, and Administrative expenses minus the sum of R&D and R&D in progress.[50]

The third and final part of the Peters and Taylor (2017) method uses the perpetual inventory method to construct the "Knowledge Capital" (K_{Know}) and "Organization Capital" (K_{Org}) stocks.

$$K_{Know,ft} = (1 - \delta_{Know})K_{Know,f,t-1} + \frac{R\&D_{ft}}{CPI_t} \tag{IA.1}$$

$$K_{Org,ft} = (1 - \delta_{Org})K_{Org,f,t-1} + (0.3)\frac{SGA_{ft}}{CPI_t} \tag{IA.2}$$

where CPI_t is the consumer price index.[51] We follow Ewens, Peters and Wang (2019) when we select δ_{Know} and δ_{Org}. Ewens, Peters and Wang (2019) show that there is heterogeneity in these parameters across industries.[52] We use their estimates from their pooled estimation, leading to $\delta_{Know} = 0.28$ and $\delta_{Org} = 0.3$.

We define intangible capital as the sum of Knowledge Capital and Organization Cap-

[50]Our results are similar using the Eisfeldt, Kim and Papanikolaou (2020) method of construction. Results are available upon request.

[51]The CPI is gathered from the Bureau of Economic Analysis.

[52]For example, their estimates of δ_{Know} range from 0.18 to 0.31.

ital, $K_{Int} = K_{Know} + K_{Org}$.[53] It follows from our definition of intangible capital that we construct intangible investment as $R\&D_{ft} + SGA_{ft}$.

IA.1.3 Execucomp

We use Execucomp to calculate the compensation to top executives at a firm.[54] Our main measure of compensation from Execucomp is total compensation (data item: TDC1). This total compensation measure includes salary, bonus, long-term incentive plans, option awards, and stock awards.

In order to capture a more direct measure of continuation utility (the variable U in the model), we also look at deferred compensation. FASB Statement NO. 123 (revised 2004), "... requires a public entity to measure the cost of employee services received in exchange for an award of equity instruments based on the grant-date fair value of the award."[55] We use this fair value of equity based compensation (e.g., stocks and options) as a measure of future promise utility. We also use the fraction of deferred compensation in the total compensation as another measure of promised utility. This latter measure is our preferred one.

IA.1.4 Process Claims Data

Our data for process claims comes from the data set compiled by Bena and Simintzi (2019).[56] The authors collect data from the U.S. Patent and Trademark Office (USPTO) up to 2021. They parse the structured-text of each patent to identify the claims section of the patent. Patent claims delineate the scope of the patent in the eyes of the law. To that end, they are important and precisely written. For example, the outcomes of patent infringement lawsuits frequently depend on these claims. Within the claims section of the patent, the authors then classify each claim as being either process or product oriented.

Though definitions are subjective, the existing literature (Bena and Simintzi (2019), Ganglmair, Robinson and Seeligson (2022)) generally defines process innovations as those that improve firm productivity/production methods or reduce costs, meanwhile

[53] If either K_{Know} or K_{Org} is less than 0, we set K_{Int} to zero.

[54] Execucomp usually includes the compensation for the top five executives at the firm. Sometimes the compensation for the top nine is included.

[55] Link to statement.

[56] We refer the reader to that paper's Internet Appendix for further details not discussed here.

product innovations introduce new products. Appendix IA.6 provides several examples of process focused patents which enhance firm's operational or production efficiency.

Within each-firm year, we compute the total number of process claims across all patents and divide that sum by the total number of claims, processes, and products. This measure aggregates information from all the patents filed by the firm that year. This measure is similar to that used by Bena and Simintzi (2019). Note that in the model process intensity, θ, is a parameter. Our measure of process intensity in the data is allowed to vary by firm-year. However, most of the variation in process intensity can be captured by a firm-level fixed effect.[57] Thus, our measures do a good job of sorting firms into different, relatively invariant, groups, which is in line with our theory.

By using patent data to construct the process intensity of the firm, we are assuming that this patent-level measure is a good proxy for the overall-firm level measure. We use the patent data because no firm-level measure of process intensity exists. If, for example, firm-level process intensity, p^f, is:

$$p^f = \beta p^p + e$$

where $\beta > 1$, p^p is the patent level measure, and e is noise, then we have classic errors-in-variables on the right-hand side. This will not lead to problems in inference, since we are interested in cross-firm comparisons.

Throughout, we drop firms with no claims of any kind (i.e., no patents). This is implicit in our measures of process intensity which are defined as the number of process claims over total claims.

IA.1.5 Summary Statistics

Tables IA.1 and IA.2 displays summary statistics. We allocate firms to different portfolios based on their process intangibility, and the averages of select variables are computed for each portfolio. The firms are assigned to a portfolio each year.

Table IA.1 shows firm balance sheet and income statement variables. The first column lists the portfolio, where a higher portfolio number indicates a larger average process intangibility. The second column lists the portfolio's average process intensity. As expected, this variable is increasing in process intangibility portfolio. Note also that the range of process intensity is non-trivial. For example, firms in the bottom portfolio are

[57] 50% of the variation in process intensity is captured by firm-fixed effects. Adding a full set of controls, including industry fixed effects, increases the R^2 of the regression by only 8%.

essentially entirely product focused. The third column shows intangibility. Again, this variable is generally increasing in process intangibility. The fourth columns shows the iB/M ratio. This acronym stands for the book-to-market ratio with intangible capital added into the book equity of the firm (Park (2019), Kazemi (2022)). The fifth column shows sales per total capital (intangible plus physical). Interestingly, sales per capital are decreasing in process intangibility. Given that high process intense firms are incentivized to invest in physical capital, we can think of these firms as plowing back earnings into the firm to build capital for the future, instead of focusing on current output. Relatedly, column six shows that physical investment is increasing in process intangibility.

Table IA.2 shows compensation and salary variables for the same portfolios.The numbers are multiplied by 100, so that they can be interpreted as percentages. The second column shows total executive compensation divided by total capital. The third column shows deferred compensation divided by total capital. The fourth column shows the fraction of compensation deferred. The fifth column shows the ratio of skilled wages to the annual average skilled wage. Broadly, all of our compensation measures are increasing in process intangibility.

Table IA.1: Summary Statistics by Process Intensity: Firm Characteristics

Bin	Process Intensity	Intangibility	iB/M Ratio	Sales / Cap.	Phys. Inv.
1	0.006	0.505	1.490	1.184	0.090
2	0.105	0.462	1.837	1.156	0.102
3	0.220	0.539	1.987	1.060	0.115
4	0.372	0.644	2.123	0.901	0.136
5	0.632	0.753	1.810	0.786	0.146

This table shows means of select variables by process intangibility level. Each year firms are sorted into five equally spaced portfolios (bins) based on their values of process intangibility. Bins are re-balanced each year. Time-series averages of the bin-year medians of variables displayed as column titles are computed for each bin. Intangibility is the intangible capital stock divided by total capital. The iB/M ratio is the book-to-market ratio with intangible capital added to firm book equity. Sales / Cap. is sales per unit total capital. Physical investment is CAPX divided by physical capital.

Bena, Ortiz-Molina and Simintzi (2022) find that process innovation has increased for all industries since 1975. In general, they find that manufacturing industries are the largest players in process innovations. That is, they tend to have the highest process intensity and the highest fraction of process innovations across the economy. We refer to that paper for more details.

Finally, Table IA.3 shows that the ratio between the compensation volatility and physical capital volatility increases with firm intangibility. This empirical observation moti-

Table IA.2: Summary Statistics by Process Intensity: Compensation

Bin	Total Comp.	Def. Comp.	Frac. Deferred	(Relative) Skilled Wage
1	0.648	0.376	45.688	68.306
2	0.560	0.399	49.461	76.796
3	0.595	0.322	51.089	80.530
4	0.948	0.757	53.383	93.550
5	1.667	1.593	53.849	103.676

This table shows means of select variables by process intangibility level. Each year firms are sorted into five equally spaced portfolios (bins) based on their values of process intangibility. Bins are re-balanced each year. Time-series averages of the bin-year medians of variables displayed as column titles are computed for each bin. Total compensation is salary plus stock and option grants at the executive level divided by total capital. Deferred compensation is stock and option grans divided by total capital. The fraction of compensation deferred is deferred compensation divded by total compensation. The relative skilled wage is the average high skill wage (see discussion of BGT in the body of the paper) divided by the annual mean high skill wage. All numbers are multiplied by 100.

vates our modelling choice in (4.15).

Table IA.3: Ratio between the compensation volatility and the physical capital volatility

Intangible quintiles	1	2	3	4	5
Volatility ratio (%)	0.15	0.30	0.46	1.07	1.85

This table presents the ratio between the compensation volatility and the physical capital volatility conditional on intangibility. This ratio increases with intangibility, but is insensitive to process intensity in each intangibility quintile. Intangiblility quintiles are constructed using ratio between intangible capital and physical capital. The volatilites are measured over five year rolling windows at the firm level. Within each intangibility quintile-year, we compute the median level of the firm-level volatilities. Then, we compute the time-series average of these medians.

IA.2 Executive Level Results

In this Appendix, we ask if better executives simply self-select into process intense firms and hence receive higher compensation. To test this, we estimate executive-level regressions on the subset of executives who change firms in our sample. That is, we estimate:

$$\begin{aligned}\text{Compensation Measure}_{if,t+1} &= \alpha_j + \alpha_t + \beta_1 \text{Process Measure}_{ft} \\ &+ \beta_2 \text{Non-Process Measure}_{ft} + \boldsymbol{\beta} \boldsymbol{X}_{ft} + \varepsilon_{f,t+1}.\end{aligned} \tag{IA.1}$$

This equation looks similar to (7.1). The key difference is in the dependent variable, which is measured at the executive-firm-date $(if, t+1)$ level. In (7.1) we looked at firm-date level regressions.

Table IA.4 displays the results. Column one uses total executive compensation over total capital. The results here are in-line with our main results. The second column, using deferred compensation over total capital, differs somewhat from previous results. Here, non-process intangibility seems to have a slightly stronger association with compensation. Our final column contains the preferred measure, the fraction of compensation deferred. The effect of process intangibility is much stronger here. Thus, we find that this restricted sample contains similar implications to our baseline. It does not appear that executives self-select into "better" jobs in a way correlated with process intangibility.

Table IA.4: Compensation and Process Intangibility (Executives who Switch Firms)

	Dependent variable:		
	100 × Tot. Comp. / Tot. Cap.	100 × Def. Comp. / Tot. Cap.	100 × Def. Comp. / Tot. Comp.
	(1)	(2)	(3)
Process Intangibility	3.727***	1.567***	31.866***
	(0.670)	(0.417)	(7.961)
Non-Process Intangibility	2.696***	1.585***	6.301
	(0.587)	(0.382)	(7.426)
Size	−0.810***	−0.363***	4.727***
	(0.064)	(0.045)	(0.742)
iB/M	−0.843***	−0.395***	−2.017**
	(0.104)	(0.068)	(0.974)
Sales / Capital	0.197***	0.088***	0.429
	(0.065)	(0.029)	(0.579)
Fixed effects	Industry + Date	Industry + Date	Industry + Date
Observations	8,621	5,029	4,910

Note: *p<0.1; **p<0.05; ***p<0.01

This table shows the relationship between executive compensation and process intangibility, conditional on the executive switching firms at least once during our sample period... Skilled labor salaries a normalized by the annual average skilled salary. The first two columns use process intangibility and non-process intangibility as the key variables. The last two use process intensity and total intangibility. The control variables are the book-to-market ratio (iB/M) inclusive of intangibles, log market capitalization, and sales per capital. Industry and year fixed effects are also included. Industry is defined as two digit SIC code before 2002 and two digit NAICS code from 2002 onward (see e.g. Belo et al. (2017)). Standard errors are clustered at the firm level.

IA.3 Proofs for Lemma 4.1 and Proposition 4.1

Proof of Lemma 4.1

Consider a probability measure $\mathbb{P}$ under which

$$d\widehat{K}_t = \sigma \widehat{K}_t dZ_t$$

with a $\mathbb{P}$-Brownian motion Z. Introduce an equivalent probability measure $\mathbb{P}^e$ such that Z^e, defined via

$$dZ^e_t = dZ_t - \frac{D(e_t, I_t, O_t) - \delta_K \widehat{K}_t}{\sigma \widehat{K}_t},$$

is a Brownian motion under $\mathbb{P}^e$. Then K follows the dynamics (3.3).

Under $\mathbb{P}$, the agent's continuation value U in (3.11) has the semimartingale decomposition

$$\mathrm{d}U_t = \mathrm{d}H_t + \varphi_t \mathrm{d}\widehat{K}_t, \tag{IA.1}$$

where φ arises from the martingale representation theorem. We will use dynamic programming to determine the finite variation process H. To this end, it follows from (3.11) and the dynamic programming that $\widetilde{U}_t = e^{-\gamma t} U_t + \int_0^t e^{-\gamma s}(\lambda \Lambda_s (1 - e_s)\mathrm{d}s + \mathrm{d}C_s)$ is a super-martingale under $\mathbb{P}^e$ for arbitrary effort e and a martingale under $\mathbb{P}^{e*}$ for the optimal effort e^*. We obtain from Itô's formula that

$$\begin{aligned}\mathrm{d}\widetilde{U}_t = e^{-\gamma t}\Big\{ &- \gamma U_t \mathrm{d}t + \mathrm{d}H_t + \lambda \Lambda_t (1 - e_t)\mathrm{d}t + \mathrm{d}C_t \\ &+ \varphi_t \widehat{K}_t \big(d(e_t, \widehat{i}_t, \theta \widehat{o}_t) - \delta_K\big)\mathrm{d}t + \varphi_t \widehat{K}_t \sigma dZ^e_t \Big\},\end{aligned}$$

where $d(e_t, \widehat{i}_t, \theta \widehat{o}_t)$ is given in (3.6). The drift of $\widetilde{U}$ is nonpositive for an arbitrary effort e and is zero for the optimal effort e^*. Therefore,

$$dH_t = \big(\gamma U_t + \varphi_t \delta_K \widehat{K}_t\big)\mathrm{d}t - \mathrm{d}C_t - \max_{e \in \{0,1\}} \Big\{ \lambda \Lambda_t (1 - e) + \varphi_t \widehat{K}_t d(e, \widehat{i}_t, \theta \widehat{o}_t) \Big\} \mathrm{d}t. \tag{IA.2}$$

The optimal effort $e^*_t = 1$ if and only if

$$\varphi_t \widehat{K}_t d(1, \widehat{i}, \theta \widehat{o}) \geqslant \lambda_t \Lambda_t + \varphi_t \widehat{K}_t d(0, \widehat{i}, \theta \widehat{o}).$$

Recall the definition of Λ_t from (3.10), the previous incentive compatibility condition is equivalent to

$$\varphi_t \geqslant \lambda_t.$$

When the previous condition holds, $e_t^* = 1$ and we obtain from (IA.1) and (IA.2) that the dynamics of U_t follows (4.1).

Proof of Proposition 4.1

We drive the HJB equation (4.6) from the dynamic programming principle. To this end, it follows from the dynamic programming principle that $\widetilde{V}_t = e^{-rt}\widehat{K}_t v(\widehat{o}_t, \widehat{u}_t) + \int_0^t e^{-rs}(Y_s \mathrm{d}s - \mathrm{d}C_s)$ is a supermartingale under arbitrary strategy $(\widehat{i}, \widehat{s}, C)$ and a martingale under the optimal strategy. Using Itô's formula, together with (4.10) and (4.11), we calculate

$$\begin{aligned}
\mathrm{d}\big(\widehat{K}v(\widehat{o},\widehat{u})\big) = &\Big\{\widehat{K}v\, d(\widehat{i},\widehat{o}) - \delta_K \widehat{K}v + \widehat{K}\partial_{\widehat{o}}v\big[\widehat{s} - (\delta_O - \delta_K)\widehat{o} - \widehat{o}\, d(\widehat{i}, \theta\widehat{o}) + \sigma^2\widehat{o}\big] \\
&+ \widehat{K}\partial_{\widehat{u}}v\big[(\gamma + \delta_K)\widehat{u} - \widehat{u}\, d(\widehat{i},\widehat{o}) + \sigma^2(\widehat{u} - \varphi)\big] \\
&+ \frac{1}{2}\widehat{K}\widehat{o}^2\sigma^2\partial^2_{\widehat{o}\widehat{o}}v + \frac{1}{2}K\sigma^2(\varphi - \widehat{u})^2\partial^2_{\widehat{u}\widehat{u}}v - \widehat{K}\widehat{o}\sigma^2(\varphi - \widehat{u})\partial^2_{\widehat{o}\widehat{u}}v \\
&+ \widehat{K}\sigma^2\big[-\widehat{o}\partial_{\widehat{o}}v + (\varphi - \widehat{u})\partial_{\widehat{u}}v\big]\Big\}\mathrm{d}t \\
&+ \widehat{K}\sigma\big[v - \widehat{o}\partial_{\widehat{o}}v + (\varphi - \widehat{u})\partial_{\widehat{u}}v\big]\mathrm{d}Z^{e^*} - \partial_{\widehat{u}}v\mathrm{d}C.
\end{aligned}$$

The drift of $\widetilde{V}$, divided throughout by $e^{-\gamma t}\widehat{K}$, is

$$\begin{aligned}
&-rv + v\, d(\widehat{i},\widehat{o}) - \delta_K v + \partial_{\widehat{o}}v\big[\widehat{s} - (\delta_O - \delta_K)\widehat{o} - \widehat{o}\, d(\widehat{i},\theta\widehat{o}) + \sigma^2\widehat{o}\big] \\
&+ \partial_{\widehat{u}}v\big[(\gamma + \delta_K)\widehat{u} - \widehat{u}\, d(\widehat{i},\widehat{o}) + \sigma^2(\widehat{u} - \varphi)\big] \\
&+ \frac{1}{2}\widehat{o}^2\sigma^2\partial^2_{\widehat{o}\widehat{o}}v + \frac{1}{2}(\varphi - \widehat{u})^2\sigma^2\partial^2_{\widehat{u}\widehat{u}}v - \widehat{o}(\varphi - \widehat{u})\sigma^2\partial^2_{\widehat{o}\widehat{u}}v + \sigma^2\big[-\widehat{o}\partial_{\widehat{o}}v + (\varphi - \widehat{u})\partial_{\widehat{u}}v\big] \\
&+ \mu\big[1 - \phi + \phi\widehat{o}^{\psi}\big]^{1/\psi} - \widehat{i} - \widehat{s} - \tfrac{Q_K}{2}(\widehat{i} - \delta_K)^2 - \tfrac{Q_O}{2}(\widehat{i}/\widehat{o} - \delta_O)^2\widehat{o} + (\partial_{\widehat{u}}v + 1)\Big(-\tfrac{1}{\widehat{K}}\tfrac{\mathrm{d}C}{\mathrm{d}t}\Big).
\end{aligned}$$

Therefore the dynamic programming principle implies that the HJB equation satisfied by v is

$$
\begin{aligned}
(r+\delta_K)v = \max_{i,s,\varphi,C} \Big\{ & \big(v - \widehat{o}\,\partial_{\widehat{o}}v - \widehat{u}\,\partial_{\widehat{u}}v\big)d(\widehat{i},\theta\widehat{o}) + \big(\widehat{s} - (\delta_O - \delta_K)\widehat{o}\big)\partial_{\widehat{o}}v + (\gamma+\delta_K)\widehat{u}\,\partial_{\widehat{u}}v \\
& + \frac{1}{2}\widehat{o}^2\sigma^2\partial^2_{\widehat{o}\widehat{o}}v + \frac{1}{2}(\varphi-\widehat{u})^2\sigma^2\partial^2_{\widehat{u}\widehat{u}}v - \widehat{o}(\varphi-\widehat{u})\sigma^2\partial^2_{\widehat{o}\widehat{u}}v \\
& + \mu\big[1-\phi+\phi\widehat{o}^{\psi}\big]^{1/\psi} - \widehat{i} - \widehat{s} - \tfrac{Q_K}{2}(\widehat{i}-\delta_K)^2 - \tfrac{Q_O}{2}(\widehat{i}/\widehat{o}-\delta_O)^2\widehat{o} \\
& + (\partial_{\widehat{u}}v+1)\Big(-\tfrac{1}{\widehat{K}}\tfrac{\mathrm{d}C}{\mathrm{d}t}\Big)\Big\}.
\end{aligned}
\tag{IA.3}
$$

Because $\mathrm{d}C/\mathrm{d}t$ can be infinite, if $\partial_{\widehat{u}}v+1<0$, the right-hand side of the previous equation can be infinite by choosing infinite $\mathrm{d}C/\mathrm{d}t$. Therefore, the wellposedness of the HJB equation requires that $\partial_{\widehat{u}}v+1\geqslant 0$. As a result, the equation (IA.3) is transformed to (4.6). In order to incentivize the full effort $e^*=1$, the incentive compatibility condition restricts $\varphi\geqslant\lambda$.

Fokker-Planck-Kolmogorov equation (4.14)

Given the state dynamics in (4.10) and (4.11), the infinitesimal generator $\mathcal{L}_{\widehat{u},\widehat{o}}$ is

$$
\begin{aligned}
\mathcal{L}_{\widehat{u},\widehat{o}} = & \big(\widehat{s} - (\delta_O-\delta_K)\widehat{o} - \widehat{o}d(\widehat{i},\theta\widehat{o}) + \widehat{o}\sigma^2\big)\partial_{\widehat{o}} + \big((\gamma+\delta_K)\widehat{u} - \widehat{u}d(\widehat{i},\theta\widehat{o}) + \sigma^2(\widehat{u}-\varphi)\big)\partial_{\widehat{u}} \\
& + \frac{1}{2}\widehat{o}^2\partial^2_{\widehat{o}\widehat{o}} + \frac{1}{2}\sigma^2(\varphi-\widehat{u})\partial^2_{\widehat{u}\widehat{u}} - \sigma\widehat{o}(\varphi-\widehat{u})\partial^2_{\widehat{u}\widehat{o}}.
\end{aligned}
$$

Its adjoint operator $\mathcal{L}^*_{\widehat{u},\widehat{o}}$, used in (4.14), is defined as

$$
\begin{aligned}
\mathcal{L}^*_{\widehat{u},\widehat{o}}g = & -\partial_{\widehat{o}}\Big(\big(\widehat{s}^* - (\delta_O-\delta_K)\widehat{o} - \widehat{o}d(\widehat{i}^*,\theta\widehat{o}) + \widehat{o}\sigma^2\big)g\Big) \\
& - \partial_{\widehat{u}}\Big(\big((\gamma+\delta_K)\widehat{u} - \widehat{u}d(\widehat{i}^*,\theta\widehat{o}) + \sigma^2(\widehat{u}-\varphi^*)\big)g\Big) \\
& + \frac{1}{2}\partial^2_{\widehat{o}\widehat{o}}\Big(\widehat{o}^2 g\Big) + \frac{1}{2}\partial^2_{\widehat{u}\widehat{u}}\Big(\sigma^2((\varphi^*)^2-\widehat{u})g\Big) - \partial^2_{\widehat{u}\widehat{o}}\Big(\sigma\widehat{o}(\varphi^*-\widehat{u})g\Big)
\end{aligned}
\tag{IA.4}
$$

for a any smooth test function g, and $\widehat{i}^*, \widehat{s}^*, \varphi^*$ are optimal investment strategies and the optimal contract sensitivity. We refer reader to Achdou et al. (2022) for more discussion on Fokker-Planck-Kolmogorov equation used for economic problems

IA.4 Numeric algorithm and model calibration

IA.4.1 Numeric algorithm

In this section, we describe the numeric procedure to solve the HJB equation (4.6) and the Fokker-Planck-Kolmogorov equation (4.14). To simplify notation, we omit $\hat{\cdot}$ throughout this section.

For the HJB equation (4.6), we employ the penalty approach to transform (4.6) into

$$
\begin{aligned}
0 = &-(r+\delta_K)v + \max_{i,s,\varphi \geqslant \lambda} \Big\{ \big(v - o\,\partial_o v - u\,\partial_u v\big) d(i, \theta o) \\
&+ \big(s - (\delta_O - \delta_K) o\big)\partial_o v + (\gamma + \delta_K) u\,\partial_u v \\
&+ \frac{1}{2} o^2 \sigma^2 \partial^2_{oo} v + \frac{1}{2}(\varphi - u)^2 \sigma^2 \partial^2_{uu} v - o(\varphi - u)\sigma^2 \partial^2_{ou} v \\
&+ \mu \big[1 - \phi + \phi o^{\psi}\big]^{1/\psi} - i - s - \tfrac{Q_K}{2}(i - \delta_K)^2 - \tfrac{Q_O}{2}(s/o - \delta_O)^2 o \Big\} \\
&- \min_{P \in [0, P_{max}]} P\big[\partial_u v + 1\big],
\end{aligned} \tag{IA.1}
$$

where P_{max} is a large positive constant (10^7 in the implementation). In the previous equation, when $\partial_u v + 1 \geqslant 0$, the optimal P is zero and the penalty term $P[\partial_u v + 1] = 0$, hence the first four lines in (IA.1) sum to zero; when $\partial_u v + 1 < 0$, the optimal $P = P_{max}$, leading to a negative penalty term $P_{max}[\partial_u v + 1]$, hence the first four lines of (IA.1) sum to be negative, consistent with the requirement in (4.6) that the first group of term on the right-hand side is always nonpositive. In (IA.1), we also set $\varphi = \lambda$ in (4.15) to obtain

$$
\begin{aligned}
0 = &-(r+\delta_K)v + \max_{i,s} \Big\{ \big(v - o\,\partial_o v - u\,\partial_u v\big) d(i, \theta o) \\
&+ \big(s - (\delta_O - \delta_K) o\big)\partial_o v + (\gamma + \delta_K) u\,\partial_u v \\
&+ \frac{1}{2} o^2 \sigma^2 \partial^2_{oo} v + \frac{1}{2}(\lambda - u)^2 \sigma^2 \partial^2_{uu} v - o(\lambda - u)\sigma^2 \partial^2_{ou} v \\
&+ \mu \big[1 - \phi + \phi o^{\psi}\big]^{1/\psi} - i - s - \tfrac{Q_K}{2}(i - \delta_K)^2 - \tfrac{Q_O}{2}(s/o - \delta_O)^2 o \Big\} \\
&- \min_{P \in [0, P_{max}]} P\big[\partial_u v + 1\big].
\end{aligned} \tag{IA.2}
$$

After the numeric solution for the previous equation is obtained, we verify whether the condition (4.7) holds. This condition is satisfied in all our numeric experiments.

We choose a domain $[0, o_{max}] \times [0, u_{max}]$ with sufficiently large o_{max} and u_{max}. Equation (IA.2) is coupled with the following boundary conditions. When $u = 0$, v satisfies

(4.12). Hence the equation (4.13) is solved numerically before (IA.2) is solved. When $o = 0$, (4.10) shows that the drift of $\mathrm{d}o$ is non-negative and the volatility vanishes. Therefore, the boundary condition at $o = 0$ is not needed in an upwind numeric scheme. When $u = u_{max}$, we impose the Neumann boundary condition

$$\partial_u v(u_{max}, o) = -1.$$

When $o = o_{max}$, we impose a technical Neumann boundary condition

$$\partial_o v(u, o_{max}) = 0.$$

With aforementioned boundary conditions, our numeric experiments show that the function v in a fixed bounded domain is not sensitive to the choice of u_{max} and o_{max} when they are sufficiently large.

We apply policy iteration methods to solve (IA.2) (see (Kushner and Dupuis, 2001, Chapter 5 and 6)).

(i) Start with initial guess $i, s, P = 0$.

(ii) For given i, s, and P, solve (IA.2) with the fixed i, s, and P using the upwind finite difference scheme in the domain $[0, o_{max}] \times [0, u_{max}]$.

(iii) Using the obtained v and their derivatives $\partial_o v$ and $\partial_u v$ (approximated by finite difference) to solve the maximization problem in (IA.2). Update i, s, and P using the corresponding maximizer.

(iv) Go back to Step (ii), until the difference between v and its value in the previous iteration is less than some small error tolerance ϵ.

To solve the Fokker-Planck-Kolmogorov equation (4.14), we use the finite difference scheme in (Achdou et al., 2022, Section 5.2). To pin down the entry rate m, we use bisection search: when the integral of the density g on $[0, u_{max}] \times [0, o_{max}]$ less than 1, m is increased; when the integral of g is larger than 1, m is decreased. Iterate a bisection search for m until the integral of g is sufficiently close to 1.

IA.4.2 Model calibration

We first solve the HJB equation (4.6) and the Fokker-Planck-Kolmogorov equation (4.14) following the procedure outlined in the previous section. The policy functions for the optimal physical investment rate $\widehat{i} = I/\widehat{K}$ and the optimal intangible investment rate $\widehat{s} = S/\widehat{K}$ are obtained as functions of state variables $\widehat{o} = O/\widehat{K}$ and $\widehat{u} = U/\widehat{K}$. The compensation boundary $\overline{u}$ is also a function of $\widehat{o}$. Using these policy functions, we simulate $\widehat{o}$ and $\widehat{u}$ using (4.10) and (4.11), respectively, starting from the stationary distribution obtained from the Fokker-Planck-Kolmogorov equation. We also simulate the physical capital efficiency X using (3.5) starting from $X_0 = 1$. We simulate 10^5 paths of $(\widehat{o}, \widehat{u}, X)$, each for 10 years, and drop the first two years in each simulation to remove dependence on initial values. The model generated quantity W_t/K_t, $t \in [2, 10]$, is obtained via $X_t W_t/\widehat{K}_t$, where $W = O, I$, or U. We use these simulations to construct quintiles of O/K and the conditional means of I/K and U/K in each quintile. Several model parameters in Table 2 are chosen so that the model-generated conditional means of I/K and U/K are close to their empirical counterparts, estimated using the full sample. Using these calibrated parameters, but changing the process intensity θ to 0.05 and 0.67, we report the model-generated conditional means of I/K and U/K for the low and high process intensity cases in Table 3.

IA.5 First best benchmark

To compare with the main model, we study in this section the first best benchmark, where the investment is not subject to agency friction. The firm's problem is

$$V(\widehat{K}, O) = \max_{I,S} \mathbb{E}\Big[\int_0^\infty e^{-rs} Y_s \mathrm{d}s \Big| \widehat{K}_0 = \widehat{K}, O_0 = O\Big], \tag{IA.1}$$

subject to (3.8), (3.2), and (3.3) with $e = 1$.

The homothetic property in $\widehat{K}$ allows us to introduce the following decomposition of the value function:

$$V(\widehat{K}, O) = \widehat{K} v(\widehat{o}), \tag{IA.2}$$

where $\widehat{o} = O/\widehat{K}$.

Proposition IA.1 *The function v in (IA.2) satisfies the following HJB equation*

$$\begin{aligned}(r+\delta_K)v = \max_{\widehat{i},\widehat{s}} \Big\{ &(v - \widehat{o}\partial_{\widehat{o}} v) d(\widehat{i}, \theta\widehat{o}) + (\widehat{s} - (\delta_O - \delta_K)\widehat{o})\partial_{\widehat{o}} v + \frac{1}{2}\widehat{o}^2\sigma^2 \partial^2_{\widehat{o}\widehat{o}} v \\ &+ \mu\big[1 - \phi + \phi \widehat{o}^{\psi}\big]^{1/\psi} - \widehat{i} - \widehat{s} - \tfrac{Q_k}{2}(\widehat{i} - \delta_K)^2 - \tfrac{Q_O}{2}\big(\widehat{s}/\widehat{o} - \delta_O\big)^2 \widehat{o}\Big\},\end{aligned} \tag{IA.3}$$

where $d(\widehat{i}, \theta\widehat{o}) = \frac{A}{a^{1/\rho}}[a\widehat{i}^{\rho} + (1-a)(\theta\widehat{o})^{\rho}]^{1/\rho}$. The optimal $\widehat{i}^* > 0$ satisfies the first order condition

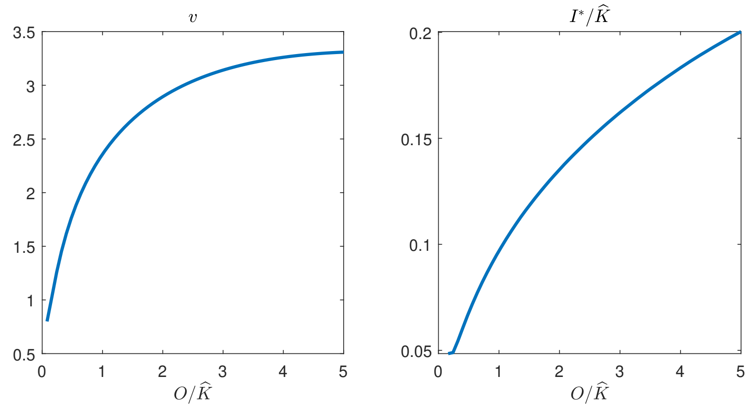
$$(v - \widehat{o}\partial_{\widehat{o}} v)\partial_{\widehat{i}} d(\widehat{i}^*, \theta\widehat{o}) = 1 + Q_K(\widehat{i}^* + \delta_K); \tag{IA.4}$$

The optimal $\widehat{s}^*$ satisfies

$$\widehat{s}^* = \widehat{o}\Big(\frac{\partial_{\widehat{o}} v - 1}{Q_o} + \delta_O\Big). \tag{IA.5}$$

Figure IA.1 provides the first best solution with the parameters in Table 2. As the efficiency-adjusted intangibility increases, investment in the physical capital becomes more efficient, hence both the principal's value and the physical capital investment rate increase. Comparing with the physical investment rate in the middle panel of Figure 3, the physical investment rate is slightly higher in the first best case without agency friction. Because the principal can dictate agent's effort in the first best case, the agent only receives his outside value as the compensation.

Figure IA.1: First best value and optimal physical investment



First best: value function v, optimal investment in the physical capital. The parameters are listed in Table 2.

Proof of Proposition IA.1

Recall the value function V in (4.4). It follows from the dynamic programming principle that $\widetilde{V}_t = e^{-rt}\widehat{K}v(\widehat{o}_t) + \int_0^t e^{-rs}(dY_s - dC_s)$ is a supermartingale for an arbitrage strategy $(\widehat{i}, \widehat{s})$ and is a martingale under the optimal strategy. Using (IA.2) and (4.10), we obtain from Itô's formula that

$$\begin{aligned} d\big(\widehat{K}v(\widehat{o})\big) =& \Big\{\widehat{K}v\, d(\widehat{i}, \theta\widehat{o}) - \delta_K\widehat{K}v + \widehat{K}\partial_{\widehat{o}}v\big[\widehat{s} - (\delta_O - \delta_K)\widehat{o} - \widehat{o}\, d(\widehat{i}, \theta\widehat{o})\big] + \frac{1}{2}K\widehat{o}^2\sigma^2\partial^2_{\widehat{o}\widehat{o}}v\Big\}\mathrm{d}t \\ &+ \widehat{K}\sigma\big(v - \widehat{o}\,\partial_{\widehat{o}}v\big)dZ_t. \end{aligned}$$

The drift of $\widetilde{V}$ (divided throughout by $e^{-rt}\widehat{K}$) is

$$\begin{aligned} &-rv + v\, d(\widehat{i}, \theta\widehat{o}) - \delta_K v + \partial_{\widehat{o}}v\big[\widehat{s} - (\delta_O - \delta_K)\widehat{o} - \widehat{o}\, d(\widehat{i}, \theta\widehat{o})\big] + \frac{1}{2}\widehat{o}^2\sigma^2\partial^2_{\widehat{o}\widehat{o}}v \\ &+ \mu\big[1 - \phi + \phi\widehat{o}^{\psi}\big]^{1/\psi} - \widehat{i} - \widehat{s} - \tfrac{Q_K}{2}(\widehat{i} - \delta_K)^2 - \tfrac{Q_O}{2}\big(\widehat{s}/\widehat{o} - \delta_O\big)^2\widehat{o}. \end{aligned}$$

Therefore the HJB equation (IA.3) follows from the fact that the drift of $\widetilde{V}$ is nonpositive for any $\widehat{i}, \widehat{s}$ and is zero for optimal $\widehat{i}^*$ and $\widehat{s}^*$. The first order conditions in $\widehat{i}^*$ and $\widehat{s}^*$ follow from the same argument as in Proposition 4.1.

IA.6 Examples of process innovation focused patents

This section provides three examples of process innovation-focused patents identified by Bena and Simintzi (2019). These processes or methods enhance the operational or production efficiency in firms.

U.S. Patent No.: 6609113.
Title: Method and system for processing internet payments using the electronic funds transfer network

Assignee: The Chase Manhattan Bank (J.P. Morgan), New York

"By combining these two trends – direct merchant to consumer distribution from independent 'intrapreneurs', and the ability to distribute products digitally – a new market place has emerged for low dollar, high volume, real-time payments with payment surety for both consumers and producers. [...] On-line merchants are currently facing a variety of problems including a low volume of on-line purchases relative to the number of site viewers; a high volume of charge-backs for on-line purchases; non-integrated 'patchwork' systems for payment processing; high fraud rates and high processing fees. [...] Furthermore, to date, there is no efficient way for consumer to make payments to other consumers using the Internet. [...]"

"The present invention represents a new paradigm for effectuating electronic payments that leverages existing platforms, conventional payment infrastructures and currently available web-based technology to enable e-commerce in both the virtual and physical marketplace. The concept provides a safe, sound, and secure method that allows users (consumers) to shop on Internet, pay bills, and pay anyone virtually anywhere, all without the consumer having to share account number information with the payee."

U.S. Patent No. 7720918
Title: System and method for interconnecting media services to an interface for transport of media assets

Assignee: Disney Enterprises, Inc., Burbank, CA

"A 'media asset' as used herein refers generally to any form of media content, including video, audio, still images and the like. [...] Organizations that create and distribute media content typically generate numerous media assets that must be managed. [...] In an effort to meet these needs, numerous computer-based media management services and systems have been developed. These services and systems use a variety of protocols and interfaces to support asset management functions such as creation, distribution, updating, storage and retrieval of media assets. [...] Integrating these solutions across disparate business units, however, has proven difficult."

"Embodiments of the present invention provide systems and methods capable of integrating media services and applications across an entire organization with multiple disparate business units."

U.S. Patent Patent No. 9640784

Title: Deposition apparatus, method of manufacturing organic light emitting display apparatus using the same, and organic light emitting display apparatus manufactured by using the method.

Assignee: Samsung Display Co., LTD., Gyeonggi-Do (KR)

"Organic light-emitting display devices have wider viewing angles, better contrast characteristics, and faster response speeds than other display devices, and thus have drawn attention as next-generationg display devices. [...] In a deposition method using a fine metal mask ('FMM'), a large FMM has to be used when manufacturing a large organic light-emitting display device [...] In this case, when such a large mask is used, the mask may bend due to self-gravity, and this may make it impossible to form an intermediate layer having a previously set and accurate pattern. Moreover, processes of aligning a substrate and an FMM to closely contact each other, performing deposition thereon, and separating the FMM from the substrate are time-consuming, resulting in a long manufacturing time and low production efficiency."

"One or more exemplary embodiments of the invention include a deposition apparatus capable of effectively preventing contamination in a deposition process. One or more exemplary embodiments of the invention include a method of manufacturing an organic light emitting display apparatus by using the deposition apparatus."