Dynamic Price Discovery: Transparency vs. Information design*

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Abstract

This paper studies how information design, via public disclosure of past trade details, affects price discovery in a dynamic market. We model that an informed forward-looking buyer sequentially trades with a series of uninformed sellers (hedgers) with heterogenous hedging motives. We discover that sellers' price discovery over the underlying hidden fundamentals is crucially affected by what they can observe about past trade details. Specifically, Post-trade *price* transparency *delays* price discovery, but once it happens, it is always perfect. In contrast, when only past *order* information is available, price discovery can never be perfect, and can even be in the *wrong direction*. Finally, we show that our findings are robust for diminishing bargaining power, non-zero outside options, and different trading positions.

Keywords: Price Discovery. Information disclosure. Asymmetric Information. Heterogenous Risk-aversion. Incentive. Public Offer vs. Private Offer.

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1 Introduction

A common concern about over-the-counter (OTC) markets is their opaqueness.[1] Therefore, the OTC market has undergone a significant change, and information on prices and orders (volumes) of completed transactions is once again publicly disclosed.[2] There is, however, a debate over the effectiveness of such regulations.[3] Therefore, in this paper we study how transparency mandates of past trade details affect market transparency as well as price discovery dynamics in the market. Specifically, we are interested in how much information is contained in price offers and how well the uninformed party learns about the private knowledge of the informed party. Can more transparency about trade history paradoxically lead to more market opacity? Moreover, to improve price discovery, does it matter what kind of past trade details the uninformed party can access?

To answer these questions, we present a parsimonious discrete-time, finite-period, dynamic trading game between an informed, forward-looking and risk-neutral buyer (informed trader) and a sequence of uninformed, single-period and heterogeneously risk-averse sellers (hedgers).[4] The informed party has private knowledge of the underlying economy, or other fundamentals affecting the risky asset. There are several examples with informed traders in these markets. For example, a firm may want to repurchase its shares to pay its shareholders. Alternatively, a firm may also want to sell some shares due to liquidity constraints. The firm, therefore, has better information than the market about its management and financial situation, as well as other fundamentals affecting its asset prices.[5]

[1] OTC markets are relatively opaque and uninformed traders are left in the dark. They do not know where the most attractive available terms are and whom to contact for those terms. For empirical evidence, see for example Green, Hollifield and Schürhoff (2007), Ashcraft and Duffie (2007), Massa and Simonov (2003), Linnainmaa and Saar (2012) and Bessembinder and Venkataraman (2004).

[2] Financial Industry Regulatory Authority (FINRA), later on the National Association of Securities Dealers (NASD), mandated transparency through the Trade Reporting and Compliance Engine (TRACE) program. The most notable reform was the U.S. Dodd-Frank Act, implemented after the 2008 financial crisis. See Duffie (2012) for an excellent review of transparency requirements of the U.S. Dodd-Frank Act. The Securities and Exchange Commission (SEC) has mandated post-trade price transparency for most U.S. corporate bonds and some other fixed-income instruments via TRACE since 2002. Similar reforms have been proposed for public transactions reporting in swap execution facilities (SEFs). Japan and Europe (in a more ambitious framework known as MiFID II and MiFIR) have followed a similar course as the United States (Duffie (2017)).

[3] See for example Duffie (2012, 2017) and Acharya and Richardson (2012).

[4] In Section 6.3 we show that all results remain unchanged if both parties switch their trading positions.

[5] As another example, specialists (market makers) can be better informed about the risky asset's value than hedgers (liquidity traders) (e.g. Gould and Verrecchia (1985), Laffont and Maskin (1990)). Since specialists often play a role in designing the securities that satisfy their clients' hedging motives, they have a better sense of how much these assets are worth. For example, following the financial crisis, regula-

To match different kinds of transparency regulations, we consider three types of market structures. The first case is called a *private history*, where both past price offers and past order decisions are not observable in future periods. This corresponds to the situation where no market transparency regulations are implemented. The second case is that all past trade details are publicly observed by both parties, including the buyer's price offer and seller's order decisions. We call it a *public history*, corresponding to regulations requiring the disclosure of both past prices and past orders. The third case, *order history*, is where the order information is publicly announced but the informed buyer's price offer are kept private between both parties in the transaction. In other words, future sellers can only observe whether there was a trade or not but not what prices the informed offered. This is the case where only post-order transparency is required but not post-price transparency. The comparison between three market structures provides insight on the effect of post-trade transparency regulations.

By comparing the equilibrium behaviors under the private, the public and the order history, we find that the structure of equilibria under all three histories are similar. All require sellers (hedgers) to be risk-averse enough for the informed buyer to conceal her information. This is because the more risk-averse the seller is, the more he wants to hedge his risky asset and the more incentive he has to trade with the informed buyer. Therefore, it becomes less costly for the buyer to sustain the opaque price that is independent of her private information.

Our first observation, derived from the comparison between the private and the public history, shows that past trade details, paradoxically, can decrease the information contained in current price offers. The intuition, which we call the *reputation building mechanism*, is as follows. The informed buyer (informed trader) has an incentive to achieve a reputation of "no-revelation-history" so that in future periods she can extract the information rent and take advantage of sellers' hedging motive. Therefore, the availability of past trade details enables such a reputation building and provides the buyer an incentive to hide her private information and maintain the "no-revelation-history".

A similar conclusion applies to the comparison between the public and the order history, but for a different reason, which we call the *belief updating mechanism*. We find that the opaque pricing strategy is harder to sustain under the order history than under

tors criticized investment banks, particularly Goldman Sachs, for betting against their customers using superior information about the value of mortgage-backed securities. Moreover, market makers can have private information about inventory imbalances, which gives them an edge in predicting the asset value (for empirical evidence see for example Cao, Evans and Lyons (2006)).

the public history. Past order information is a sufficient statistic for buyer's revelation of private information, therefore, reputation building story fails to hold here. However, sellers' belief updating process differs under both histories. Specifically, under the order history, in equilibrium when the economy is bad, with a higher chance the informed buyer offers discriminating prices and decline trade with sellers with low risk-aversion coefficients. Hence, observing no decline of trade in previous periods can increase uninformed sellers' confidence that the economy is good. As a result, these sellers ask for a higher price, which makes the opaque equilibrium harder to sustain.

By comparing the evolution of sellers' posterior beliefs about the underlying economy under the public history and the order history, we show the following patterns of price discovery. (i) Under the public history, price discovery is perfect whenever it happens. In fact, sellers mainly learn about the fundamentals through the informed buyer's current price offer. Therefore, they can perfectly identify the economic state once the informed buyer starts offering discriminating prices. (ii) Under the order history, price discovery is never perfect but in the right direction in a good economy, and in the wrong direction at first, then followed by a perfect learning in a bad economy. The intuition is as follows. Due to the belief updating mechanism, the learning can also happen even when the buyer is offering opaque prices. As discussed above, no decline of trade itself is an informative event for future sellers and under such a history they start to update their posteriors towards a good economy. In contrast, once the trade has been declined, one can derive that in equilibrium it only happens when the economy is bad and when the seller's risk-aversion coefficient is low. Therefore, price discovery is perfect right after such an event. (iii) The last but not the least observation is that under the order history, price discovery happens in a weakly earlier period than under the public history.

We also discuss some comparative statistics about the fraction of more risk-averse sellers (hedgers). Intuitively, under all histories, as a higher fraction of sellers becomes more risk-averse, the opaque pricing scheme becomes easier to sustain. Moreover, under the order history, as more sellers come with a high risk-aversion coefficient, price discovery is slowed down. This is due to the belief updating mechanism. A higher fraction of more risk-averse sellers impairs the order signals and slows down future sellers' learning about buyer's private information.

As regulators have an interest in increasing market transparency and the information contained in informed party's price offers, we compare the "opaqueness" of equilibria discussed in this paper. We formally define the degree of ignorance of a PBE and use it as a selection rule while dealing with equilibrium multiplicity in such a signaling game. Intuitively, we compare how long the informed party can offer opaque prices and choose the one where she can hide her private information to the most extent. With such a measure, we are able to show that our equilibrium construction achieves the *maximal degree of ignorance among all pure-strategy PBE*, under some regularity assumptions. As a consequence, our analysis can be viewed as the worst case one for regulators who are concerned about market opaqueness.

In addition, we show that our results are robust even when the informed buyer does not have full bargaining power. One concern for the reputation building mechanism is that the disclosure of past trade details may decrease informed buyer's bargaining power, and therefore lowers buyer's payoff and impairs the reputation building mechanism. However, we show that the former, the reputation building mechanism, dominates the latter, the bargaining power concern. In general the disclosure of past trade details is harmful for market transparency, price informativeness and price discovery in these markets. The introduction of bargaining power does not alter our main messages.

Finally, we show our results are robust to other natural extensions. First, we discuss the case where the informed buyer has non-zero outside options. The informed buyer may have access to other markets, so if she fails to successfully acquire the risky asset from the market, she can always purchase it from another channel at a fixed price. Second, we show that our results do not depend on trading positions. Our main conclusions will remain unchanged when the informed party (informed trader) is a seller who possesses some risky assets and uninformed players (hedgers) are a series of buyers who need one unit of those assets to hedge their other investments.

The paper is organized as follows. Section 2 presents the model and discusses the equilibrium concept. Section 3 analyzes equilibrium behaviors. Price discovery results are presented in Section 4. The degree of ignorance and the equilibrium selection criterion are discussed in Section 5, followed by the discussion of the bargaining power mechanism in Section 6.1. Other extensions of the model to informed buyer (informed trader) with outside options is in Section 6.2, and different trading positions are presented in Section 6.3. Section 7 concludes. Finally, Appendix A provides some additional results for other parameter conditions, and Appendix B contains all omitted proofs.

1.1 Related literature

This paper is related to several bodies of work.

The transparency feature of our model contributes to the literature on information design. The recent literature has a variety of focuses: optimal design of contests (e.g., Bimpikis, Ehsani and Mostagir (2015)), design of crowdfunding campaigns (e.g., Alaei, Malekian and Mostagir (2016)), inspection and information disclosure (e.g., Papanastasiou, Bimpikis and Savva (2018)), information design and diffusion in networks (e.g., Acemoglu, Ozdaglar and ParandehGheibi (2010), Ajorlou, Jadbabaie and Kakhbod (2018), Candogan and Drakopoulos (2019)), among others. In contrast to these important works, our paper particularly focuses on how information design, via dissemination of past trade details, affect the learning dynamics of uninformed liquidity traders (hedgers).

This work also contributes to the literature on market microstructure. There is an earlier literature that considers the liquidity demand side to be more informed (e.g., Milgrom and Glosten (1985), Easley and O'Hara (1987), Admati and Pfleiderer (1988)). In contrast to these works we consider the strategy of an "large" informed buyer (inside trader) and a sequence of uninformed liquidity traders. This framework is adopted in other works as well, e.g., Gould and Verrecchia (1985), Laffont and Maskin (1990)).[6] Importantly, however, in contrast to these important works, we aim to consider how the public disclosure of past trade details affects price discovery in dynamic markets.[7]

This paper also contributes to the growing literature on dynamic trading in OTC markets. Duffie (2012) has an excellent review of studies about OTC markets. Recent literature has a variety of focuses. For example, Guerrieri and Shimer (2014) study adverse selection with search frictions and discrete trading opportunities, Babus and Parlatore (2017) study welfare effects of decentralized trading; Duffie, Gârleanu and Pedersen (2005) and Lagos and Rocheteau (2009) look at random search and matching in large markets with a continuum of traders, Zhu (2014) shows how adding a dark pool improves market price discovery.[8]

[6] For empirical evidence that liquidity traders might be less informed than specialists (big traders) see, for example, Cao, Evans and Lyons (2006)).

[7] This paper is also related to the literature of the market efficiency in strategic informed trading, dated back to Kyle (1985*a*,*b*)'s seminal articles. Wang (1993, 1994) consider an infinite-horizon model where competitive insiders receive information on a firm's dividends over time in steady-state. They show that risk-neutral competitive insiders will reveal their private information instantly whereas risk-aversion can reduce their trading aggressiveness, leading to a slower information revelation. Back and Pedersen (1998) consider a finite-horizon model with a monopolistic informed insider and show that the insider reveals her information gradually. Chau and Vayanos (2008) consider the market efficiency in an infinite-horizon model with a monopolistic insider trading with competitive dealers and noisy traders as well. They discover that the insider chooses to reveal her information quickly, as the market approaches continuous trading.

[8] See also the literature on how information design via information production and choice may be market destabilizing, e.g., Bebchuk and Goldstein (2011); Goldstein (2012); Gorton and Ordonez (2014); Ahnert and Kakhbod (2017).

The comparison between public offers and private offers in our paper is related to the repeated game literature with perfect and imperfect monitoring (Abreu, Pearce and Stacchetti (1990), Abreu, Milgrom and Pearce (1991), Fudenberg, Levine and Maskin (1994), Chapter 15 of Mailath and Samuelson (2006)) and the one with reputation (Kreps et al. (1982)). However, rather than focusing on the characterization of all possible perfect public equilibrium payoffs, or have a Folk-Theorem-like result, we are more interested in when opaque equilibria can hold. We show that hidden past orders can encourage the informed buyer (informed trader) to share her private information. A similar phenomenon has been found in distinctly different settings such as consumer-information-based price discrimination (Fudenberg and Villas-Boas (2006); Bonatti and Cisternas (2018)) and bargaining models (Hörner and Vieille (2009)). We examine whether specific past trade details can encourage or discourage more disclosure of private information in securities markets.

2 Model

In this section we present a discrete-time, finite-period, random match model of the dynamic OTC market with a buyer and a series of sellers. Section 2.1 describes the basic setup and the timeline. Section 2.2 discusses three kinds of past trade histories and sellers' posterior beliefs. Buyer's and sellers' payoffs can be found in Section 2.3, and Section 2.4 defines our solution concept. We conclude this section with a discussion of several potential extensions, which are illustrated further in Sections 6.

2.1 The Setup and the Timeline

We consider a discrete-time, finite-period, dynamic trading game between an informed, risk-neutral and forward-looking **buyer** (informed trader) and a series of uninformed, risk-averse and single-period **sellers** (hedgers). There are N periods. At the beginning of each period, the buyer is randomly matched with a seller, who possesses one unit of risky asset to trade with the buyer.

The asset value $V_t = V(\theta, z_t)$ depends on the underlying economy state θ and an idiosyncratic, independently and identically distributed shock z_t. At the beginning of the game, θ is stochastically drawn from $\Theta = \{g, b\}$, corresponding to high or low risky asset value respectively. For example, θ can represent the underlying economy, whether it is in good or bad times. Another interpretation of θ can be the financial situation of the company owned by the buyer. Throughout the paper, we assume that θ is fixed throughout the following N periods. The stochastic term z_t can be interpreted as the friction of the outside market where the asset can be traded, the stochastic arrival of good or bad news[9], the uncertainty of dividend payment, or that the buyer faces a seasonal liquidity and inventory shock.[10]

The state θ is chosen by Nature at the beginning of the game, and privately observed by the buyer. The buyer, who is institutional trader in our settings, usually can hire agents to analyze the economic trend or have better knowledge about the situation of her company. The value of this risky asset is never directly observed by sellers in all future periods, but kept private between both parties involved in the trade. Without loss of generality and consistent with the empirical literature, we assume that[11]

$$\begin{aligned} \mathbf{E}(V(g,z_t)) &> \mathbf{E}(V(b,z_t)), \\ \mathrm{Var}(V(g,z_t)) &\leq \mathrm{Var}(V(b,z_t)). \end{aligned} \tag{1}$$

For example, the asset value given by the following dynamic process satisfies all above assumptions:

$$V_t = V(\theta, z_t) = J_\theta + \sigma_\theta z_t; \quad z_t \overset{iid}{\sim} F, J_g > J_b, \sigma_g \leq \sigma_b.$$

The timeline of the game is as follows, which is also illustrated in Figure 1. At the beginning of the game, Nature chooses θ, which is fixed thereafter. At the beginning of each period, the buyer offers a take-it-or-leave-it offer p_t to the matched seller. The seller then decides whether to accept it and sell his asset to her ($o_t = -1$) or not ($o_t = 0$). After he makes a decision, the risky asset value is realized and both parties collect their payoffs. At the end of each period, depending on which history we are studying, trade details are publicly announced for future sellers.

[9] There are several empirical works suggesting that asset values response to news. For example, Kothari and Warner (1997), Fama (1998), Daniel et al. (1998) and Hong et al. (2000) have excellent synopses of the literature on value reactions to various events.

[10] The informed market makers (firms) sometimes have to liquidate or buyback shares within a particular trading window, which brings another independent and identical shock to the asset value. For example, Gould and Verrecchia (1985), Laffont and Maskin (1990) cover this issue and Cao et al. (2006) discusses it empirically.

[11] For empirical evidence see Kothari and Warner (1997), Fama (1998), Daniel, Hirshleifer and Subrahmanyam (1998) and Hong, Lim and Stein (2000).

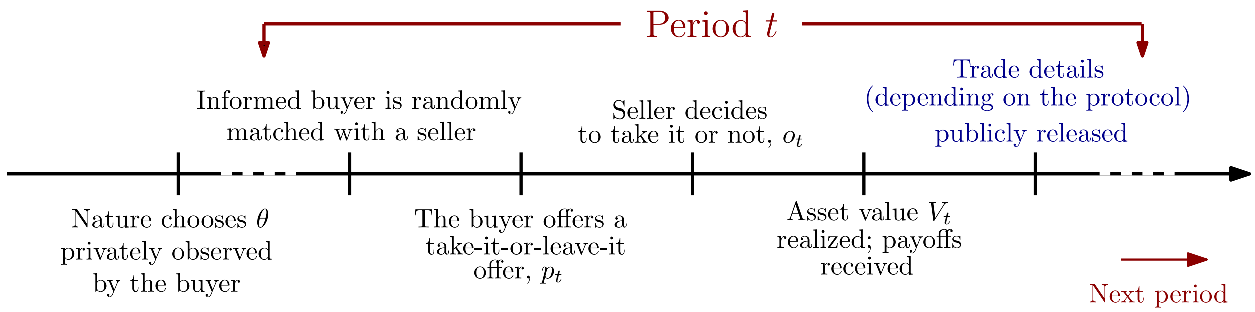

Figure 1: Timeline of the trading game.

2.2 Past Trade Histories and Sellers' Beliefs

In this paper we compare the equilibrium behaviors and the effect on price discovery and price informativeness under three kinds of past trade histories h^t. Under all three kinds of past trade histories, the asset value is never observed by future sellers. What distinguishes the three kinds of histories is whether the past prices and the past orders are available or not.

The first case is that all past trade details are publicly observed by the buyer and the seller who receives it, including buyer's price offer, p_t and seller's decision, o_t. In this paper we call it a **public history**. This corresponds to the regulations requiring the disclosure of past prices and past orders. The second case is called **private history**, where both buyer's price offers and sellers' decisions are not observable in future periods. This corresponds to the situation where no market transparency regulations are implemented. The third case, **order history**, is where the order information is publicly announced but buyer's price offer is kept private between parties in the transaction. In other words, future sellers can only observe whether there is a trade or not but not on what prices the buyer and a previous seller agree. This is the case where only post-order transparency is required but not post-price transparency.

We now look at sellers' beliefs. At the beginning of the game, Nature chooses $\theta = g$ with probability $\alpha^* \in (0,1)$, which consists of the common prior of all sellers. In the following periods, we denote $\alpha_t \equiv \Pr(\theta = g|h^{t-1})$ and $\mu_t \equiv \Pr(\theta = g|h^{t-1}, p_t)$ as seller's corresponding posterior belief before and after observing buyer's current offer. That is, before observing any past trade details and the current price offer, each seller believes that $\theta = g$ with probability $\mu_0 = \alpha^*$. Then, α_t is seller's posterior belief about θ given only h^{t-1}, whereas μ_t is seller's belief given both h^{t-1} and p_t.

The sellers Bayesian update their beliefs, whenever possible. For example, when making his decision, the seller updates his belief given both h^{t-1} and p_t in the following way.

$$\begin{aligned}\mu_t(p_t, h^{t-1}) &\equiv \Pr(\theta = g|p_t, h^{t-1}) \\ &= \frac{\alpha_{t-1}(h^{t-1})\Pr(p_t|\theta = g, h^{t-1})}{\alpha_{t-1}(h^{t-1})\Pr(p_t|\theta = g, h^{t-1}) + (1-\alpha_{t-1}(h^{t-1}))\Pr(p_t|\theta = b, h^{t-1})}.\end{aligned}$$

We use μ_t when we analyze sellers' decisions, while the evolution of α_t tells how well sellers learn about buyer's private information, θ, via past trade histories. In other words, the change in α_t enables us to discuss the effect on price discovery.

2.3 Payoffs

We model that all sellers are risk-averse and have a mean-variance utility function. However, the degree of their risk-aversion is heterogeneous. We assume that there are two types of sellers: those who are highly risk-averse (**high-type** sellers) and those who are not so risk-averse (**low-type** sellers), with corresponding risk-aversion coefficients ρ_H and ρ_L ($\rho_H > \rho_L \geq 0$). At the beginning of the game, each seller's type is independently drawn by Nature. With probability ϵ, he becomes a high-type. A seller's risk-averse attitude (type) is privately observed only by himself and the buyer, and is not learned by future sellers. [12]

The seller's end-of-period wealth from the trade is p_t, or V_t if there is no trade. The seller is single-period but risk-averse, with risk-aversion coefficient $\rho_t \in \{\rho_H, \rho_L\}$ and a posterior belief of $\mu_t(p_t, h^{t-1})$. Hence, his utility in period t is

$$w_t = (o_t + 1)\cdot[\mathbf{E}(V(\theta, z_t)|\mu_t(p_t, h^{t-1})) - \frac{\rho_t}{2}\mathrm{Var}(V(\theta, z_t)|\mu_t(p_t, h^{t-1}))] - o_t \cdot p_t. \tag{2}$$

The buyer's expected end-of-period payoff from the trade is $\mathbf{E}(V(\theta, z_t)|\theta) - p_t$. The buyer is forward-looking, so we discount her future payoffs by a factor δ and her average

[12] A trader's hedging motive can be linked to his size or identity. For example, several empirical studies show that public firms (e.g., pension funds or mutual funds) are less risk averse than small private firms (e.g., hedge funds). Since a seller's identity is public information, the buyer knows whether he is institutional (e.g. big public firm with high risk aversion) or retail (e.g. a small hedge fund with low risk aversion). The contemporary big data technology also enables institutional traders to learn the risk attitude about their counter-parties via surveys, past transactions and credit histories.

payoff from period t to the end of the game is

$$U_t = \sum_{s=t}^{N} \delta^{s-t} o_s \cdot [p_s - \mathbf{E}(V(\theta, \sigma_s)|\theta)]. \tag{3}$$

2.4 Equilibrium Concept

The solution concept in this paper is pure-strategy perfect Bayesian equilibrium, which is formally defined below.

Definition 1. *A perfect Bayesian equilibrium $\{p^*(\cdot), o^*(\cdot), \mu^*(\cdot)\}$ consists of the buyer's (trader's) price offer, $p^*(\cdot)$, the seller's (hedger's) order decision, $o^*(\cdot)$, and the seller's (hedger's) posterior belief about the underlying economy θ, $\mu^*(\cdot)$, such that the following properties hold:*

- *The buyer chooses her optimal price offer $p_t = p^*(\theta, h^{t-1})$ to maximize her expected utility (see Eq. (3)) given the available trade history h^{t-1}, her private information about θ, and seller's order decision $o^*(\cdot)$.*
- *Given any trade history h^{t-1} and the price offer p_t proposed by the buyer, the seller updates his posterior belief about θ according to the Bayes's rule, whenever it applies.*
- *The seller chooses his optimal order decision $o^*(p_t, \mu^*(\cdot); h^{t-1}) \in \{0, -1\}$ to maximize his expected utility (see Eq. (2)), given the price offer p_t, the available trade history h^{t-1}, and his posterior belief $\mu^*(\cdot)$ about θ.*

We notice that there are multiple equilibria in the dynamic trading game, but we are interested in under what conditions can a buyer hide her private information as long as possible. That is, when there are multiple equilibria, we always choose the one where the buyer can hide her private information to the most extent. As the regulators want to enhance the market transparency and prefers more informative price from the buyer, such analysis can be viewed as a worst case analysis for regulators in terms of price discovery. In other words, we are interested when she can offer a uniform price that is same when $\theta = g$ and $\theta = b$. We call such an equilibrium a fully pooling one. In contrast, in a fully separating equilibrium, the buyer can not hide her private information anymore and needs to offer different prices. We also focus on an intermediate case where the buyer can hide her private information until period k. The comparison of these equilibria allows us to identify the effect of past trade histories in the worst case, that is, the buyer hides her private information to the most extent. The detailed discussion of our selection criteria is reserved for Section 5.

In Section 6, we extend the above model in the following directions. Section 6.1 considers sellers to have some bargaining power. Section 6.2 considers the case where in each period, if the buyer fails to purchase a unit of the risky asset in the OTC market, she can always purchase it at a fixed cost from her secondary choice or outside option. Section 6.3 switches different trading positions of players. There is a seller and a series of buyers; each is in demand of one unit of the risky asset from the seller.

3 Equilibrium Behaviors

In this section we characterize equilibrium behaviors under three kinds of past trade histories. We start with the private history case as a bench mark. Then in Section 3.2 and Section 3.3, we study what happens if the past trade history is publicly disclosed to future sellers. We show that the availability of past trade details enables the buyer to build a reputation of "no-revelation-history". With such a reputation, in the future periods the buyer can extract the information rent. Therefore, the buyer has an incentive to offer opaque price and hide her private information.We call this intuition the "reputation building mechanism".

Our results for the dynamic trading game (public history and order history) depend on the following assumptions. Assumption 1 says that when $\theta = b$, the more optimistic the sellers are, the higher their reservation price. Hence, it becomes harder for the buyer to extract profit from the trade and hence there is lower static social welfare. Assumption 2 says that if the buyer ever reveals her private information, then she can not hide it anymore and should behave in the same way for all future high-type sellers.

Assumption 1 (monotonicity)**.** *Denote $\pi(\rho,\alpha)$ as the static surplus from the trade when $\theta = b$ if seller's (hedger's) risk-aversion coefficient is ρ and posterior belief is α. Specifically,*

$$\underbrace{\pi(\rho,\alpha)}_{\text{ex-post surplus from the trade}} \equiv \underbrace{\mathbf{E}(V(b,z_t))}_{\text{buyer's evaluation of the risky asset}} - \underbrace{[\mathbf{E}(V(\theta,z_t)|\alpha) - \frac{\rho}{2}\mathrm{Var}(V(\theta,z_t)|\alpha)]}_{\text{seller's evaluation of the risky asset}}. \tag{4}$$

Then $\pi(\rho,\alpha)$ monotonically decreases in α for any $\rho > 0$.

One example can be that the underlying asset value follows the following process: $a_t = J_\theta + \sigma_\theta z_t$ where the drift and the volatility of the stochastic shock depend on the underlying economy θ. In this example, $\pi(\rho,\alpha) = \frac{\rho}{2}[\alpha\sigma_g^2 + (1-\alpha)\sigma_b^2 + \alpha(1-\alpha)(J_g - J_b)^2] -$

$\alpha(J_g - J_b)$. One can verify that the monotonicity assumption holds if $J_g - J_b$ is small enough or $\sigma_g^2 - \sigma_b^2$ is large enough.

Assumption 2 (consistency)**.** *If the buyer (informed trader) ever reveals her private information before (offering a discriminating price that depends on θ under the public history, or declining the trade under the order history), then she will propose the same offer p_t for all future high-type sellers, no matter whether the revelation happens on-path or off-path.*

Assumption 2 is a natural assumption, because it says that if the informed buyer ever reveals her private information to the market (i.e., uninformed sellers fully observes the true value of the fundamental θ) then it becomes very costly for her to manipulate prices anymore.[13]

3.1 Private History

In this section we consider a situation where future sellers learn nothing about trade details in a previous period. The lack of past trade details de-links the dynamics between periods and makes both players to act as if this is a one-shot game. In other words, we can treat $\delta = 0$. Therefore, sellers update their beliefs about θ based only on buyer's current period price offer.

The buyer now can choose to hide her private information about θ by offering the same price when $\theta = g$ and $\theta = b$, or to reveal the underlying economy state by pricing discriminatingly. As we will show in the following Proposition, when sellers are risk-averse enough, the buyer can hide her private information and a fully pooling equilibrium exists. In contrast, if sellers have relatively low risk-aversion coefficient, then the buyer starts pricing opaquely and a fully separating equilibrium exists. The specific cutoffs are characterized in the proof of Proposition 1.

Proposition 1. *There exists $\rho^{pooling} < \rho^{separating}$ such that:*

(1) ***if and only if*** *the seller's (hedger's) risk-aversion coefficient $\rho \geq \rho^{pooling}$, there exists a fully pooling equilibrium where the buyer (informed trader) can hide her private information and offer a uniform price that is independent of θ;*

[13] Particularly, it can be due to high fines imposed by regulators, for example, SEC (securities and exchange commission) imposes high fines for price manipulation, and securities laws and related SEC rules broadly prohibit fraud in the purchase and sale of securities, and the Securities Exchange Act of 1934, Section 9, specifically makes it unlawful to manipulate security prices.

(2) ***if and only if*** *the seller's (hedger's) risk-aversion coefficient* $\rho \leq \rho^{separating}$*, there exists a fully separating equilibrium where the buyer (informed trader) can offer a discriminating price that depends on her private information about* θ*. Moreover, in the separating equilibrium, when* $\theta = b$ *the buyer (informed trader) will offer a price low enough such that no trade occurs.*

The intuition is as follows. For the first part of Proposition 1, opaque pricing strategy requires a strong hedging motive from sellers. That is, sellers need to sufficiently dislike their volatility in their asset values, and have a low enough reservation price. Otherwise it becomes too costly for the buyer to buy the asset from them, especially when $\theta = b$. As a result, a high reservation price provides an incentive for the buyer to decline the trade and avoid the loss. If buyer's evaluation of the asset when $\theta = b$ is lower than that of the seller, the fully pooling equilibrium breaks. For the second part, in a fully separating equilibrium, no trade will occur when $\theta = b$ because otherwise when $\theta = g$ the buyer has an incentive to pretend it is in bad times, and buyer's incentive compatibility (IC) constraint fails. For this reason, when the buyer starts to price discriminatingly, she always offers a low enough price when $\theta = b$ so that no trade occurs.

Throughout the rest of the paper, we focus on a non-trivial case where $\rho_L < \rho^{pooling} \leq \rho_H$. From the analysis in this section, we know that if both types of sellers are risk-averse enough ($\rho^{pooling} \leq \rho_L < \rho_H$), then opaque pricing can always be sustained in a one-shot game, therefore in any period of the dynamic game. If both types sellers are not risk-averse ($\rho_L < \rho_H < \rho^{pooling}$), then opaque pricing is never sustained. Hence we pay attention to the case where under the private history, the buyer can hide her private information until the first low-type seller arrives.

In addition, for the ease of presentation, we will assume in the main paper that $\rho_L < \rho^{pooling} \leq \rho^{separating} < \rho_H$, i.e., $\pi(\rho_H, \alpha) \geq 0, \forall \alpha$. In Section A, we show that the analysis for the case $\rho_L < \rho^{pooling} < \rho_H \leq \rho^{separating}$ exactly mirrors the one in the main text and does not change our main conclusions.

3.2 Public History

In this section we show that compared to the private history, under the public history the opaque pricing strategy is easier to sustain. Hence, the availability of past trade details actually decreases market transparency. Without loss of generality, we fix ρ_H as a parameter throughout this paper and characterize such conditions in terms of ρ_L.

For finite-period dynamic game, we can solve this by backward induction. Period N is the last period where the economy state is θ; therefore there is no dynamic concern from the buyer and all players behave as if in the private history. From the analysis in the previous section, we know that the buyer will reveal her private information about θ if there arrives a low-type seller and will otherwise continue to hide her private information.

Before the last period, buyer's forward-looking concern provides her with another incentive to hide her private information. If she releases her private information and declines trade when $\theta = b$ in the current period, then in all future periods, sellers can observe this action and adjust their beliefs to punish her. Therefore, the buyer has an incentive to hide her private information and build a reputation of "no-revelation-history". Nevertheless, as we show in the Section 3.1, hiding information is costly for her. When the economy is bad and the seller is of low-type, the buyer incurs a loss by offering the opaque price. Therefore, there is a trade-off between avoiding immediate loss and a decline of future profits.

Such a trade-off enables the existence of an equilibrium where the buyer hides her private information for a while and then releases it in a certain period. At earlier periods, the forward-looking buyer has more incentive to price opaquely. This is because at earlier periods, she expects more future payoffs, and longer punishment is triggered after the deviation. Hence, the buyer is more and more prone to price discriminatingly.

We now formalize this idea. We start by characterizing a such kind of equilibrium, which we call "k-pooling equilibrium". In a "k-pooling equilibrium", the buyer prices opaquely in the first k period, after which she starts pricing discriminatingly if a low-type seller comes. The formal definition is below.

Definition 2. *In the dynamic trading game (under the public history or order history), a PBE is called a "k–pooling equilibrium" if and only if*

- *(hide before period k) the buyer's (trader's) on-path price offer is independent of θ for $t = 1, \ldots, k$, no matter whether the seller (hedger) is high- or low-type;*
- *(hide for high-type) the buyer's (trader's) on-path price offer is independent of θ for periods $t = k+1, \ldots, N$ if the seller (hedger) is high-type;*
- *(reveal for low-type after period k) the buyer's (trader's) on-path price offer is different when $\theta = g$ and $\theta = b$ for periods $t = k+1, \ldots, N$ if the seller (hedger) is low-type;*

In fact, "k-pooling equilibrium" is a more general concept that nests both a fully pooling and a fully separating equilibrium. When $k = N$, the buyer hides her private information in all periods $t = 1, .., N$, and this is actually a fully pooling equilibrium. When $k = 0$, the buyer reveals her information whenever a low-type seller arrives since the first period, coinciding with the fully separating equilibrium.

Proposition 2 characterizes the necessary and sufficient condition for the existence of a "k-pooling equilibrium". From the proposition we learn that the sustainability of a "k-pooling equilibrium" under the public history is similar to that of a fully pooling equilibrium under the private history. Both exist if and only if low-type sellers are risk-averse enough. Moreover, the more risk-averse sellers are, the longer the buyer can hide her private information about the underlying economy and offers a uniform price.

Proposition 2. *Under the public history, that is, when future sellers (hedgers) can observe past orders as well as prices, there exists a sequence* $\{\rho_k\}_{k=1}^N$ *such that*

- $\rho_N = \rho^{pooling}$
- $\{\rho_k\}_{k=1}^N$ *increases in* k;
- *for* $k = 1, ..., N$, *there exists a "k-pooling equilibrium" if and only if* $\rho_L \geq \rho_k$.

We notice that with the public disclosure of past trade details, the opaque pricing strategy becomes easier to sustain. As previously discussed, the intuition is that the forward-looking buyer now also cares about her future profits. If $\theta = g$, then the buyer never wants to deviate. However, when $\theta = b$, if she defects and refuses to offer the opaque price specified in the equilibrium, then such deviation is observed by all future sellers and the buyer loses her reputation of "no-revelation-history". All future sellers can now adjust their beliefs to punish her for this deviation. For example, if they adjust their belief to be the most optimistic one,[14], then the highest feasible profit the buyer can extract from future sellers becomes $\max\{\pi(\rho, 1), 0\}$, which is less than her on-path payoff $\pi(\rho, \alpha^*)$. Hence, the access to past trade details enables the buyer to build up a reputation of "no-revelation-history" and decreases buyer's incentive to reveal her private information.

Remark 1. *We note that the analysis for the public history case can be applied to a more realistic case. In such a case, if the trader declines the offer and no trade happens, then trade details of*

[14]Specifically, future sellers hold a belief that the economy state $\theta = g$ if an off-path action is observed.

that period are not publicly announced for future traders. That is, results of this section can still hold even if future traders do not observe the offered price in an earlier, no-trade, period. This is because observing no information of a period implies that no trade happened in that period. Such order information is a sufficient static for the informed buyer's deviation and future uninformed traders (sellers) do not need the price information to distinguish whether or not the buyer follows the on-path equilibrium strategy. Therefore, the reputation building mechanism is not weakened and the constructed equilibria in this section can still hold.

3.3 Order History

In this section we discuss what happens under the order history, where future sellers can only observe in a previous period whether there is a trade or not but not what transaction prices are. We still focus on "$k-$pooling equilibrium". To characterize the equilibrium behaviors under the order history, we provide a similar result as Proposition 2. We then compare cutoffs between public history and order history, as illustrated in Figure 2.

Proposition 3. *Under the order history, that is, when future sellers (hedgers) can only observe whether there is a trade or not in a previous period but not on which prices both parties agree, there exists a sequence $\{\hat{\rho}_k\}_{k=1}^N$ such that*

- *$\{\hat{\rho}_k\}_{k=1}^N$ increases in k and $\hat{\rho}_N = \rho^{pooling}$;*
- *for $k = 1, ..., N$, there exists a "k-pooling equilibrium" if and only if $\rho_L \geq \hat{\rho}_k$.*
- *$\hat{\rho}_N = \rho_N = \rho^{pooling}$, $\hat{\rho}_{N-1} = \rho_{N-1}$, and for any $k = 1, ..., N-2$, $\hat{\rho}_k > \rho_k$.*

From Proposition 3, the sustainability of a "k-pooling equilibrium" under the order history is similar to that under the public history. Both require sellers risk-averse enough for the opaque pricing strategy to sustain. Moreover, Proposition 3 implies that restricting the information to which future sellers can get access increases the chance of a discriminating and informative price from the buyer. In other words, at a certain period, releasing past orders can also make the opaque pricing strategy easier to sustain and leads to less transparency in the current market price.

The intuition behind this major distinction between public and order history is the seller's belief updating process. In the "k-pooling equilibrium" under the order history, after period $k+1$, if a decline of the trade has not been observed yet, then simply observing whether there is a transaction or not now becomes informative for sellers. In fact, the decline of the trade happens only when $\theta = b$ and the seller is low-type. Because future sellers can not infer previous sellers' types, they can not distinguish whether the transaction happens is due to $\theta = g$, or due to the fact that a high-type seller comes when $\theta = b$. As the two scenarios happen with different probabilities, Bayesian sellers start updating their beliefs after period k. As time goes by, if the decline of trade is still not observed, sellers tend to be more optimistic about the underlying economy. The monotonicity assumption then implies that buyer's on-path payoff is smaller, making the opaque pricing harder to sustain under the order history than under the public history.

Another observation of Proposition 3 is that when $k = N$, the two cutoffs $\hat{\rho}_N$ and ρ_N coincide, implying that if we focus on the fully pooling equilibrium ("N-pooling equilibrium"), then the order information is a sufficient statistic for buyer's past information revelation.

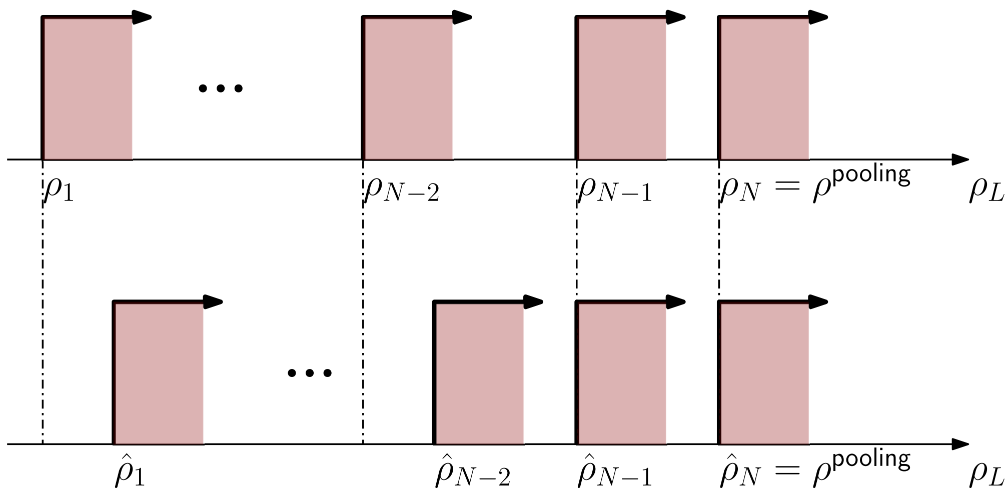

Figure 2: Cutoffs for a pooling equilibrium to sustain under different past trade histories.

4 Price Discovery

So far we have discussed the effect of past trade details on price informativeness, that is, how much information buyer's current offer contains. We now change our focus towards price discovery, that is, how much information sellers learn via only past trade details. To study this, we compare the evolution of sellers' posterior beliefs about the underlying economy θ under the public history and order history. Specifically, we look at how close sellers' posteriors are to the true economy state in equilibrium. The following definition formalizes such a metric, which is in the same spirit of Zhu (2014).

Definition 3. *The degree of price discovery at period t is defined as the absolute difference between $\mathbb{1}(\theta = g)$ and sellers' (hedgers') posterior belief at period t α_t, that is, $|\alpha_t - \mathbb{1}(\theta = g)|$. If*

$$|\alpha_t - \mathbb{1}(\theta = g)| \begin{cases} < |\alpha^* - \mathbb{1}(\theta = g)|, & \text{then there is positive price discovery (correct learning).} \\ = |\alpha^* - \mathbb{1}(\theta = g)|, & \text{then there is no price discovery (no learning).} \\ > |\alpha^* - \mathbb{1}(\theta = g)|, & \text{then there is negative price discovery (wrong learning).} \end{cases}$$

If $|\alpha_t - \mathbb{1}(\theta = g)| = 0$, then we say there is perfect price discovery (perfect learning).

Theorem 1 then describes price discovery under both public and order histories. To summarize, under the public history, price discovery is perfect whenever it happens. Under the order history, in contrast, when $\theta = g$, positive price discovery happens exactly from period $k + 1$, but is never perfect. When $\theta = b$, starting from period $k + 1$, there can be negative price discovery for a while, followed by a perfect discovery.

Theorem 1. *Fix a risk-aversion coefficient ρ_L,*

- *under the public history, there is no price discovery for a while, then there is perfect price discovery at a certain period; specifically, if a set of parameters where a "k-pooling equilibrium" exists, then price discovery do not occur until the first low-type seller after period k comes;*
- *under the order history, if a set of parameters where a "k-pooling equilibrium" exists, then in such "k-pooling equilibrium"*
  - *no price discovery occurs until period $k + 1$;*
  - *if $\theta = b$ and there comes a low-type seller (hedger) after period $k + 1$, then there is perfect price discovery immediately;*
  - *otherwise there is imperfect price discovery; it is positive price discovery in good times and negative price discovery in bad times;*

Here is what happens under the public history. In a "k-pooling equilibrium", the buyer does not reveal her private information about θ until the first low-type seller after period k comes, upon which price discovery starts to happen. Once she starts to price discriminatingly, future sellers learn exactly the true state of world, θ, and price discovery becomes perfect. Figure 3 illustrates this.

Under the order history, in a "k-pooling equilibrium", similarly there is still no price discovery for the first k periods. This is because the buyer offers uniform prices during

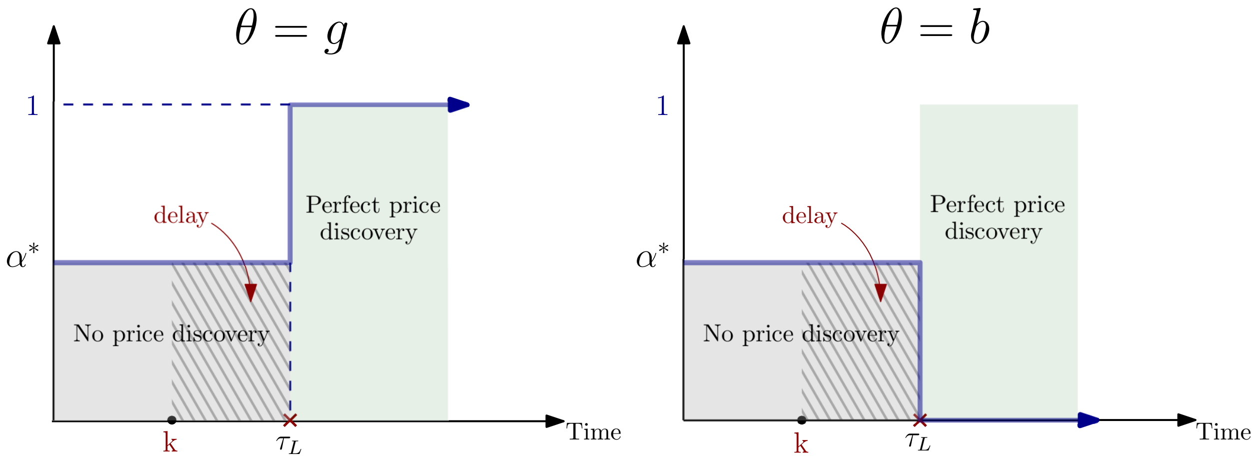

Figure 3: Sellers' posterior under the public history.

these "pooling" periods. After period k, in contrast, sellers start to update their posteriors toward 1, for the reason discussed in Section 3. When $\theta = g$, such price discovery is positive and in the right direction; whereas when $\theta = b$, such price discovery is negative and in the wrong direction. This process continues until there is a decline of trade, which happens in equilibrium only when $\theta = b$ and a low-type seller comes after period k. If such declines occurs, then future sellers precisely identify that $\theta = b$ and perfect price discovery occurs. Moreover, as such decline never happens in equilibrium when $\theta = g$, so in good times price discovery is never perfect. Figure 4 shows this.

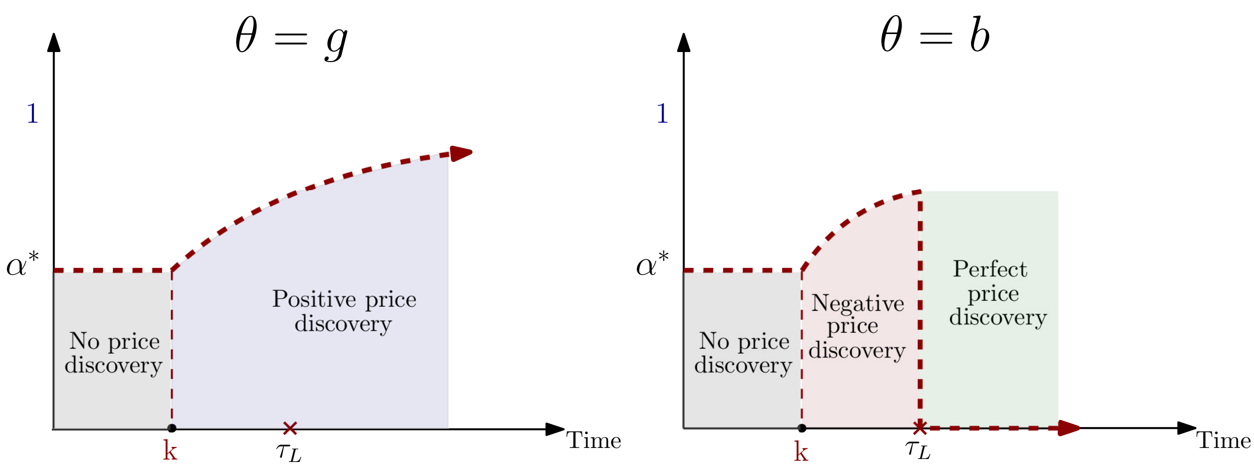

Figure 4: Sellers' posterior under the order history.

Given Theorem 1, now we are able to discuss the effect of post-trade price transparency on price discovery. First notice that whenever a "k–pooling equilibrium" exists under the order history, it also exists under the public history. Hence, the buyer can hide her private information for both types of sellers for a longer period under the public history. Suppose that, to the most extent, the buyer can hide her private information for both types till period $k' \geq k$. Moreover, from the above analysis, price discovery happens immediately after period k under the order history, whereas under the public history it does not happen until there comes a low-type seller after period k'. In conclusion, price discovery happens in a later period under the public history. Theorem 2 summarizes and Figure 5 illustrates the comparison of price discovery under both the public and order history.

Theorem 2. *Fix a risk-aversion coefficient ρ_L; then under the order history, price discovery always occurs weakly earlier than that under the public history. However, under the public history, once price discovery occurs, it is a perfect one.*

We also discuss the comparative statistics over the fraction of high-type sellers, displayed in the following Corollary. It is obvious that under both histories, as more sellers become high-type (more risk-averse), the opaque pricing scheme becomes easier to sustain. Hence price discovery comes later under both histories. Moreover, under the order history, a higher fraction of high-type sellers makes future seller more pessimistic about the underlying economy when no decline of trade is observed. Thus price discovery is delayed as well as slowed down. Finally, in extreme cases, if all sellers are high-type, then the buyer always hides her private information and only the fully pooling equilibrium exists; if all sellers are low-type, then the buyer starts revealing her private information from the first period, and only the fully separating equilibrium exists.

Corollary 1. *Keep all other things fixed, if ϵ increases, then*

- *under the public history, the cutoff ρ_k decreases for all $k = 1, \ldots, N$;*
- *under the order history, the cutoff $\hat{\rho}_k$ decreases for all $k = 1, \ldots, N$;*
- *under the order history, sellers' posterior in period t, given that the buyer has not yet revealed her private information, α_{N-t}, decreases for all $t = 1, \ldots, N$.*

Moreover,

- *if $\epsilon = 0$, then $\rho_k = \hat{\rho}_k = \rho^{pooling}$, and by the definition of ρ_L, only "N-opaque equilibrium" (i.e., the fully separating equilibrium) exists. In other words, when all coming sellers are low-type, then the buyer starts revealing her private information from the first period and there is perfect price discovery at the beginning of the game.*

- *If $\epsilon = 1$, then only "0-pooling equilibrium" (i.e., the fully pooling equilibrium) exists. That is, when all coming sellers are high-type, the buyer always hides her private information and there is no price discovery throughout the game.*

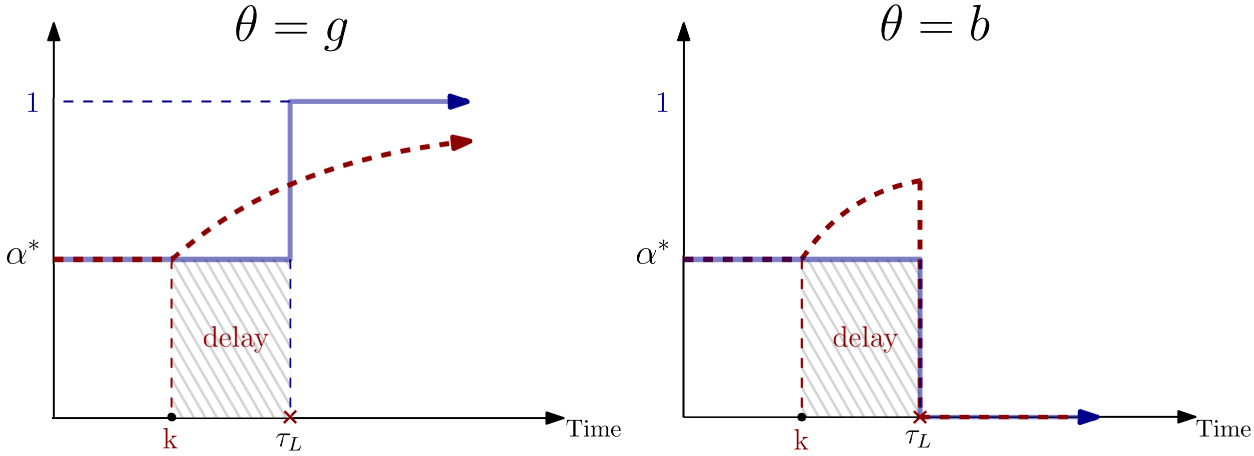

Figure 5: Sellers' posteriors under the public history (blue solid line) vs the order history (red dash line).

5 Degree of Market Ignorance

In previous sections, we have characterized multiple equilibria in the dynamic trading environment. In this section, we present our selection criterion and demonstrate that it is consistent with a worst case analysis for regulators who care about market transparency. In other words, in case of equilibrium multiplicity, we choose the one where the buyer can hide her private information and offer opaque prices to the most extent. We also show that according to such a standard, the one chosen among all PBE with consistency assumption is always in the form of "k-pooling equilibrium".

We first need a more precise measure to quantify the "opaqueness" of an equilibrium. Here we count the expected number of periods where the buyer continues to hide her private information. This is the expected length of the period where the seller does not update his prior belief, or no price discovery occurs. Definition 4 formalizes this measure.

Definition 4. *Under the public or order history, the* ***degree of ignorance*** *of a PBE is defined as follows:*

$$\mathbf{E}(\max_{\alpha_T=\alpha^*} T).$$

In other words, it is the expected number of periods along the equilibrium path where there is no price discovery.

From the analysis in Section 4, we also learn that under the order history, there can be negative price discovery when $\theta = b$. If we also count these periods into our measure, then we get another version of the degree of ignorance, defined below.

Definition 5. *Under the order history, the* ***degree of ignorance*** *of a PBE is defined as follows:*

$$\mathbf{E}(\max_{|\alpha_T - \mathbb{1}(\theta=g)| \geq |\alpha^* - \mathbb{1}(\theta=g)|} T).$$

In other words, it is the expected number of periods along the equilibrium path where there is no price discovery or negative price discovery.

With these two measures, we can compare the degree of ignorance of all equilibria discussed above. In fact, as Proposition 4 shows below, our construction achieves the maximal degree of ignorance among all PBE with the consistency assumption. Hence, analysis in Section 3 and Section 4 can be viewed as the worst case for regulators who care about market transparency and how much information contained in buyer's offer.

Proposition 4. *Given Assumption 1 and Assumption 2, suppose $\rho_L \in [\rho_k, \rho_{k+1})$ under the public history, or $\rho_L \in [\hat{\rho}_k, \hat{\rho}_{k+1})$ under the order history, then there exists a "k-pooling equilibrium" that has the* ***maximal degree of ignorance*** *among all PBE with consistency, no matter which version of the degree of ignorance measure is chosen.*

6 Robustness and Extensions

In this section we extend our model to relax buyer's bargaining power, buyer's outside options and different trading positions. We show that the main message we have delivered in previous sections remains unchanged.

6.1 Bargaining Power

In this section we extend our model to relax the full bargaining power assumption.[15]

One concern of the reputation building mechanism is that in reality the disclosure of past trade details may decrease buyer's bargaining power. If so, then buyer's future

[15]For empirical evidences supporting imperfect (monopolisitic) competition in securities (OTC) market, see e.g., Green, Hollifield and Schürhoff (2007), Ashcraft and Duffie (2007) and Massa and Simonov (2003).

periods' payoffs decrease, which impairs the benefits of reputation building and makes the opaque pricing strategy harder to sustain. As a result, such bargaining power concern potentially diminish the negative effect of the post-trade transparency on price informativeness and price discovery in the OTC market. In other words, one may worry that the major mechanism we discussed above, the reputation building mechanism, is overturned by such bargaining power concern. As a result, the disclosure of past trade information may be beneficial to the price transparency in the current period.

In this section, nevertheless, we mechanically model the fact that past trade details may affect the bargaining process between two parties and show that the reputation building mechanism still dominates. Specifically, we assume that the buyer can collect γ_k fraction of the total surplus if there are k periods of past trade details available to both parties. For example, under the private history, the buyer collects γ_0 fraction of the total payoff in every period. With more past trade details available, the seller may have a stronger bargaining power while trading with the buyer. Therefore, we make the following assumption. Similar negative correlation between market transparency and buyer's net profit can also be found in the theoretical model of Di Maggio and Pagano (2017).

Assumption 3. $\gamma_0 \geq \gamma_1 ... \geq \gamma_{N-1}$.

We redo the analysis of equilibrium sustainability under the private history, the public history and the order history. We start with the private history. As past trade history is still unavailable, the buyer still lacks the tool to build a "no-revelation-history" reputation and extract the information rent in the future period. Thence, the cutoff $\rho^{pooling}$ in Proposition 1 remains unchanged.

We now look at the dynamic trading game. Although future periods' payoffs are decreased by a lower bargaining power, the on-path ones are still higher than the off-path ones. Thus, the benefit of reputation building still exists, and the post-trade transparency can still promote opaque pricing, compared to the static or the private history case. Proposition 5 formalizes this observation, and Figure 6 illustrates it.

Proposition 5. *Given Assumption 3, the following statements are true.*

1. *Under the private history, if and only if $\rho \geq \rho^{pooling}$, there exists a fully pooling equilibrium where the buyer can hide her private information about θ and offer a uniform price in each period.*

2. *Under the public history, for any $k = 1, ..., N$, there exists $\rho_k^{bargain} \in [\rho_k, \rho^{pooling}]$ such that if and only if $\rho_L \geq \rho_k^{bargain}$, there is a "$k-$pooling" equilibrium;*

3. *Under the order history, for any* $k = 1, \ldots, N$*, there exists* $\hat{\rho}_k^{bargain} \in [\min\{\rho_k^{bargain}, \hat{\rho}_k\}, \rho^{pooling}]$ *such that if and only if* $\rho_L \geq \hat{\rho}_k^{bargain}$*, there is a "k−pooling" equilibrium.*

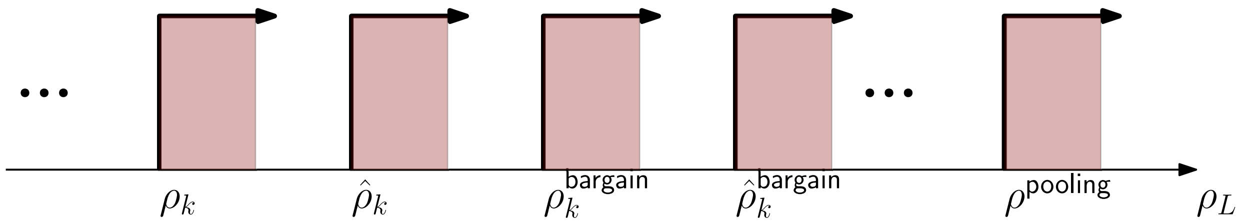

Figure 6: Cutoffs with Diminishing Bargaining Power.

Proposition 5 implies that the disclosure of past trade details have two-fold effect. On the one hand, it enables the buyer to build a "no-revelation-history" and extract future information rent and therefore facilitates the opaque equilibrium. This effect reduces market transparency, price informativeness and price discovery in the OTC market. On the other hand, the availability of past trade details decreases buyer's bargaining power and therefore, lowering future payoffs and impairing her benifit from reputation building. As a result, it partially cancels out the former effect. However, Proposition 5 shows that the former dominates the latter, and in general the disclosure of past trade details is harmful for the current price transparency. Finally, we also want to point out that the introduction of bargaining power does not alter our main results, including the structure of "k−pooling" equilibria and the comparison of the sustainability of such equilibria under different histories.

6.2 Buyer's Outside Option

In this subsection we consider the case where the buyer has an outside option if she can not trade her risky asset with the seller. Specifically, we assume that in each period t, if $o_t = 0$, that is, if the buyer fails to successfully acquire the risky asset from the OTC market, then she can always buy from another channel at a fixed price p^*. Denote buyer's profit from trading in this secondary channel when $\theta = b$ as $\pi^* \equiv \mathbf{E}(V(b, z_t)) - p^*$. Then when analyzing buyer's behavior, we can compare her payoff from trading in the OTC market versus π^*. We show in Proposition 6 that under the private history, we have the same structure of a fully pooling equilibrium and a fully separating equilibrium.

Proposition 6. *If the buyer has constant outside option, then there exist cutoffs* $\rho_{outside}^{pooling} < \rho_{outside}^{separating}$ *such that*

- *a fully pooling equilibrium exists if and only if* $\rho \geq \rho_{outside}^{pooling}$;
- *a fully separating equilibrium exists if and only if* $\rho \geq \rho_{outside}^{separating}$. *Moreover, in such a fully separating equilibrium, when* $\theta = b$ *no trade occurs and the buyer gets an expected payoff of* π^* *from the secondary channel.*

We then look at the dynamic trading game. Again, all the analysis remain unchanged, except for one difference. Now when there is no trade in the OTC market, the buyer obtains a non-negative payoff. Nevertheless, one can simply normalize $\pi^* = 0$ and buyer's problem remains the same. Proposition 7 shows that all previous results are unaffected.

Proposition 7. *If the buyer has constant outside option, then there exists* $\{\rho_{k,outside}\}_k$ *and* $\{\hat{\rho}_{k,outside}\}_k$ *such that*

- $\rho_{N,outside} = \hat{\rho}_{N,outside} = \rho_{outside}^{pooling}$;
- $\rho_{N-1,outside} = \hat{\rho}_{N-1,outside} \leq \rho_{outside}^{pooling}$;
- *for* $k = 1, \ldots, N$, $\hat{\rho}_{k,outside} \geq \rho_{k,outside}$;
- $\{\rho_{k,outside}\}_k$ *and* $\{\hat{\rho}_{k,outside}\}_k$ *increase in* k;
- *under the public history, there exists a "$k-$pooling equilibrium" if and only if* $\rho_L \geq \rho_{k,outside}$, $k = 1, \ldots, N$;
- *under the order history, there exists a "$k-$pooling equilibrium" if and only if* $\rho_L \geq \hat{\rho}_{k,outside}$, $k = 1, \ldots, N$;

6.3 Different Trading Positions

In this subsection, we alter the trading positions between two parties and show that our result is robust against such a variation. That is, there is a seller with risky assets and a series of buyers who need one unit of these assets to hedge their other investments. We start with some notations. In each period, given buyer's risk-aversion coefficient is ρ and his posterior belief about the economy is $\Pr(\theta = g) = \alpha$, by accepting seller's offer ($o_t = 1$), his payoff is $-p_t$. If he rejects the offer ($o_t = 0$), then he needs to pay the realization of the risky asset, which is $-V_t$. Hence, his payoff in period t becomes

$$w_t = (1 - o_t)[\mathbf{E}(-V(\theta, z_t)|\alpha) - \frac{\rho_t}{2}\mathrm{Var}(-V(\theta, z_t)|\alpha)] - o_t p_t, \tag{5}$$

where α is buyer's posterior belief after observing past trade details and the current price offer. Therefore, the price at which he is indifferent between accepting ($o_t = 1$) and rejecting ($o_t = 0$) now becomes

$$\mathbf{E}(V(\theta, z_t)|\alpha) + \frac{\rho_t}{2}\mathrm{Var}(V(\theta, z_t)|\alpha).$$

The analysis of the equilibrium behavior where there is a seller and a sequence of buyers aligns with the one where there is a buyer and a sequence of sellers. We first present what happens under the private history.

Proposition 8. *There exists* $0 < \rho^{pooling}_{\chi_t=-1} \leq \rho^{separating}_{\chi_t=-1}$ *such that:*

(1) if and only if the buyer's risk-aversion coefficient $\rho \geq \rho^{pooling}_{\chi_t=-1}$*, there exists a fully pooling equilibrium;*

(2) if and only if the buyer's risk-aversion coefficient $\rho \leq \rho^{separating}_{\chi_t=-1}$*, there exists a fully separating equilibrium. Moreover, in such an equilibrium, when* $\theta = g$*, the seller will offer a price high enough so no trade occurs.*

Now the binding IC constraint is the $\theta = g$ one. Hence we denote $\pi_{\chi_t=-1}(\rho, \alpha)$ as the static surplus from the trade when $\theta = g$ if buyer has a risk-aversion coefficient of ρ and a posterior belief of α.

$$\underbrace{\pi_{\chi_t=-1}(\rho, \alpha)}_{\substack{\text{ex-post surplus} \\ \text{from the trade}}} \equiv \underbrace{\left[\mathbf{E}(V(\theta, z_t)|\alpha) + \frac{\rho}{2}\mathrm{Var}(V(\theta, z_t)|\alpha)\right]}_{\text{buyer's reservation price for the risky asset}} - \underbrace{\mathbf{E}(V(g, z_t))}_{\substack{\text{seller's reservation} \\ \text{price for the asset}}}. \tag{6}$$

As with Assumption 1, we assume that the more optimistic a buyer about the underlying economy and the more risky the asset value is, the higher the seller can sell her asset to him, and the more benefit and static surplus from the trade.

Assumption 4. $\pi_{\chi_t=-1}(\rho, \alpha)$ *increases in* α*.*

Then the analysis of equilibrium behavior exactly mirrors the one with one seller and a sequence of buyers. Specifically, in Proposition 9, we can show that the equilibrium structure under each past trade history and the comparison between them remain the same.

Proposition 9. *There exists* $\{\rho_{k,\chi_t=-1}\}_k$ *and* $\{\hat{\rho}_{k,\chi_t=-1}\}_k$ *such that*

- $\rho_{N,\chi_t=-1} = \hat{\rho}_{N,\chi_t=-1} = \rho^{pooling}_{\chi_t=-1}$;
- $\rho_{N-1,\chi_t=-1} = \hat{\rho}_{N-1,\chi_t=-1} \leq \rho^{pooling}_{\chi_t=-1}$;
- *for* $k = 1, \ldots, N$, $\hat{\rho}_{k,\chi_t=-1} \geq \rho_{k,\chi_t=-1}$
- $\{\rho_{k,\chi_t=-1}\}_k$ *and* $\{\hat{\rho}_{k,\chi_t=-1}\}_k$ *increase in k;*
- *under the public history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \rho_{k,\chi_t=-1}$, $k = 1, \ldots, N$;
- *under the order history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \hat{\rho}_{k,\chi_t=-1}$, $k = 1, \ldots, N$;

Finally, under the order history, price discovery can be imperfect when $\theta = b$ and negative when $\theta = g$. Moreover, it always comes weakly later than that under the public history, where price discovery, if occurs, is always perfect. In fact, under the order history, with opposite trading positions, trade occurs less frequently when $\theta = g$ than $\theta = b$. Therefore, when $\theta = g$, the longer the trade lasts, the more pessimistic the buyers and the lower their posteriors are. In other words, we have negative price discovery when the economy is good. In contrast, when $\theta = b$, the seller never declines the trade so buyers' posteriors keeps decreasing but never reaches to 0, resulting in imperfect price discovery.

7 Conclusion

We develop a tractable, yet rich, model to study the dynamics of price discovery in a securities market with asymmetric information and heterogenous hedging motives. In this model a large informed buyer trades sequentially with a series of uninformed and heterogenous sellers (hedgers). We characterize the pure-strategy perfect Bayesian equilibrium in which the informed buyer can hide her private information and offer opaque prices to the greatest extent. In other words, we conduct a worst case analysis for regulators who worry about the opaqueness of OTC markets.

Particularly, we analyze how transparency mandates, such as the Dodd-Frank Act implemented through the TRACE after the 2008 financial crisis, affect price discovery for uninformed traders. We show that post-trade *price* transparency delays price discovery, but once it happens, it is always perfect. With only the past *order* details but not the past *price* ones, price discovery can never be perfect, or even in the wrong direction. Therefore, price discovery dynamics crucially depends on what uninformed sellers (traders) can observe.

We further discuss the effect on market transparency. We establish that the availability of past trade details, paradoxically, makes it easier for the informed party to hide her private information and offer opaque prices. The intuition behind the public disclosure of past *prices* is the *reputation building mechanism*, whereas that of past *orders* is due to the *belief updating mechanism*.

To wrap up, we demonstrate that our findings are robust when the informed party's bargaining power decreases with the disclosure of trade histories. Moreover, we extend our results to the case where the informed buyer can have access to a secondary market, and the one where both parties switch their trading positions.

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A The case of $\rho_L < \rho^{pooling} < \rho_H \leq \rho^{separating}$.

If $\rho_H \leq \rho^{separating}$, then the fully separating equilibrium always exists, no matter whether there comes a low or high-type seller. Therefore, we can use this fully separating equilibrium as the off-path punishment for the buyer. One can simply show that the fully separating equilibrium payoff actually consists of a min max payoff for the buyer. Thence, in the proof of Proposition 2 and Proposition 3, by replacing the off-path punishment with the fully separating equilibrium, we can still impose the harshest feasible punishment. Then we can show below the equilibrium structures and the comparison of them under different histories can still hold.

Proposition A.1. *If $\rho_L < \rho^{pooling} < \rho_H \leq \rho^{separating}$, then there exists $\{\underline{\rho}_k\}_k$ and $\{\underline{\hat{\rho}}_k\}_k$ such that*

- $\underline{\rho}_N = \underline{\hat{\rho}}_N = \rho^{pooling}$;
- $\underline{\rho}_{N-1} = \underline{\hat{\rho}}_{N-1} \leq \rho^{pooling}$;
- *for $k = 1, \ldots, N$, $\underline{\hat{\rho}}_k \geq \underline{\rho}_k$;*
- $\{\underline{\rho}_k\}_k$ *and* $\{\underline{\hat{\rho}}_k\}_k$ *increase in k;*
- *under the public history, there exists a "k–pooling equilibrium" if and only if $\rho_L \geq \underline{\rho}_k$, $k = 1, \ldots, N$;*
- *under the order history, there exists a "k–pooling equilibrium" if and only if $\rho_L \geq \underline{\hat{\rho}}_k$, $k = 1, \ldots, N$;*

Appendix B: Proofs

Proposition 1. *There exists $\rho^{pooling} < \rho^{separating}$ such that:*

(1) ***if and only if*** *the seller's (hedger's) risk-aversion coefficient $\rho \geq \rho^{pooling}$, there exists a fully pooling equilibrium where the buyer (informed trader) can hide her private information and offer a uniform price that is independent of θ;*

(2) ***if and only if*** *the seller's (hedger's) risk-aversion coefficient $\rho \leq \rho^{separating}$, there exists a fully separating equilibrium where the buyer (informed trader) can offer a discriminating price that depends on her private information about θ. Moreover, in the separating equilibrium, when $\theta = b$ the buyer (informed trader) will offer a price low enough such that no trade occurs.*

Proof. We first show the necessary condition of the existence for a fully pooling equilibrium and characterize $\rho^{pooling}$. We then prove that $\rho \geq \rho^{pooling}$ is sufficient for the existence of a fully pooling equilibrium by construction. Next, we analyze seller's strategy under a fully separating equilibrium. Finally, we prove the comparison relation between $\rho^{pooling}$ and $\rho^{separating}$.

(i) **Necessity of $\rho \geq \rho^{pooling}$**

If the buyer offers an opaque price, then after observing it, the seller will still hold his prior belief that $\theta = g$ with probability α^*. To sustain such an opaque pricing strategy and guarantee that trade occurs, buyer's evaluation about the asset should be weakly higher than that of the seller, in both good and bad times. Specifically, it is the $\theta = b$ case that binds and such a fully pooling equilibrium exists if

$$\begin{aligned} & \mathbf{E}(V(b,z_t)) \geq \mathbf{E}(V(\theta,z_t)|\alpha^*) - \frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*), \\ \Leftrightarrow \quad & \rho_t \geq \rho^{pooling} \equiv 2\frac{\mathbf{E}(V(\theta,z_t)|\alpha^*) - \mathbf{E}(V(b,z_t))}{\mathrm{Var}(V(\theta,z_t)|\alpha^*)}, \end{aligned}$$

which characterizes $\rho^{pooling}$.

(ii) **Sufficiency of $\rho \geq \rho^{pooling}$**

We construct the following strategies and beliefs and verify that they consist of a PBE given $\rho \geq \rho^{pooling}$. Specifically, in period t,

- the buyer offers a price $p_t^{opaque} \equiv \mathbf{E}(V(\theta,z_t)|\alpha^*) - \frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*)$;

- Conditional on observing p_t^{opaque}, the seller holds his prior belief that $\Pr(\theta = g) = \alpha^*$. Otherwise he believes that the economy is good for sure;
- the seller accepts a price offer if and only if it is weakly above his evaluation of the risky asset given the above belief.

It is easy to check that the seller's belief updating follows Bayes' rule whenever it applies and the seller is maximizing his expected utility given his belief. To see that the buyer has no incentive to deviate, let us consider her options. She can

- offers the equilibrium price p_t^{opaque} and earns non-negative payoff, guaranteed by condition $\rho \geq \rho^{pooling}$;
- offers a price lower than p_t^{opaque}; however doing so leads to the rejection by the seller, as he adjusts his belief to $\Pr(\theta = g) = 1$ and requires at least $\mathbf{E}(V(g, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(g, z_t)) > p_t^{opaque}$;
- offers a price higher than p_t^{opaque}; but no matter how seller responds to it, such a deviation leads to lower payoff of the buyer.

Hence it is also suboptimal for the buyer to deviate.

(iii) **Buyer's Strategy under a Fully Separating Equilibrium**

First, in a fully separating equilibrium, trade can only occur when $\theta = g$, because otherwise in good times the buyer can offer a bad times price offer and convince the seller to sell his asset at a lower price. Such profitable deviation leads to the break of the equilibrium. Next, after observing the good times price offer, the seller will update his posterior to believe that $\theta = g$ for sure. Notice that this is the most optimistic belief he can ever hold and the seller is willing to accept any price offer weakly above

$$p_t^g = \mathbf{E}(V(g, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(g, z_t)).$$

Therefore, there is no reason for a rational buyer to offer something strictly higher. In conclusion, in a fully separating equilibrium, in good times the buyer will offer exactly p_t^g and in bad times offer something low enough to deter trade. Eventually, the only thing left to check is that buyer in bad times should not have an incentive to offer a good times price and induce a profitable trade. Thence, her incentive-compatibility (IC) constraint when $\theta = g$ implies that

$$\begin{aligned} \mathbf{E}(V(b,z_t)) \le p_t^g &= \mathbf{E}(V(g,z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(g,z_t)); \\ \Leftrightarrow \quad \rho_t \le \rho^{separating} &\equiv 2\frac{\mathbf{E}(V(g,z_t)) - \mathbf{E}(V(b,z_t))}{\mathrm{Var}(V(g,z_t))}, \end{aligned}$$

which characterizes $\rho^{separating}$.

To see the sufficiency, construct seller's belief such that they believe $\theta = g$ unless they observe a price lower than their bad times evaluations. It is easy to check that buyer's pricing strategy above, along with this belief, forms a PBE.

(iv) $\rho^{pooling} \le \rho^{separating}$

To see that $\rho^{pooling} \le \rho^{separating}$, we observe that $\mathbf{E}(V(g,z_t)) > \mathbf{E}(V(\theta,z_t)|\alpha^*)$ and $\mathrm{Var}(V(\theta,z_t)|\alpha^*) \ge \mathrm{Var}(V(g,z_t))$. The inequality then follows immediately.

□

Proposition 2. *Under the public history, that is, when future sellers (hedgers) can observe past orders as well as prices, there exists a sequence $\{\rho_k\}_{k=1}^N$ such that*

- $\rho_N = \rho^{pooling}$
- $\{\rho_k\}_{k=1}^N$ *increases in k;*
- *for $k = 1,\dots,N$, there exists a "k-pooling equilibrium" if and only if $\rho_L \ge \rho_k$.*

Proof. We show the proof in three steps. First, we construct a set of strategies and beliefs and show sufficient conditions for them to consist of a PBE. We characterize ρ_k from the binding constraint. Second, the necessity of $\rho_L \ge \rho_k$ comes from the fact that in the construction we have already used the harshest possible punishment. Finally, we show comparison relationships between different bounds.

(I) Characterize ρ_k

First, let us consider the following set of strategies and beliefs:

- if there is no deviation in previous periods, then
  - for $t = 1,\dots,k$, the buyer offers $p_t = \mathbf{E}(V(\theta,z_t)|\alpha^*) - \frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*)$;

- for $t = k+1, \dots, N$, the buyer offers

$$p_t \begin{cases} = \mathbf{E}(V(\theta, z_t)|\alpha_t) - \frac{\rho_H}{2}\mathrm{Var}(V(\theta, z_t)|\alpha_t) & \text{if } \rho_{t'} = \rho_H, t' = 1, \dots, t \\ = \mathbf{E}(V(g, z_t)) - \frac{\rho_H}{2}\mathrm{Var}(V(g, z_t)) & \text{if } \rho_t = \rho_H \text{ and } \rho_{t'} = \rho_L \\ & \text{for some } t' \in [k+1, t-1]\,; \\ = \mathbf{E}(V(g, z_{t,})) - \frac{\rho_L}{2}\mathrm{Var}(V(g, z_t)) & \text{if } \rho_t = \rho_L \text{ and } \theta = g \\ < \mathbf{E}(V(b, z_t)) - \frac{\rho_L}{2}\mathrm{Var}(V(b, z_t)) & \text{if } \rho_t = \rho_L \text{ and } \theta = b \end{cases}$$

- after observing an on-path price offer, the seller updates his posterior based on past trade history as well as the current period price offer;
- sellers accept any offer weakly above their evaluation about the asset, given their posterior belief;

• if there is deviation observed in previous periods, then sellers will hold the belief that $\theta = g$ and the buyer offers

$$p_t \begin{cases} = \mathbf{E}(V(g, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(g, z_t)) & \text{if } \rho_t = \rho_H \text{ or } \theta = g \\ < \mathbf{E}(V(b, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(b, z_t)) & \text{if } \rho_t = \rho_L \text{ and } \theta = b \end{cases}; \tag{7}$$

sellers update their beliefs whenever the Bayes' rule applies, or believe that $\theta = g$ otherwise; sellers accept any offer weakly above their evaluation about the asset, given the above constructed beliefs.

To check the validity of "k-pooling" equilibrium, notice that sellers' beliefs already follow the Bayes' rule and sellers' actions are sequential rational. The only condition to check is that buyer's behavior is sequential rational. We look at all kinds of histories.

Let us first focus on off-path histories. If the buyer is ever observed to deviate before, then by offering the price in equation (7), she can earn non-negative profits if the seller is high-type or $\theta = g$, and zero if the seller is low-type and $\theta = b$. Hence, when $\theta = g$ or there comes a high-type, the buyer has no incentive to offer $\mathbf{E}(V(b, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(b, z_t))$. Similarly as in the proof of Proposition 1, offering something higher will lower her current period payoff. If she offers something strictly lower than $\mathbf{E}(V(b, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(b, z_t))$, then Bayes' rule does not apply and the seller will believe that the economy is good. Hence, he will reject the trade and also lead to a loss of buyer's current period payoff. Hence, deviating will cause the loss of current payoff and no change in future payoffs. A rational buyer will not deviate off-path.

We then consider possible deviations during period $t = 1, ..., k$. Again, we rule out possible deviations such as offering a higher price. But given seller's beliefs, offering a lower price will lead to his rejection. In good times or if the seller is high-type, such deviations will result in a loss of current profit and trigger future punishment. Hence, only when facing a low-type seller in bad times, can the buyer have an incentive to decline trade and avoid current period loss. In other words, these cases are the only incentive compatibility (IC) constraints we need to check. Denote V_n as buyer's continuation payoff when $\theta = b$ if she keeps offering opaque prices in previous periods and there are n sellers coming. In these periods V_n follows the dynamics:

$$V_n = \epsilon\pi(\rho_H, \alpha^*) + (1-\epsilon)\pi(\rho_L, \alpha^*) + \delta V_{n-1}, \; n = N-k+1, ..., N. \tag{8}$$

buyer's IC constraints are

$$\pi(\rho_L, \alpha^*) + \delta V_{n-1} \geq 0 + \sum_{i=1}^{n-1} \delta^i \epsilon\pi(\rho_H, 1), \quad n = N-k+1, ..., N. \tag{9}$$

We now look at on-path history h^{t-1} where $t \geq k+1$. Let us look at this case by case.

(i) If no low-type seller comes between period $k+1$ and t (inclusive), then in period t, the buyer offers the uninformative price. Similarly as in the proof of Proposition 1, as $\rho_H > \rho^{pooling}$, deviation will cause a current loss and will trigger future punishment.

(ii) The second case is that at least one low-type seller comes between period $k+1$ and period $t-1$ (inclusive), but the current period comes a high-type seller. According to the construction, the buyer is supposed to offer $p_t = \mathbf{E}(V(g, z_t)) - \frac{\rho_H}{2}\mathrm{Var}(V(g, z_t))$ and will make positive profit in the current period. Offering higher price will lower the current profit. Offering lower price will let the seller adjust his belief to $\Pr(\theta = g) = 1$ and reject the offer. So all possible deviation leads to the loss of current profit and trigger future punishment.

(iii) The third case is when in the current period comes a low-type seller. Similarly, then given sellers' belief construction, the buyer can not buy the asset at a price lower than $\mathbf{E}(V(g, z_t)) - \frac{\rho_L}{2}\mathrm{Var}(V(g, z_t))$ and the equilibrium strategy is already optimal.

Hence, condition (24) is a sufficient condition for the validity of the constructed PBE. Now let us show that $n = N-k+1$ is a binding one. We will prove this by induction.

let us first finalize the dynamics of V_n: for $n = 1,...,N-k$, the dynamics of V_n follows

$$V_1 = \epsilon\pi(\rho_H,\alpha^*) \tag{10}$$

$$V_n = \epsilon\pi(\rho_H,\alpha^*) + \epsilon\delta V_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1). \tag{11}$$

We then introduce an auxiliary sequence $\{\underline{V}_n\}_{n=1}^N$, show that $V_n \geq \underline{V}_n, \forall n = 1,...,N$ and explicitly characterize the value of this auxiliary sequence.

$$\begin{aligned}
\underline{V}_n &= V_n, \quad n = 1,...,N-k \\
\underline{V}_n &= \epsilon\pi(\rho_H,\alpha^*) + \epsilon\delta\underline{V}_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1), \quad n = N-k+1,...,N
\end{aligned}$$

Then, one can easily check by induction that for $n = 1,...,N$,

$$\underline{V}_n = [1+(\delta\epsilon)+...+(\delta\epsilon)^{n-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] + (1+\delta+...+\delta^{n-1})\epsilon\pi(\rho_H,1).$$

Claim 1. *If* $\pi(\rho_L,\alpha^*) + \delta V_{N-k} = \pi(\rho_L,\alpha^*) + \delta\underline{V}_{N-k} \geq \sum_{i=1}^{N-k}\delta^i\epsilon\pi(\rho_H,1)$*, then following claims are true.*

- *Claim* $A_n : V_n \geq \underline{V}_n, \quad n = 1,...,N;$
- *Claim* $B_n : \pi(\rho_L,\alpha^*) + \delta V_n \geq \pi(\rho_L,\alpha^*) + \delta\underline{V}_n \geq \sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1), \quad n = N-k,...,N$

Proof of the Claim. We prove by induction. First, the statement B_{N-k} and statements $A_1,..,A_{N-k}$ are true. We then prove by induction the rest of claims are true.

First, suppose A_{n-1} and B_{n-1} are true, where $n > N-k$, then

$$\begin{aligned}
V_n &= \epsilon\pi(\rho_H,\alpha^*) + (1-\epsilon)\pi(\rho_L,\alpha^*) + \delta V_{n-1} \\
&\overset{(i)}{\geq} \epsilon\pi(\rho_H,\alpha^*) + \epsilon\delta V_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1) \\
&\overset{(ii)}{\geq} \epsilon\pi(\rho_H,\alpha^*) + \epsilon\delta\underline{V}_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1) \\
&\overset{(iii)}{=} \underline{V}_n,
\end{aligned}$$

where inequality (i) is due to statement B_{n-1}, inequality (ii) comes from statement A_{n-1} and equality (iii) is the definition of $\underline{V}_n$. Hence, statement A_n is true.

Next suppose that statement A_n and B_{n-1} are true, $n > N-k$, then

$$\begin{aligned}
& \pi(\rho_L,\alpha^*)+\delta V_n-\sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) \\
\overset{(iv)}{\geq}\quad & \pi(\rho_L,\alpha^*)+\delta\underline{V}_n-\sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) \\
\overset{(v)}{=}\quad & \pi(\rho_L,\alpha^*)+\delta[1+(\delta\epsilon)+\ldots+(\delta\epsilon)^{n-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\
& \quad +(\delta+\ldots+\delta^n)\epsilon\pi(\rho_H,1)-\sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) \\
=\quad & \pi(\rho_L,\alpha^*)+\delta[1+(\delta\epsilon)+\ldots+(\delta\epsilon)^{n-2}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\
& \quad +\delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\
\overset{(vi)}{=}\quad & \pi(\rho_L,\alpha^*)+\delta\underline{V}_{n-1}-\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1)+\delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\
\overset{(vii)}{\geq}\quad & \delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\
\overset{(viii)}{\geq}\quad & 0,
\end{aligned}$$

where inequality (iv) is from statement A_n, equalities (v) and (vi) use the explicit form of $\underline{V}_n$, inequality (vii) is because of statement B_{n-1} and finally inequality (viii) comes from the monotonicity of $\pi(\cdot,\alpha)$. Thence, statement B_n is true.

Therefore, by induction, statements $A_1,\ldots,A_N$ and $B_{N-k},\ldots,B_N$ are all true. $\square$

From the above claim, we can then conclude that the binding constraint is

$$\pi(\rho_L,\alpha^*)+\delta V_{N-k}\geq\sum_{i=1}^{N-k}\delta^i\epsilon\pi(\rho_H,1),$$

which gives a lower bound of ρ_L, denoted as ρ_k:

$$\pi(\rho_k,\alpha^*)+\delta[1+(\delta\epsilon)+\ldots+(\delta\epsilon)^{N-k-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)]=0. \tag{12}$$

(II) The Necessity of $\rho_L \geq \rho_k$

To show the necessity of $\rho_L \geq \rho_k$, we only need to show that, in the construction of equilibrium, the harshest deviation punishment has already been imposed. In fact, we can show that the off-path payoffs consist of the min-max payoff for the buyer.

Claim 2. *For any conditional belief $\mu(p_t;\cdot)$, the buyer's off-path payoffs consist of the min-max static payoffs given that the seller best responses to her strategy. Specifically, for any h^{t-1},*

$$\min_{\mu(p_t;h^{t-1})}\max_{p_g,p_b}\min_{o_t\in \mathsf{BR}_{\mu(p_t;h^{t-1})}}(\mathbf{E}(V(g,z_t))-p_g)\cdot o_t(p_g,\mu(\cdot),h^{t-1})=\frac{\rho_t}{2}\mathrm{Var}(V(g,z_t));$$

and

$$\min_{\mu(p_t;h^{t-1})}\max_{p_g,p_b}\min_{o_t\in \mathsf{BR}_{\mu(p_t;h^{t-1})}}(\mathbf{E}(V(b,z_t))-p_b)\cdot o_t(p_b,\mu(\cdot),h^{t-1})=$$
$$\max\{\mathbf{E}(V(b,z_t))-\mathbf{E}(V(g,z_t))+\frac{\rho_t}{2}\mathrm{Var}(V(g,z_t)),0\}.$$

Proof. We prove the claim case by case.

- Case I: when $\theta=b$ and the seller is low-type.

  First observe that the buyer can always offer a low enough price to decline the trade and collect an ex-post payoff of 0.

  Next, we show that for the following belief, she can do no better than that. Suppose the seller believes that the current period is in good times unless $\mathbf{E}(V(b,z_t))$ is offered, at which he believes it is in bad times, then he will only accept any offer above $\mathbf{E}(V(g,z_t))-\frac{\rho_L}{2}\mathrm{Var}(V(g,z_t))$ or exactly at $\mathbf{E}(V(b,z_t))$. Offering the former leads to non-positive payoff, while offering the latter leads to zero payoff. Hence, conditional on this seller's belief and seller's best response function, buyer's highest ex-post payoff is 0. Combining with the first observation,

$$\min_{\mu(p_t;h^{t-1})}\max_{p_b}\min_{o_t\in \mathsf{BR}_{\mu(p_t;\cdot)}}(\mathbf{E}(V(b,z_t))-p_b)\cdot o_t(p_b,\mu(\cdot),h^{t-1})=0.$$

- Case II: when $\theta=g$.

  First observe that the buyer's ex-post payoff will be at least $\frac{\rho_t}{2}\mathrm{Var}(V(g,z_t))$. In fact, for any $\mu(p_t;\cdot)$,

$$\begin{aligned}\mathbf{E}(V(\theta,z_t)|\mu(\cdot|p))-\frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\mu(\cdot|p)) &= \mathbf{E}(V(b,z_t))-\pi(\rho_t,\mu(\cdot|p))\\ &\le \mathbf{E}(V(b,z_t))-\pi(\rho_t,1)\\ &= \mathbf{E}(V(g,z_t))-\frac{\rho_t}{2}\mathrm{Var}(V(g,z_t)).\end{aligned}$$

Therefore, the buyer can always offer a price $p_t^g = \mathbf{E}(V(g,z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(g,z_t))$ to confirm the order from the seller and guarantee an ex-post payoff of $\frac{\rho_t}{2}\mathrm{Var}(V(g,z_t))$ for sure. Next, this is the best she can get if the seller believes that the current period is always in good times. As only prices weakly above p_t^g will be accepted by the seller. So together we have,

$$\min_{\mu(p_t;h^{t-1})} \max_{p_g,p_b} \min_{o_t \in \mathrm{BR}_{\mu(p_t;h^{t-1})}} (\mathbf{E}(V(g,z_t)) - p_g) \cdot o_t(p_g, \mu(\cdot), h^{t-1}) = \frac{\rho_t}{2}\mathrm{Var}(V(g,z_t)).$$

- Case III: when $\theta = b$ and the seller is high-type.

  First observe that the buyer can guarantee a non-negative payoff $\mathbf{E}(V(b,z_t)) - \mathbf{E}(V(g,z_t)) + \frac{\rho_t}{2}\mathrm{Var}(V(g,z_t))$ by offering a price at the level of seller's highest feasible evaluation p_t^g (defined above).

  Next, consider the following belief where the seller always believe $\theta = g$ and rejects any offer lower than p_t^g, then this is the lowest price the buyer can successfully acquire the asset. Thence, similarly we have

$$\begin{aligned}\min_{\mu(p_t;h^{t-1})} \max_{p_g,p_b} \min_{o_t \in \mathrm{BR}_{\mu(p_t;h^{t-1})}} &(\mathbf{E}(V(g,z_t)) - p_g) \cdot o_t(p_g, \mu(\cdot), h^{t-1}) \\ &= \mathbf{E}(V(b,z_t)) - \mathbf{E}(V(g,z_t)) + \frac{\rho_t}{2}\mathrm{Var}(V(g,z_t)).\end{aligned}$$

□

With the availability of Claim 2, let us finish the proof of necessity. In any "k-pooling equilibrium" with consistency assumption, σ, consider a following history h^{k-1}, when the buyer has always offered uniform price in period 1,2,...,$k-1$. We will show that buyer's IC constraint in period k when $\theta = b$ leads to $\rho_L \geq \rho_k$. If the buyer ever prices discriminatingly, then in any future period, she can get a profit of $\hat{\pi}_H$ from the high-type and $\hat{\pi}_L$ from the low-type seller. Then,

$$\hat{\pi}_H \geq \pi(\rho_H, 1), \hat{\pi}_L \geq 0.$$

As otherwise, the buyer can deviate to offer $p_t^g = \mathbf{E}(V(g,z_t)) - \frac{\rho_H}{2}\mathrm{Var}(V(g,z_t))$ for the high-type and decline the trade for low-type after revealing her private information. Then the dynamics of V_n in equilibrium σ is as follows.

$$V_n \;=\; \epsilon\pi(\rho_H,\alpha^*)+\epsilon\delta V_{n-1}+(1-\epsilon)\sum_{i=1}^{n-1}\delta^i[\epsilon\hat{\pi}_H+(1-\epsilon)\hat{\pi}_L],\; n=1,...,N-k.$$

One can verify by induction that

$$\begin{aligned} V_n \;=\;& [1+(\delta\epsilon)+...+(\delta\epsilon)^{n-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\hat{\pi}_H-(1-\epsilon)\hat{\pi}_L] \\ &+(1+\delta+...+\delta^{n-1})[\epsilon\hat{\pi}_H+(1-\epsilon)\hat{\pi}_L], \quad n\le N-k. \end{aligned}$$

Then buyer's IC constraint in period k when $\theta=b$ becomes

$$\begin{aligned} &\pi(\rho_L,\alpha^*)+\delta V_{N-k}\ge\sum_{i=1}^{N-k}\delta^i[\epsilon\hat{\pi}_H+(1-\epsilon)\hat{\pi}_L]; \\ \Rightarrow\; &\pi(\rho_L,\alpha^*)+\delta[1+(\delta\epsilon)+...+(\delta\epsilon)^{N-k-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\hat{\pi}_H-(1-\epsilon)\hat{\pi}_L]\ge 0; \\ \Rightarrow\; &\pi(\rho_L,\alpha^*)+\delta[1+(\delta\epsilon)+...+(\delta\epsilon)^{N-k-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)]\ge 0, \end{aligned}$$

which exactly results in $\rho_L\ge\rho_k$.

(III) Comparisons between ρ_k

Last, we show that $\rho_N=\rho^{pooling}$ and $\rho_{k+1}\ge\rho_k$ for $k=1,...,N-1$. The former is trivial as when $k=N$, the characterization equations for ρ_N and ρ^* coincides with each other. To see the latter, notice that

$$\begin{aligned} \pi(\rho_{k+1},\alpha^*) \;&=\; -\delta[1+(\delta\epsilon)+...+(\delta\epsilon)^{N-k-2}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\ &\ge\; -\delta[1+(\delta\epsilon)+...+(\delta\epsilon)^{N-k-1}]\cdot[\epsilon\pi(\rho_H,\alpha^*)-\epsilon\pi(\rho_H,1)] \\ &=\; \pi(\rho_k,\alpha^*). \end{aligned}$$

Therefore, $\rho_{k+1}\ge\rho_k$ comes from the monotonicity of $\pi(\cdot,\alpha)$.

□

Proposition 3. *Under the order history, that is, when future sellers (hedgers) can only observe whether there is a trade or not in a previous period but not on which prices both parties agree, there exists a sequence $\{\hat{\rho}_k\}_{k=1}^N$ such that*

- *$\{\hat{\rho}_k\}_{k=1}^N$ increases in k and $\hat{\rho}_N=\rho^{pooling}$;*
- *for $k=1,...,N$, there exists a "k-pooling equilibrium" if and only if $\rho_L\ge\hat{\rho}_k$.*

- $\hat{\rho}_N = \rho_N = \rho^{pooling}$, $\hat{\rho}_{N-1} = \rho_{N-1}$, *and for any* $k = 1, ..., N-2$, $\hat{\rho}_k > \rho_k$.

Proof. The proof aligns closely with that of Proposition 2. We adopt the same construction and present sufficient conditions for it to form a PBE. We then show the binding constraint and characterize $\hat{\rho}_k$. Next, the necessity follows the same argument as in Claim 2 in the proof of Proposition 2, that is, we already implemented the harshest deviation punishment on buyer's deviations. Finally, we show comparison relationships between different bounds.

The proof of "if part" is same as that in Proposition 2 with only one exception. After period k, for the history where trade has never been terminated so far (so the buyer has not revealed her private information about θ), sellers posterior beliefs will no longer be α^*. Bayes' rule implies that,

$$\begin{aligned} \alpha_n &= \alpha^*, n = 1, ..., k+1; \\ \alpha_n &= \frac{\alpha_{n-1}}{\alpha_{n+1} + (1-\alpha_{n-1})\epsilon}, \quad n = k+2, ..., N. \end{aligned} \tag{13}$$

The intuition is as follows. According to our construction, after period k, trade occurs under two circumstances. The first case is in good times, when trade always occurs, no matter whether there comes a low-type or high-type seller. The second case is in bad times and when there comes a high-type seller. Due to this probabilistic asymmetry, observing trade occurs, even without any further knowledge about trade details such as transaction price, is informative for future sellers, who will Bayesian update her posterior belief. Since it is more likely for the trade to happen in good times than in bad times, future sellers will tend to have more optimistic beliefs about θ if they keep observing the trade.

Therefore, $\{\alpha_n\}_n$ is a monotonically increasing sequence and $\pi(\rho_H, \alpha_n)$ monotonically decreasing in n.

The dynamics of V_n now becomes

$$\begin{aligned} V_1 &= \epsilon\pi(\rho_H, \alpha_N) \\ V_n &= \epsilon\pi(\rho_H, \alpha_{N+1-n}) + \epsilon\delta V_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H, 1), \quad n = 2, ..., N-k \\ V_n &= \epsilon\pi(\rho_H, \alpha_{N+1-n}) + (1-\epsilon)\pi(\rho_L, \alpha_{N+1-n}) + \delta V_{n-1} \\ &= \epsilon\pi(\rho_H, \alpha^*) + (1-\epsilon)\pi(\rho_L, \alpha^*) + \delta V_{n-1}, \ n = N-k+1, ..., N \end{aligned}$$

and the auxiliary sequence $\{\underline{V}_n\}_{n=1}^N$ defined as below,

$$\begin{aligned}
\underline{V}_1 &= \epsilon\pi(\rho_H, \alpha_N) \\
\underline{V}_n &= \epsilon\pi(\rho_H, \alpha_{N+1-n}) + \epsilon\delta\underline{V}_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H, 1), \quad n = 2, \ldots, N.
\end{aligned}$$

Notice that $\underline{V}_n$ takes the same value as those in Proposition 2.

We will show that Claim 1 still holds.

Proof of Claim 1. Here we check that inductive steps still hold. It is obvious that $A_1, \ldots, A_{N-k}$ and B_{N-k} holds by construction and the assumption of the theorem.

(I) From A_{n-1} and B_{n-1} to A_n, where $n > N-k$,

$$\begin{aligned}
V_n &= \epsilon\pi(\rho_H, \alpha^*) + (1-\epsilon)\pi(\rho_L, \alpha^*) + \delta V_{n-1} \\
&\overset{(i)}{\geq} \epsilon\pi(\rho_H, \alpha^*) + \epsilon\delta V_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H, 1) \\
&\overset{(ii)}{\geq} \epsilon\pi(\rho_H, \alpha^*) + \epsilon\delta\underline{V}_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H, 1) \\
&\overset{(iii)}{=} \underline{V}_n,
\end{aligned}$$

Again, inequality (i) is from B_{n-1} and inequality (ii) is from A_{n-1}. Since $\underline{V}_n$ has the same value as that in the proof of Proposition 2, equality (iii) still holds. Hence, statement A_n is true.

(II) From A_n and B_{n-1} to B_n, $n > N-k$:

One can show by induction that

$$
\begin{aligned}
\underline{V}_n = & \sum_{i=0}^{n-1}(\delta\epsilon)^i[\epsilon\pi(\rho_H,\alpha_{N+1-n+i})-\epsilon\pi(\rho_H,1)]+(1+\ldots+\delta^{n-1})\epsilon\pi(\rho_H,1) \\
\implies & \pi(\rho_L,\alpha^*)+\delta V_n-\sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) \\
\overset{(iv)}{\geq} & \pi(\rho_L,\alpha^*)+\delta\underline{V}_n-\sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) \\
\overset{(v)}{=} & \pi(\rho_L,\alpha^*)+\delta\sum_{i=0}^{n-1}(\delta\epsilon)^i[\epsilon\pi(\rho_H,\alpha_{N+1-n+i})-\epsilon\pi(\rho_H,1)] \\
& +(\delta+\ldots+\delta^n)\epsilon\pi(\rho_H,1)-\sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) \\
= & \pi(\rho_L,\alpha^*)+\delta\sum_{i=0}^{n-2}(\delta\epsilon)^i[\epsilon\pi(\rho_H,\alpha_{N+1-n+i})-\epsilon\pi(\rho_H,1)]+\delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha_N)-\epsilon\pi(\rho_H,1)] \\
\overset{(vi)}{\geq} & \pi(\rho_L,\alpha^*)+\delta\sum_{i=0}^{n-2}(\delta\epsilon)^i[\epsilon\pi(\rho_H,\alpha_{N+1-(n-1)+i})-\epsilon\pi(\rho_H,1)] \\
& +\delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha_N)-\epsilon\pi(\rho_H,1)]
\end{aligned}
$$

$$
\begin{aligned}
\overset{(vii)}{=} & \pi(\rho_L,\alpha^*)+\delta\underline{V}_{n-1}-\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1)+\delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha_N)-\epsilon\pi(\rho_H,1)] \\
\overset{(viii)}{\geq} & \delta(\delta\epsilon)^{n-1}\cdot[\epsilon\pi(\rho_H,\alpha_N)-\epsilon\pi(\rho_H,1)] \\
\overset{(ix)}{\geq} & 0
\end{aligned}
$$

where inequality (iv) still comes from statement A_n and inequality $(viii)$ is because of statement B_{n-1}. Equalities (v) and (vii) are due to the explicit form of $\underline{V}_n$. Inequalities (vi) and (ix) are due to the monotonicity of $\pi(\rho_H,\cdot)$ and $\{\alpha_n\}_n$. □

The proof of the necessity then follows the same step as those in Proposition 2. In fact, one can easily show that the only deviation that provides current gain is to decline the trade with a low-type seller in bad times. For this reason, order information itself is a sufficient statistics for buyer's deviation. In other words, by monitoring whether there is a trade or not in a previous period, future sellers can identify a previous deviation and Claim 2 guarantees that the harshest punishment is triggered in our construction.

Finally we will prove that $\hat{\rho}_k \geq \rho_k$. Denote V_n^{public} as buyer's bad times continuous payoff under public history, and she offers opaque prices in all previous periods with n sellers left. That is, these are the V_n we derived in the proof of Proposition 2. As $\{\alpha_n\} \geq \alpha^*$ and $\pi(\rho_H, \cdot)$ decreases in α, it can be easily shown by induction that $V_n^{\text{public}} \geq V_n$. From the proof of Proposition 2 and Proposition 3, we learn that the binding IC constraints are at $n = N - k$ and thence $\hat{\rho}_k$ and ρ_k are characterized by the following conditions

$$\begin{aligned} \pi(\rho_k, \alpha^*) + \delta V_{N-k}^{\text{public}} &= \sum_{i=1}^{N-k} \delta^i \epsilon \pi(\rho_H, 1), \\ \pi(\hat{\rho}_k, \alpha^*) + \delta V_{N-k} &= \sum_{i=1}^{N-k} \delta^i \epsilon \pi(\rho_H, 1). \end{aligned} \tag{14}$$

Then $V_{N-k}^{\text{public}} \geq V_{N-k}$ implies that $\hat{\rho}_k \geq \rho_k$. The intuition is as follows. Post-price transparency disables sellers' posterior updating and increases buyer's on-path payoff. This makes IC constraints more slack and opaque pricing strategies easier to sustain. $\square$

Theorem 1. *Fix a risk-aversion coefficient ρ_L,*

- *under the public history, there is no price discovery for a while, then there is perfect price discovery at a certain period; specifically, if a set of parameters where a "k-pooling equilibrium" exists, then price discovery do not occur until the first low-type seller after period k comes;*

- *under the order history, if a set of parameters where a "k-pooling equilibrium" exists, then in such "k-pooling equilibrium"*

  - *no price discovery occurs until period $k + 1$;*
  - *if $\theta = b$ and there comes a low-type seller (hedger) after period $k + 1$, then there is perfect price discovery immediately;*
  - *otherwise there is imperfect price discovery; it is positive price discovery in good times and negative price discovery in bad times;*

Proof. The first bullet point is obvious from the analysis of equilibrium behaviors under the public history. We now show price discovery under the order history is as described in the second bullet point. In fact, from the proof of Proposition 3, we learn that under the order history, in a "k–pooling equilibrium" the buyer offers the opaque price and hides her private information till period k, no matter whether there comes a low or high-type seller. Hence during these periods, sellers' posteriors are still at the prior level and no price discovery occurs.

After period k, if a low-type seller ever comes, then according to the definition of "k-pooling equilibrium", the buyer will offer him different prices when $\theta = g$ and $\theta = b$. Thence, under the order history, future sellers can distinguish different economy states from such an offer. In other words, all future sellers immediate learn about θ and there is perfect discovery.

Finally, if all sellers coming between period $k+1$ and the current period are high-type, then the buyer keeps offering opaque price and has not revealed her private information. According to the Bayesian updating rule in Equation (13), sellers' posteriors $\{\alpha_t\}$ increase with t. Hence, sellers' learning is closer to the true economy when $\theta = g$ and is further away when $\theta = b$. □

Theorem 2. *Fix a risk-aversion coefficient ρ_L; then under the order history, price discovery always occurs weakly earlier than that under the public history. However, under the public history, once price discovery occurs, it is a perfect one.*

Proof. We only need to show that price discovery under the order history is no later than that under the public history. First augment sequences $\{\rho_k\}_{k=1}^N$ and $\{\hat{\rho}_k\}_{k=1}^N$ with $\rho_0 = \hat{\rho}_0 = -\infty$ and $\rho_{N+1} = \hat{\rho}_{N+1} = \infty$. Fix a risk-aversion coefficient ρ_L, then there exists integers m and n in {0,1,...,N} such that $\rho_L \in [\rho_m, \rho_{m+1})$ and $\rho_L \in [\hat{\rho}_n, \hat{\rho}_{n+1})$. According to Proposition 3, $\hat{\rho}_k \geq \rho_k, \forall k$. It follows immediately that $0 \leq n \leq m \leq N$.

By Proposition 3, under the order history, there does not exist a PBE with consistency assumption where the buyer can hide her private information and offer opaque price for both low and high-type seller in period $n+1$. But there exists an "m-pooling equilibrium" as constructed in the proof of Proposition 2. In such an equilibrium the buyer can hide her private information in period $n+1$ under the public history. Before period n, under both histories, in the worst case for the regulator, sellers' posteriors remain as the prior and there is no price discovery. Between period $n+1$ and period m, in the "m-pooling equilibrium" under the public history, no price discovery occurs and sellers' posterior still remain at the prior level. Under the order history, however, if a low-type seller comes, the buyer can not hide her private information and sellers start updating their posteriors after. In other words, price discovery happens weakly earlier under the order history than that under the public history.

□

Corollary 1. *Keep all other things fixed, if ϵ increases, then*

- *under the public history, the cutoff* ρ_k *decreases for all* $k = 1, ..., N$*;*
- *under the order history, the cutoff* $\hat{\rho}_k$ *decreases for all* $k = 1, ..., N$*;*
- *under the order history, sellers' posterior in period* t*, given that the buyer has not yet revealed her private information,* α_{N-t}*, decreases for all* $t = 1, ..., N$*.*

Moreover,

- *if* $\epsilon = 0$*, then* $\rho_k = \hat{\rho}_k = \rho^{pooling}$*, and by the definition of* ρ_L*, only "N-opaque equilibrium" (i.e., the fully separating equilibrium) exists. In other words, when all coming sellers are low-type, then the buyer starts revealing her private information from the first period and there is perfect price discovery at the beginning of the game.*
- *If* $\epsilon = 1$*, then only "0-pooling equilibrium" (i.e., the fully pooling equilibrium) exists. That is, when all coming sellers are high-type, the buyer always hides her private information and there is no price discovery throughout the game.*

Proof. We prove each bullet point one by one.

First, under the public history, from the proof of Proposition 2, cutoffs $\{\rho_k\}_{k=1}^N$ are characterized by Equation (12):

$$\pi(\rho_k, \alpha^*) = -\delta[1 + (\delta\epsilon) + ... + (\delta\epsilon)^{N-k-1}] \cdot [\epsilon\pi(\rho_H, \alpha^*) - \epsilon\pi(\rho_H, 1)]. \tag{15}$$

Because $\pi(\rho_H, \alpha^*) \geq \pi(\rho_H, 1)$, as ϵ increases, the RHS decreases. Due to the fact that $\pi(\rho, \alpha)$ increases with ρ, ρ_k decreases and the "k-pooling equilibrium" is easier to sustained with a higher fraction of high-type sellers.

Next, from the proof of Proposition 3, cutoffs $\{\hat{\rho}_k\}_{k=1}^N$ are characterized by Equation 14:

$$\pi(\hat{\rho}_k, \alpha^*) = -[\delta V_{N-K} - \sum_{i=1}^{N-k} \delta^i \epsilon\pi(\rho_H, 1)], \tag{16}$$

where the dynamics of V_n is:

$$\begin{aligned} V_1 &= \epsilon\pi(\rho_H, \alpha_1) \\ V_n &= \epsilon\pi(\rho_H, \alpha_n) + \epsilon\delta V_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1} \epsilon\delta^i \pi(\rho_H, 1), n \leq N - k. \end{aligned}$$

In other words,

$$\begin{aligned}\delta V_1 - \delta\epsilon\pi(\rho_H,1) &= \delta\epsilon[\pi(\rho_H,\alpha_1)-\pi(\rho_H,1)]\\ \delta V_n - \sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1) &= \delta\epsilon[\pi(\rho_H,\alpha_n)-\pi(\rho_H,1)]+\delta\epsilon[\delta V_{n-1}-\sum_{i=1}^{n-1}\delta^i\epsilon\pi(\rho_H,1)].\end{aligned}$$

Because of $\pi(\rho_H,\alpha) > \pi(\rho_H,1)$ for all $\alpha \in [0,1)$, one can then easily prove by induction that $\delta V_n - \sum_{i=1}^{n}\delta^i\epsilon\pi(\rho_H,1)$ is non-negative and increases with ϵ. Therefore, as ϵ increases, the RHS of Equation (16) decreases and $\hat{\rho}_k$ decreases.

Then, the monotonicity of α_n over ϵ is obvious from the dynamics of $\{\alpha_t\}_{t=1}^{N}$, characterized by Equation (13).

Finally to see results of extreme cases, notice that when $\epsilon = 1$, all sellers have risk-aversion coefficient above $\rho^{pooling}$, therefore, a fully pooling equilibrium can be sustained and the buyer can hide her private information till the end of the game.

On the other extreme, when $\epsilon = 0$, the RHS of Equations (15) and (16) are all 0. So for any k, both ρ_k and $\hat{\rho}_k$ coincide with $\rho^{pooling}$. But by the choice of ρ_L, it is always below $\rho^{pooling}$. Hence, the buyer reveals her private information from the first period of the game. In other words, only a fully separating equilibrium can be sustained. □

Proposition 4. *Given Assumption 1 and Assumption 2, suppose $\rho_L \in [\rho_k,\rho_{k+1})$ under the public history, or $\rho_L \in [\hat{\rho}_k,\hat{\rho}_{k+1})$ under the order history, then there exists a "k-pooling equilibrium" that has the* ***maximal degree of ignorance*** *among all PBE with consistency, no matter which version of the degree of ignorance measure is chosen.*

Proof. We first show the statement is true for the public history. From the discussion in Section 4, as future sellers perfectly observe past prices, there is either no price discovery (when the buyer keeps pooling) or perfect price discovery (when the buyer starts separating) in any pure strategy PBE. Thence, both measures coincide with each other.

When $\rho_L < \rho_{k+1}$, due to the necessity proof in Proposition 2, there does not exist a "$(k+1)$-pooling equilibrium" under the public history. Therefore, in any PBE, the buyer starts revealing her private information in period $k+1$ if there comes a low-type seller. If the buyer is lucky enough to not have a low-type seller until period $t' > k+1$, then in period t', there are less future periods for the buyer than in period $k+1$. A buyer who can not afford opaque pricing for a low-type seller in period $k+1$ will not be able to do this in period t'. Thence, in any PBE, the buyer reveals her private information no later than the first low-type seller after period k comes. In other words, in any PBE with consistency assumption,

$$
\begin{aligned}
\mathbf{E}(\max_{\alpha_T \neq \alpha^*} T) &= \mathbf{E}(\max_{|\alpha_T - \mathbb{1}(\theta=g)| \geq |\alpha^* - \mathbb{1}(\theta=g)|} T) \\
&\leq k + \mathbf{E}(\text{first arrival of a low type}) \\
&= k + \frac{1}{1-\epsilon}. \qquad (17)
\end{aligned}
$$

The construction in the proof of Proposition 2, however, forms a PBE given $\rho_L \geq \rho_k$. In that construction, buyer starts revealing exactly when the first low-type seller after period k comes, which achieves the upper bound in equation (17).

The proof for the order history case is similar. In any PBE, the buyer always offer opaque price for the high-type seller. So for the first version measure, sellers start updating their posteriors once the buyer start offering discriminating price for low-type sellers. The necessity proof in Proposition 3 implies that there does not exist a PBE where the buyer can hide her private information and offer opaque price for both types in period $k+1$. Hence, in any PBE,

$$
\mathbf{E}(\max_{\alpha_T \neq \alpha^*} T) \leq k, \qquad (18)
$$

which is achieved by our construction in the proof.

For the second version of measure, in any PBE with consistency, if the buyer starts pricing discriminatingly for the low-types, but none of them has arrived yet, then there is negative price discovery when $\theta = b$. Such negative price discovery exist until the first low-type seller after the pooling period (when the buyer offers opaque prices for both types) comes. The necessity of $\rho_L \geq \rho_k$ implies that, in any PBE, the pooling period is no longer than k.

$$
\begin{aligned}
\mathbf{E}(\max_{|\alpha_T - \mathbb{1}(\theta=g)| \geq |\alpha^* - \mathbb{1}(\theta=g)|} T) &\leq k + \mathbf{E}(\text{first arrival of a low type}) \\
&= k + \frac{1}{1-\epsilon}, \qquad (19)
\end{aligned}
$$

which is also achieved by the same construction in the proof.

In conclusion we show that the upper bound of the degree of ignorance can be achieved by our consturcted "k-pooling equilibrium". □

Proposition 5. *Given Assumption 3, the following statements are true.*

1. *Under the private history, if and only if $\rho \geq \rho^{pooling}$, there exists a fully pooling equilib-*

rium where the buyer can hide her private information about θ and offer a uniform price in each period.

2. *Under the public history, for any $k = 1,...,N$, there exists $\rho_k^{bargain} \in [\rho_k, \rho^{pooling}]$ such that if and only if $\rho_L \geq \rho_k^{bargain}$, there is a "k−pooling" equilibrium;*

3. *Under the order history, for any $k = 1,...,N$, there exists $\hat{\rho}_k^{bargain} \in [\min\{\rho_k^{bargain}, \hat{\rho}_k\}, \rho^{pooling}]$ such that if and only if $\rho_L \geq \hat{\rho}_k^{bargain}$, there is a "k−pooling" equilibrium.*

Proof. We prove each bullet point one by one.

I: private history

To see this, we only need to show that the opaque price $\hat{p}_t^{\text{opaque}}$ in equilibrium is weakly lower than buyer's worst case evaluation $\mathbf{E}(V(b, z_t))$. Notice that the opaque price now is uniquely determined by the bargaining process. Specifically, when $\theta = b$, $\hat{p}_t^{\text{opaque}}$ divides the surplus from the trade between two parties by a ratio of $\gamma_k : (1 - \gamma_k)$. Thence, if and only if buyer's evaluation of the risky asset is weakly higher than that of the seller, the buyer can collect non-negative profit from the transaction and afford not revealing her private information. As a result, the IC constraint stays the same and we have exactly same cutoff $\rho^{pooling}$ for the opaque pricing to be sustained.

II: public history and order history

Without loss of generality, we can focus on a case with a more general notations of $\{\alpha_n\}$ where $\alpha_1 = ... = \alpha_{k+1} = \alpha^*$. To see that statements are true for the public history case, we can then restrict $\alpha_n = \alpha^*$ for all $n = 1,...,N$. Similarly to see that statements hold under the order history, we can restrict that $\alpha_1 = ... = \alpha_{k+1} = \alpha^*$ and $\{\alpha_n\}$ follows the evolving rule in (13) for $n = k+2,...,N$.

First of all, we adopt the same construction as that in the proof of Proposition 2. Similarly, we now characterize buyer's IC constraint where she is willing to hide her private information. We adopt similar notations and let $V_n^{\text{bargaining}}$ denotes her continuation payoff when $\theta = b$ and when she has not revealed her private information (either by pricing differentiately under the public history or declining trade under the order history).

The dynamics of $V_n^{\text{bargaining}}$ is as follows.

$$
\begin{aligned}
V_1^{bargain} &= \gamma_{N-1}\epsilon\pi(\rho_H, \alpha_N) \\
V_n^{bargain} &= \epsilon[\gamma_{N-n}\pi(\rho_H, \alpha_{N+1-n}) + \delta V_{n-1}^{bargain}] + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\gamma_{N-n+i}\epsilon\pi(\rho_H, 1), \quad n = 2, \ldots, N-k. \\
V_n^{bargain} &= \gamma_{N-n}[\epsilon\pi(\rho_H, \alpha_{N+1-n}) + (1-\epsilon)\pi(\rho_L, \alpha_{N+1-n})] + \delta V_{n-1}^{bargain} \\
&= \gamma_{N-n}[\epsilon\pi(\rho_H, \alpha^*) + (1-\epsilon)\pi(\rho_L, \alpha^*)] + \delta V_{n-1}^{bargain}, \quad n = N-k+1, \ldots, N.
\end{aligned}
$$

Buyer's IC constraints, which are also sufficient conditions for the constructed strategies and beliefs to form a PBE, are

$$
\gamma_{N-n}\pi(\rho_L, \alpha^*) + \delta V_{n-1}^{bargain} \geq 0 + \sum_{i=1}^{n-1}\delta^i \cdot \gamma_{N-n+i}\epsilon\pi(\rho_H, 1), \quad n = N-k+1, \ldots, N. \tag{20}
$$

Each inequality in condition (20) provides a lower bound of ρ_L. Then $\rho_k^{bargain}$ and $\hat{\rho}_k^{bargain}$ are characterized by the greatest lower bound.

The necessity proof follows exactly from the fact that we have already implemented the harshest deviation punishment possible (min-max payoffs) in our construction. The comparison between $\rho_k^{bargain}$ and $\hat{\rho}_k^{bargain}$ also follows the similarly step as in the proof of Proposition 3. In fact, under the order history sellers' posterior belief start increasing after period $k+1$, implying less continuation payoff $V_n^{bargain}$ than under the public history. Thence any lower bound in equation (20) is weakly greater under the order history than that under the public history. As a result, the maximum of those lower bounds under the order history is also weakly higher than that under the public history, which is exactly $\hat{\rho}_k^{bargain} \geq \rho_k^{bargain}$.

It is then obvious that $\rho_k^{bargain}$ and $\hat{\rho}_k^{bargain}$ are no greater than $\rho^{pooling}$. To see this, notice that if $\rho_L \geq \rho^{pooling}$, then $\pi(\rho_L, \alpha^*) \geq 0$, and hence inequalities (20) are all satisfied. In other words, when $\rho_L \geq \rho^{pooling}$, there exists a fully pooling equilibrium and the opaque pricing strategies can always be sustained under the public history or the order history.

Finally, we want to show that $\rho_k^{bargain} \geq \rho_k$ and $\hat{\rho}_k^{bargain} \geq \hat{\rho}_k$. In fact, we can show that if

$$
P_k \equiv \gamma_{N-k}\pi(\rho_L, \alpha^*) + \delta V_{k-1}^{bargain} - \sum_{i=1}^{k-1}\delta^i \cdot \gamma_{N-k+i}\epsilon\pi(\rho_H, 1) \geq 0,
$$

then

$$Q_k \equiv \pi(\rho_L, \alpha^*) + \delta V_{k-1} - \sum_{i=1}^{k-1} \delta^i \cdot \epsilon\pi(\rho_H, 1) \geq 0.$$

We prove this by induction. One can prove by induction that for $n = 2, ..., k$

$$V_n^{bargain} = \sum_{i=0}^{n-1} (\delta\epsilon)^i \gamma_{N-n+i}[\epsilon\pi(\rho_H, \alpha_{N+1-n+i}) - \epsilon\pi(\rho_H, 1)] + \sum_{i=0}^{n-1} \delta^i \gamma_{N-n+i} \epsilon\pi(\rho_H, 1)$$

Then we have

$$\begin{aligned}
Q_k &= \pi(\rho_L, \alpha^*) + \delta V_{k-1} - \sum_{i=1}^{k-1} \delta^i \cdot \epsilon\pi(\rho_H, 1) \\
&= \pi(\rho_L, \alpha^*) + \delta \sum_{i=0}^{k-1} (\delta\epsilon)^i [\epsilon\pi(\rho_H, \alpha_{N+1-k+i}) - \epsilon\pi(\rho_H, 1)] \\
&\geq \pi(\rho_L, \alpha^*) + \delta \sum_{i=0}^{k-1} (\delta\epsilon)^i \frac{\gamma_{N-k+i}}{\gamma_{N-k}} [\epsilon\pi(\rho_H, \alpha_{N+1-k+i}) - \epsilon\pi(\rho_H, 1)] \\
&= \pi(\rho_L, \alpha^*) + \frac{1}{\gamma_{N-k}} \delta V_{k-1}^{bargain} - \frac{1}{\gamma_{N-k}} \sum_{i=1}^{k-1} \delta^i \cdot \gamma_{N-k+i} \epsilon\pi(\rho_H, 1) \\
&= \frac{1}{\gamma_{N-k}} P_k \geq 0,
\end{aligned}$$

which completes the proof.

□

Proposition 6. *If the buyer has constant outside option, then there exist cutoffs* $\rho_{outside}^{pooling} < \rho_{outside}^{separating}$ *such that*

- *a fully pooling equilibrium exists if and only if* $\rho \geq \rho_{outside}^{pooling}$*;*
- *a fully separating equilibrium exists if and only if* $\rho \geq \rho_{outside}^{separating}$*. Moreover, in such a fully separating equilibrium, when* $\theta = b$ *no trade occurs and the buyer gets an expected payoff of* π^* *from the secondary channel.*

Proof. We repeat the analysis in the proof of Proposition 1. The indifferent price for the seller to accept and reject the offer still remains as $\mathbf{E}(V(\theta, \alpha)) - \frac{\rho_t}{2}\text{Var}(V(\theta, z_t)|\alpha)$. We use the same construction. Then, buyer's binding constraint now becomes that when $\theta = b$, her expected profit from trading in the OTC market is no less than π^*. Hence, we have

$$\begin{aligned} & \mathbf{E}(V(b,z_t)) - \mathbf{E}(V(\theta,z_t)|\alpha) + \frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha) \geq \pi^*, \\ \Rightarrow \quad & \rho_t \geq \rho_{outside}^{pooling} \equiv \frac{2[\mathbf{E}(V(\theta,z_t)|\alpha) - \mathbf{E}(V(b,z_t)) + \pi^*]}{\mathrm{Var}(V(\theta,z_t)|\alpha)}. \end{aligned} \tag{21}$$

Similarly, for the fully separating equilibrium, same analysis applies. Now the binding constraint becomes that the buyer does not want to pretend it is $\theta = g$ when it is actually $\theta = b$. That is,

$$\begin{aligned} & \mathbf{E}(V(b,z_t)) - p_t^g \leq \pi^*, \\ \Rightarrow \quad & \mathbf{E}(V(b,z_t)) - \mathbf{E}(V(g,z_t)) + \frac{\rho_t}{2}\mathrm{Var}(V(g,z_t)) \leq \pi^*, \\ \Rightarrow \quad & \rho_t \leq \rho_{outside}^{separating} \equiv \frac{2[\mathbf{E}(V(g,z_t)) - \mathbf{E}(V(b,z_t)) + \pi^*)}{\mathrm{Var}(V(g,z_t))]}. \end{aligned} \tag{22}$$

The necessity proof for both equilibria follows exactly same steps as in Proposition 1. It is then obvious that $\rho_{outside}^{pooling} < \rho_{outside}^{separating}$. □

Proposition 7. *If the buyer has constant outside option, then there exists* $\{\rho_{k,outside}\}_k$ *and* $\{\hat{\rho}_{k,outside}\}_k$ *such that*

- $\rho_{N,outside} = \hat{\rho}_{N,outside} = \rho_{outside}^{pooling}$;
- $\rho_{N-1,outside} = \hat{\rho}_{N-1,outside} \leq \rho_{outside}^{pooling}$;
- *for* $k = 1,...,N$, $\hat{\rho}_{k,outside} \geq \rho_{k,outside}$;
- $\{\rho_{k,outside}\}_k$ *and* $\{\hat{\rho}_{k,outside}\}_k$ *increase in* k;
- *under the public history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \rho_{k,outside}$, $k = 1,...,N$;
- *under the order history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \hat{\rho}_{k,outside}$, $k = 1,...,N$;

Proof. As in the proof of Proposition 5, to simplify our presentation, here we focus on the more general version of notation for $\{\alpha\}_n$: $\alpha_1 = ... = \alpha_{k+1} = \alpha^*$. To see that statements are true for the public history case, we can then restrict $\alpha_n = \alpha^*$ for all $n = 1,...,N$. Similarly to see that statements hold under the order history, we can restrict that $\alpha_1 = ... = \alpha_{k+1} = \alpha^*$ and $\{\alpha_n\}$ follows the evolving rule in (13) for $n = k+2,...,N$.

The dynamics of V_n, the continuation payoff for the buyer who has not revealed her private information when $\theta = b$, now becomes

$$\begin{aligned} V_1 &= \epsilon\pi(\rho_H, \alpha_N) + (1-\epsilon)\pi^* \\ V_n &= \epsilon\pi(\rho_H, \alpha_{N+1-n}) + (1-\epsilon)\pi^* + \epsilon\delta V_{n-1} + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i[\epsilon\pi(\rho_H, 1) + (1-\epsilon)\pi^*], n = 2, \dots, N-k \\ V_n &= \epsilon\pi(\rho_H, \alpha^*) + (1-\epsilon)\pi(\rho_L, \alpha^*) + \delta V_{n-1}, \quad n = N-k+1, \dots, N. \end{aligned}$$

Buyer's IC constraint becomes

$$\pi(\rho_L, \alpha^*) + \delta V_{n-1} \geq \pi^* + \sum_{i=1}^{n-1}\delta^i \epsilon\pi(\rho_H, 1), \quad n = N-k+1, \dots, N. \tag{23}$$

Simply replace $V_n' = V_n - (1 + \delta + \dots + \delta^{n-1})\pi^*$ and $\pi'(\rho, \alpha) = \pi(\rho, \alpha) - \pi^*$, we obtain

$$\begin{aligned} V_1' &= \epsilon\pi'(\rho_H, \alpha_N) \\ V_n' &= \epsilon\pi'(\rho_H, \alpha_{N+1-n}) + \epsilon\delta V_{n-1}' + (1-\epsilon)\sum_{i=1}^{n-1}\delta^i\epsilon\pi'(\rho_H, 1), \quad n = 2, \dots, N-k \\ V_n' &= \epsilon\pi'(\rho_H, \alpha^*) + (1-\epsilon)\pi'(\rho_L, \alpha^*) + \delta V_{n-1}', \qquad n = N-k+1, \dots, N. \end{aligned}$$

With buyer's IC constrains:

$$\pi'(\rho_L, \alpha^*) + \delta V_{n-1}' \geq 0 + \sum_{i=1}^{n-1}\delta^i\epsilon\pi'(\rho_H, 1), \quad n = N-k+1, \dots, N. \tag{24}$$

Notice this is exactly the same problem as those in Proposition 2 and Proposition 3. Hence, the rest of the proof immediately follows. □

Proposition 8. *There exists $0 < \rho_{\chi_t=-1}^{pooling} \leq \rho_{\chi_t=-1}^{separating}$ such that:*

(1) if and only if the buyer's risk-aversion coefficient $\rho \geq \rho_{\chi_t=-1}^{pooling}$, there exists a fully pooling equilibrium;

(2) if and only if the buyer's risk-aversion coefficient $\rho \leq \rho_{\chi_t=-1}^{separating}$, there exists a fully separating equilibrium. Moreover, in such an equilibrium, when $\theta = g$, the seller will offer a price high enough so no trade occurs.

Proof. We first prove both bullet points one by one, followed by the proof of comparison relationship between $\rho_{\chi_t=-1}^{pooling}$ and $\rho_{\chi_t=-1}^{separating}$.

First, in a fully pooling equilibrium, buyers learn nothing from seller's offer. Therefore, they all hold the prior belief and the indifferent price for them to accept the ask offer from the seller is

$$\mathbf{E}(V(\theta,z_t)|\alpha^*)+\frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*).$$

Thence, for a seller to have no incentive to deviate and decline a trade, we need this indifferent price to be weakly higher than her evaluation of the risky asset, especially when $\theta=g$. Hence, $\rho^{pooling}_{\chi_t=-1}$ is characterized by

$$\begin{aligned}&\mathbf{E}(V(\theta,z_t)|\alpha^*)+\frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*)\geq \mathbf{E}(V(g,z_t)),\\ \Rightarrow\quad &\rho_t\geq\rho^{pooling}_{\chi_t=-1}\equiv\frac{2[\mathbf{E}(V(g,z_t))-\mathbf{E}(V(\theta,z_t)|\alpha^*)]}{\mathrm{Var}(V(\theta,z_t)|\alpha^*)}.\end{aligned}$$

The proof of sufficiency mirrors exactly that in the proof of Proposition 1, with the replacement of "$\theta=b$" with "$\theta=g$" and p_t^{opaque} as $\mathbf{E}(V(\theta,z_t)|\alpha^*)+\frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*)$.

We now characterize $\rho^{separating}_{\chi_t=-1}$ and show the second bullet point. Again, the proof mirrors exactly that in Proposition 1. The seller should not have any incentive to offer an equilibrium price of $\theta=g$ when the underlying economy is actually bad. Therefore, no trade can occur when $\theta=g$ for seller's IC constraint to hold. Seller's IC constraint, in the other direction, requires that when $\theta=g$, the seller has no incentive to pretend $\theta=b$ and offer the bad times price, p_t^b. For that reason, seller's evaulation for the risky asset when $\theta=g$ should be weakly higher than p_t^b, which is also weakly higher than buyer's evaluation after learning that $\theta=b$. After observing the bad times offer, a Bayesian buyer will update his belief and is convinced that $\theta=b$. The indifferent price for this buyer to accept the ask offer now becomes

$$\mathbf{E}(V(b,z_t))+\frac{\rho_t}{2}\mathrm{Var}(V(b,z_t)).$$

In other words, one can derive that:

$$\begin{aligned}&\mathbf{E}(V(b,z_t))+\frac{\rho_t}{2}\mathrm{Var}(V(b,z_t))\leq \mathbf{E}(V(g,z_t)),\\ \Rightarrow\quad &\rho_t\leq\rho^{separating}_{\chi_t=-1}\equiv\frac{2[\mathbf{E}(V(g,z_t))-\mathbf{E}(V(b,z_t))]}{\mathrm{Var}(V(b,z_t))}.\end{aligned}$$

The sufficiency proof uses the exact construction in the proof of Proposition 1, with the switch between $\theta=g$ and $\theta=b$ and p_t^b as $\mathbf{E}(V(b,z_t))+\frac{\rho_t}{2}\mathrm{Var}(V(b,z_t))$.

The last but not the least, $0<\rho^{pooling}_{\chi_t=-1}\leq\rho^{separating}_{\chi_t=-1}$ due to our parameter assumptions of the asset value in inequalities (1). $\square$

Proposition 9. *There exists* $\{\rho_{k,\chi_t=-1}\}_k$ *and* $\{\hat{\rho}_{k,\chi_t=-1}\}_k$ *such that*

- $\rho_{N,\chi_t=-1} = \hat{\rho}_{N,\chi_t=-1} = \rho^{pooling}_{\chi_t=-1}$;
- $\rho_{N-1,\chi_t=-1} = \hat{\rho}_{N-1,\chi_t=-1} \leq \rho^{pooling}_{\chi_t=-1}$;
- *for* $k = 1,\dots,N$, $\hat{\rho}_{k,\chi_t=-1} \geq \rho_{k,\chi_t=-1}$
- $\{\rho_{k,\chi_t=-1}\}_k$ *and* $\{\hat{\rho}_{k,\chi_t=-1}\}_k$ *increase in k;*
- *under the public history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \rho_{k,\chi_t=-1}$, $k = 1,\dots,N$;
- *under the order history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \hat{\rho}_{k,\chi_t=-1}$, $k = 1,\dots,N$;

Proof. We now denote $\pi_{\chi_t=-1}(\rho,\alpha)$ as the static surplus from the trade when $\theta = g$ if buyer's risk-aversion coefficient is ρ and posterior belief is α.

$$\underbrace{\pi_{\chi_t=-1}(\rho,\alpha)}_{\substack{\text{ex-post surplus}\\ \text{from the trade}}} \equiv \underbrace{\left[\mathbf{E}(V(\theta,z_t)|\alpha) + \frac{\rho}{2}\mathrm{Var}(V(\theta,z_t)|\alpha)\right]}_{\text{buyer's reservation price for the risky asset}} - \underbrace{\mathbf{E}(V(g,z_t))}_{\substack{\text{seller's reservation price}\\ \text{for the risky asset}}}. \tag{25}$$

We then repeat the same steps as those of Proposition 2 and Proposition 3. For instance, the construction now becomes:

- if there is no deviation in previous periods, then
  - for $t = 1,\dots,k$, the seller offers $p_t = \mathbf{E}(V(\theta,z_t)|\alpha^*) + \frac{\rho_t}{2}\mathrm{Var}(V(\theta,z_t)|\alpha^*)$.
  - for $t = k+1,\dots,N$, the seller offers

$$p_t \begin{cases} = \mathbf{E}(V(\theta,z_t)|\alpha_t) + \frac{\rho_H}{2}\mathrm{Var}(V(\theta,z_t)|\alpha_t) & \text{if } \rho_{t'} = \rho_H, t' = 1,\dots,t \\ = \mathbf{E}(V(b,z_t)) + \frac{\rho_H}{2}\mathrm{Var}(V(b,z_t)) & \text{if } \rho_t = \rho_H \text{ and } \rho_{t'} = \rho_L \\ & \text{for some } t' \in [k+1, t-1]\,; \\ = \mathbf{E}(V(b,z_t)) + \frac{\rho_L}{2}\mathrm{Var}(V(b,z_t)) & \text{if } \rho_t = \rho_L \text{ and } \theta = b \\ > \mathbf{E}(V(g,z_t)) + \frac{\rho_L}{2}\mathrm{Var}(V(g,z_t)) & \text{if } \rho_t = \rho_L \text{ and } \theta = g \end{cases}$$

  - after observing an on-path price offer, the buyer updates his posterior based on past trade history as well as the current period price offer;

- buyers accept any offer weakly below their evaluations about the asset, given their posterior belief;

- if there is deviation observed in previous periods, then buyers will hold the belief that $\theta = b$ and the seller offers a price

$$p_t \begin{cases} = \mathbf{E}(V(b,z_t)) + \frac{\rho_t}{2}\text{Var}(V(b,z_t)) & \text{if } \rho_t = \rho_H \text{ or } \theta = b \\ > \mathbf{E}(V(g,z_t)) + \frac{\rho_t}{2}\text{Var}(V(g,z_t)) & \text{if } \rho_t = \rho_L \text{ and } \theta = g \end{cases};$$

  buyers update their beliefs whenever the Bayes' rule applies, or believe that $\theta = b$ otherwise; buyers accept any offer weakly below their evaluations about the risky asset, given the constructed beliefs.

Notice that under the order history, buyers' posteriors monotonically decrease over the time given that the seller has not declined trade yet. Therefore,

$$\pi_{\chi_t=-1}(\rho,\alpha^*) \geq \pi_{\chi_t=-1}(\rho,\alpha_{N+1-n+i}) \geq \pi_{\chi_t=-1}(\rho,\alpha_{N+1-(n-1)+i}) \geq \pi_{\chi_t=-1}(\rho,0), \quad \forall \rho \geq 0$$

still holds. The rest of the proof then exactly mirrors, with the switch of $\theta = b$ and $\theta = g$, and the replacement of $\pi(\rho_H, 1)$ with $\pi_{\chi_t=-1}(\rho_H, 0)$. □

Proposition A.1. *If $\rho_L < \rho^{pooling} < \rho_H \leq \rho^{separating}$, then there exists $\{\underline{\rho}_k\}_k$ and $\{\underline{\hat{\rho}}_k\}_k$ such that*

- $\underline{\rho}_N = \underline{\hat{\rho}}_N = \rho^{pooling}$;
- $\underline{\rho}_{N-1} = \underline{\hat{\rho}}_{N-1} \leq \rho^{pooling}$;
- *for* $k = 1, \ldots, N$, $\underline{\hat{\rho}}_k \geq \underline{\rho}_k$;
- $\{\underline{\rho}_k\}_k$ *and* $\{\underline{\hat{\rho}}_k\}_k$ *increase in k;*
- *under the public history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \underline{\rho}_k$, $k = 1, \ldots, N$;
- *under the order history, there exists a "k–pooling equilibrium" if and only if* $\rho_L \geq \underline{\hat{\rho}}_k$, $k = 1, \ldots, N$;

Proof. We repeat the same analysis as those in the proof of Proposition 2 and Proposition 3. Specifically, we re-construct the "k–pooling equilibrium" with slight modification:

In a "k-pooling equilibrium", the buyer does not reveal her private information until period $k+1$. Specifically,

- if there is no deviation in previous periods, then
  - for $t = 1, \ldots, k$, the buyer offers $p_t = \mathbf{E}(V(\theta, z_t)|\alpha^*) - \frac{\rho_t}{2}\mathrm{Var}(V(\theta, z_t)|\alpha^*)$, where ρ_t is the risk-aversion coefficient of seller coming in period t;
  - for $t = k+1, \ldots, N$, the buyer offers

$$p_t \begin{cases} = \mathbf{E}(V(\theta, z_t)|\alpha^*) - \frac{\rho_H}{2}\mathrm{Var}(V(\theta, z_t)|\alpha^*) & \text{if } \rho_{t'} = \rho_H, t' = 1, \ldots, t \\ = \mathbf{E}(V(g, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(g, z_t)) & \text{if } \rho_{t'} = \rho_L \text{ for some } t' \in [k+1, t] \text{ and } \theta = g \\ < \mathbf{E}(V(b, z_t)) - \frac{\rho_t}{2}\mathrm{Var}(V(b, z_t)) & \text{if } \rho_{t'} = \rho_L \text{ for some } t' \in [k+1, t] \text{ and } \theta = b \end{cases};$$

  - after observing an on-path price offer, the seller updates his posterior based on past trade history as well as the current period price offer;
  - sellers accept any offer weakly above their evaluation about the asset, given their posterior belief;
- if there is deviation observed in previous periods, then a fully separating equilibrium is played.

The rest of the proof are exactly same with the additional restriction that $\pi(\rho_H, 1) = 0$.

□