The Role of Managerial Protections in the Effect of Post-crisis Stress-testing on U.S. Bank Lending*

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Abstract

We show that bank stress-testing after the 2008 financial crisis led to a relative increase in lending for banks with strong managerial protections compared to banks with weak protections. We then explain this finding with a model in which managerial protections can provide an incentive for banks to increase lending in response to lower debt ratios, depending on other firm characteristics. We show that the model accurately predicts when protections are likely to generate a stronger a pro-investment effect. The model is also consistent with other documented facts concerning the effects of managerial protections on capital structure and investment.

Keywords: D24, G32.

JEL Classification: Managerial protections, Stress testing, Banking, Capital structure, Ownership structure, Investment.

*This Draft: March 2020. We thank Daron Acemoglu, Ricardo Caballero, Andrey Malenko, Rob Townsend and Ivan Werning for very helpful and constructive comments. Bosshardt: Department of Economics, Massachusetts Institute of Technology (MIT), Cambridge, MA 02139, USA. Email: jbosshar@mit.edu, Kakhbod: Department of Economics, Massachusetts Institute of Technology (MIT), Cambridge, MA 02139, USA. Email: akakhbod@mit.edu.

1 Introduction

There are numerous conflicting perspectives on the implications of bank capital ratios for lending. On the one hand, the safety from a large capital buffer can lead banks to decrease lending because they have more severe financing frictions (Diamond and Rajan (2000)) or higher charter values that motivate them to lend more cautiously (Keeley (1990)). On the other hand, a greater capital stock can increase the capacity to make loans by improving monitoring (Holmstrom and Tirole (1997)) or by mitigating debt overhang (Admati et al. (2013)). These channels are relevant for understanding the effects of policies that constrain bank debt ratios. Several recent empirical studies have focused on the introduction of the bank stress tests after the 2008 financial crisis, which function like forward-looking capital requirements (Acharya, Berger and Roman (2018)). Some of these studies have shown that the stress tests were negatively associated with lending (Acharya, Berger and Roman (2018) and Cortés et al. (2018)), while others have found weaker or contrary effects (Bassett and Berrospide (2018)). To understand the effect of capital requirements on lending, it is important to consider how they interact with other major determinants of investment more generally.

A relevant consideration is the degree of managerial protections, which refers to antitakeover laws, restricted shareholder rights, and other laws and provisions that could enable a manager to obtain a private benefit from controlling a firm. Managerial protections have been shown to affect investment through a variety of channels. On the one hand, they have been associated with a greater levels of potentially inefficient investments (Gompers, Ishii and Metrick (2003), Giroud and Mueller (2011)), possibly by providing greater freedom for managers to build "empires". On the other hand, Bertrand and Mullainathan (2003) show that managerial protections can lead to reductions in potentially efficient investments in a way that suggests that they allow managers to enjoy a "quiet life". However, less is known about the role of managerial protections in the banking industry, which is a unique context due to its relatively high degree of regulation.

This paper first provides empirical evidence that the introduction of stress testing led to a relative increase in lending for bank holding companies (BHCs) with strong managerial protections compared to BHCs with weak managerial protections. We then explain this finding with a general model of firm investment in the presence of managerial protections. Managerial protections provide an incentive for managers to pursue unprofitable projects to obtain a private benefit of control. Debt and the level, or scale, of investment both reduce the tendency for managers to pursue unprofitable projects. Consequently, if regulations restrict debt, then investment may increase as a partial substitute for debt in disciplining managers. However, debt-constraining regulations can also reduce the return on investment by requiring firms to choose a suboptimal capital structure. The dominant effect in the model depends on other firm characteristics, such as the distribution of earnings. Finally, we show that the model's prediction of when the pro-investment effect is likely to be relatively strong is qualitatively consistent with the empirical findings.

To empirically examine the interaction between managerial protections and regulations targeting capital structure, we consider how BHCs adjusted lending in response to the stress tests conducted by the Federal Reserve. The stress tests require a subset of large banks to maintain sufficient capital to lend under potential adverse scenarios. The stress tests function like forward-looking capital requirements and have been associated with increased capital ratios (Acharya, Berger and Roman (2018)). We use a difference-in-differences specification to compare how the introduction of stress-testing in 2009 differentially affected lending for BHCs with strong managerial protections relative to BHCs with weak managerial protections. We represent the degree of managerial protections with the "Governance Index" or G-index constructed in Gompers, Ishii and Metrick (2003), which represents the number of firm provisions and state laws that provide defenses against shareholder actions and takeover threats. Using regulatory FR Y-9C data covering the subset of BHCs that were included in the initial stress tests conducted by the Federal Reserve, we find that lending increased for stress-tested BHCs with a high G-index relative to stress-tested BHCs with a low G-index. Additionally, this effect is stronger among firms with a relatively concave earnings distribution cdf.

We then explain these results with a general model of capital structure, investment, and managerial protections. In the model, a firm chooses its value-maximizing capital structure and level of investment while taking into account the incentives of managers as well as regulatory restrictions that constrain debt ratios. The manager of the firm then learns about the quality of the investment project during development and can choose to terminate or complete it. Note that in applying the model to the context of the banking industry, investment is interpreted as lending.

More specifically, the firm's optimal capital structure depends on three frictions. First, private information about the firm's return on investment substantiates a distinction between equityholders and creditors. In particular, in addition to investing his or her own wealth, the manager can finance the firm with external funds from informed investors, who are given equity shares, and uninformed investors, who are given debt claims. Second, the benefit of debt extends from allowing the manager to obtain a private benefit from controlling projects that are brought to completion. In particular, debt disciplines managers from completing unprofitable projects in order to obtain the private benefit of control. Third, the cost of debt extends from the existence of liquidation costs that are incurred when the firm defaults.

The firm's optimal level of investment depends on its capital structure through two channels. On the one hand, distortions to the firm's capital structure reduce the return on investment. On the other hand, policies that constrain firm debt ratios, such as the stress tests and capital requirements more generally, can undermine the effect of debt on disciplining the manager. At the same time, under the canonical assumptions of a linear liquidation cost and a concave expected return, the manager's incentive to liquidate unprofitable projects is increasing in the level of investment. As a result, the optimal level of investment can increase in response to such policies since it serves as a partial substitute for debt in disciplining the manager.

The dominant channel determining the firm's investment response to policies that constrain debt ratios depends on other firm characteristics. For example, we show that the pro-investment channel dominates when the distribution of the firm's earnings has a concave cdf, whereas the anti-investment channel dominates when the distribution of earnings has a convex cdf. In applying this result to the context of the bank stress tests, we interpret it to imply that the relative increase in investment for BHCs with strong managerial protections after the introduction of the stress tests should be stronger among the subset of BHCs with concave earnings distributions. This prediction is consistent with the empirical finding that the stress tests were associated with a greater relative increase in investment for BHCs with a relatively concave return distribution cdf.

Finally, we show that an extended model allowing for managerial choice of capital structure and investment can speak to other facts from the corporate governance literature concerning the effects of managerial protections on debt and investment. We consider two additional results of the extended model. First, the model illustrates channels by which protections can cause managers to issue either too much or too little debt. On the one hand, protections can intensify the incentive for managers to increase their voting power by financing the firm with debt rather than issuing shares to outside equityholders. On the other hand, protections can lead managers to issue less debt in order to reduce the probability of liquidation. The first channel is consistent with the positive association between managerial protections and debt ratios documented in John and Litov (2010).

Second, the model offers a novel explanation for why managerial protections can lead to reduced investment, which is consistent with the evidence in Bertrand and Mullainathan (2003).[1] In particular, if managers can only obtain the private benefit of control from projects that are brought to completion, and if additionally the probability of liquidating a project is increasing in the level of investment, then managers may underinvest in order to increase their expected private benefit of control.[2]

Literature Review

This paper relates to four strands of the literature.

This paper contributes to the literature on the effect of bank regulations on lending by presenting a novel channel through which policies that constrain debt ratios, such as bank stress tests and capital requirements, affect investment. The theoretical literature on the role of bank capital offers mixed predictions regarding the effect of a tightening. One perspective is that the safety from a larger capital buffer could result in more severe financing frictions (Diamond and Rajan (2000)) or higher charter values (Keeley (1990)), leading banks to lend more conservatively.[3] Another perspective is that a greater capital stock can improve monitoring (Holmstrom and Tirole (1997)) and mitigate debt overhang (Admati et al. (2013)), allowing banks to lend more. It could also increase the

[1] Note that this result differs from the earlier result concerning the pro-investment effect of the stress tests in two major respects. First, this result allows the manager to choose the level of investment, whereas the previous result considered the firm's value-maximizing level of investment. Second, this result focuses on the level of investment conditional on a given capital structure, whereas the previous result focuses on how distortions to capital structure affect the level of investment.

[2] Note that the model can also be extended to incorporate "empire building" preferences that would provide an incentive for managers to overinvest, similar to the empirical findings in Gompers, Ishii and Metrick (2003) and Giroud and Mueller (2011).

[3] Additionally, Thakor (1996) shows that risk-based capital requirements that increase the cost of loans relative to safe investments can also decrease lending.

value of loans by increasing the probability that a bank remains operational to collect the cashflows (Bahaj and Malherbe (2018)). Our model is similar to the latter view in that we illustrate a channel by which policies that constrain debt ratios can increase lending. However, we propose a novel mechanism based on the effect of debt on corporate governance.

This paper relates more generally to a stream of papers on the advantages and disadvantages of bank capital requirements (Thakor (2014)). For example, Diamond and Rajan (2000), Diamond and Rajan (2001) and DeAngelo and Stulz (2015) provide theoretical evidence that tightening capital requirements may distort banks' provision of liquidity services, while Dewatripont and Tirole (2012) show that stricter capital requirements may introduce governance problems. However, Hellmann and Stiglitz (2000), Morrison and White (2005), Repullo (2004) and Acharya and Thakor (2015) argue that stringent capital regulation can induce prudent behavior by banks. We add to this literature by showing that the effect of capital requirements could crucially depend on the extent of managerial protections.[4]

This paper also relates to the empirical literature on the effect of tightening capital requirements on lending. Some papers find a negative effect. For example, Acharya, Berger and Roman (2018) and Cortés et al. (2018) find that the U.S. stress tests led banks to decrease credit supply. Fraisse, Lé and Thesmar (2020) and Gropp et al. (2019) present additional evidence from other contexts that increasing capital requirements can reduce lending. Other work has found a weak effect or even a positive effect in some specifications (Bassett and Berrospide (2018)). Rather than estimating the overall effect of capital requirements on lending, we isolate the role of managerial protections by showing that stress-tested BHCs with strong protections increased lending relative to those with weak protections.

This paper also contributes to the literature on the effects of managerial protections on firm performance by illustrating its distortionary effect on capital structure and invest-

[4] This paper also relates more generally to a vast literature on banking. See, for example, work on banking failures and crises (Caballero (2010), Caballero and Krishnamurthy (2008), Ashcraft (2005)), Baltensperger (1980), contagion (Acemoglu, Ozdaglar and Tahbaz-Salehi (2015)), shadow banking (Gennaioli, Shleifer and Vishny (2013)), loan supply effects of monetary policy and bank financing constraints (Kashyap, Stein and Wilcox (1993), Paravisini (2008)), corporate governance (Ivashina et al. (2009)), the role of supervisory policies on bank lending and external financing (Beck, Demirguc-Kunt and Levine (2006)), and monopoly banking with uncertainty (Prisman, Slovin and Sushka (1986)).

ment. Gompers, Ishii and Metrick (2003) show that managerial protections are associated with worse firm performance across a variety of indicators, including stock returns, firm valuation, profits, and sales growth. Giroud and Mueller (2011) show that managerial protections are additionally associated with lower labor productivity, higher input costs, and value-destroying acquisitions. Bertrand and Mullainathan (2003) present causal evidence suggesting that the introduction of antitakeover laws led to reduced firm productivity and profitability, possibly due to weaker incentives for managers to keep costs down and initiate value-enhancing changes. We contribute to this literature by considering additional mechanisms related to how managerial protections distort capital structure and investment decisions.[5]

2 Empirical motivation

To motivate the model presented in Section 3, this section presents empirical evidence indicating that the introduction of the stress tests was associated with a relative increase in lending for banks with relatively strong managerial protections.

2.1 Data: G-index and FR Y-9C

To examine how policies that constrain debt ratios interact with managerial protections in determining investment, we combine two data sources: the "Governance Index" or G-index constructed by Gompers, Ishii and Metrick (2003) and quarterly regulatory FR Y-9C reports filed by bank holding companies (BHCs) that were included in the initial stress tests conducted by the Federal Reserve.

The G-index represents the extent to which managers are protected by firm-level provisions and state laws that protect managers from shareholder actions and takeover threats. Following Gompers, Ishii and Metrick (2003), we omit firms with dual-class stock.

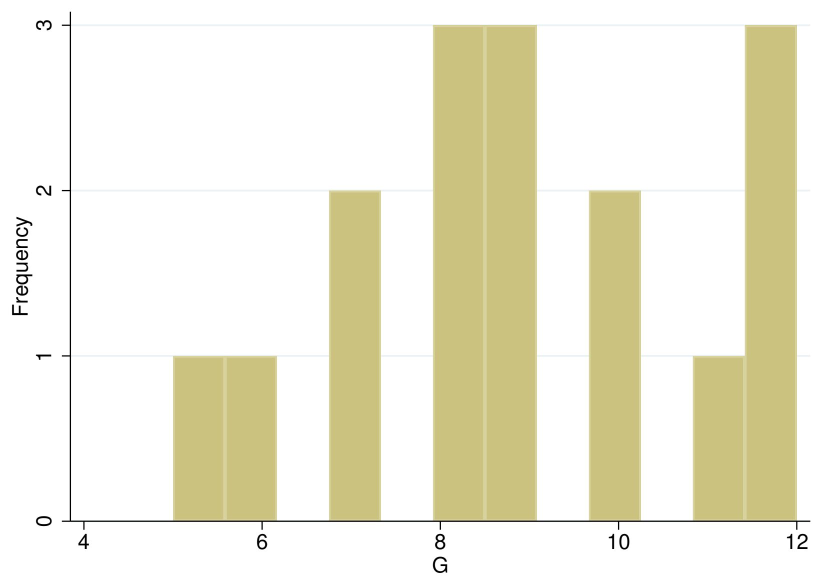
The stress tests conducted by the Federal Reserve, particularly the Supervisory Capital Assessment Program (SCAP) in 2009 and Comprehensive Capital Analysis Review (CCAR) conducted annually since 2011, require a subset of large BHCs to maintain sufficient capital to lend under potential adverse scenarios (Goldstein and Sapra (2014)). The

[5] For surveys of the literature on corporate governance see Shleifer and Vishny (1997) and Tirole (2006).

SCAP and the first three rounds of the CCAR were conducted for 19 BHCs with assets exceeding $100 billion as of 2009.[6] Acharya, Berger and Roman (2018) find evidence that the stress tests led to higher capital levels, which is consistent with interpreting them as constraining debt ratios.

To construct the sample, we restrict to the 16 of these BHCs that could be merged to the G-index dataset in 2006, the last year in which G-index data is available, and that did not experience a large merger after 2006. Figure 1 shows the distribution of the G-index among these firms.

Figure 1: Distribution of G-index for stress-tested bank holding companies. This figure presents the frequencies of the G-index constructed in Gompers, Ishii and Metrick (2003), as of 2006 for our sample BHCs that were included in the 2009 SCAP and 2011, 2012, and 2013 CCAR stress tests.



We designate a BHC as having strong managerial protections if its G-index is greater than the median among this sample, and we similarly designate a BHC as having weak managerial protections if its G-index is less than the median. BHCs whose G-index is equal to the median are excluded, which leaves 13 BHCs for the analysis.

Our sample covers the period from 2004Q1 to 2017Q3. We consider how the ratio of loans and unused commitments to assets varies with managerial protections while controlling for fundamentals corresponding to the CAMELS rating system, which includes

[6] For more detailed background information about the stress tests, see Acharya, Berger and Roman (2018) or Connolly (2018).

capital adequacy (C) as measured by the fraction of tier 1 equity capital to assets, asset quality (A) as measured by the fraction of non-performing loans, manager quality (M) as measured by non-interest expenses to total assets, earnings (E) as measured by return on assets, liquidity (L) as measured by the fraction of liquid assets, sensitivity to market risk (S) as measured by the absolute value of the difference between short-term assets and short-term liabilities divided by total assets, and size as measured by the logarithm of total assets.[7] Nominal variables are normalized to 2010Q4 dollars using the GDP deflator. Summary statistics are presented in Table 1.

Table 1: Summary statistics for stress-tested bank holding companies. This table presents summary statistics for the sample of bank holding company-quarter observations obtained by matching bank holding companies that were included in the 2009 SCAP stress test with the G-index constructed in Gompers, Ishii and Metrick (2003).

	N	Mean	SD	P25	P75
G-index (2-18)	645	8.95	2.36	7.00	11.00
Loans/assets (%)	645	96.02	54.39	78.23	107.64
Tier 1 capital/assets (%)	645	7.94	1.95	6.47	9.30
NPLs/loans (%)	645	1.19	1.15	0.38	1.55
Non-interest expenses/assets (%)	645	1.99	2.47	0.86	1.48
Net income/assets (%)	645	0.97	1.21	0.65	1.31
Liquid assets/assets (%)	645	12.76	11.60	3.26	19.79
Sensitivity to market risk (%)	645	7.69	7.90	1.78	11.84
Log assets	645	19.79	1.16	18.78	21.07

2.2 Specification: difference-in-differences

This section uses the introduction of the bank stress tests to examine how managerial protections affect the relationship between capital structure and investment.

Similar to Acharya, Berger and Roman (2018), we estimate a difference-in-difference specification during a window around the introduction of stress testing in 2009:

$$Loans/assets_{it} = \beta highG_i \times STactive_t + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it} \quad (1)$$

[7] Since commitments are partly chosen by customers, we consider the sum of loans and unused commitments to better reflect the lending choices of banks, similar to Acharya, Berger and Roman (2018).

where $Loans/assets_{it}$ is the ratio of loans and unused commitments to assets for BHC i at quarter t, $highG_i$ is an indicator for whether BHC i's 2006 G-index is above the median among the set of stress-tested BHCs, $STactive_t$ is an indicator for whether a quarter occurs on or after $2010Q1$, $\mathbf{X}_{it-1}$ represents a vector of lagged control variables corresponding to the CAMELS rating system, and ψ_i represents BHC fixed effects, and ϕ_t represents quarter fixed effects. The difference-in-differences coefficient β summarizes the relative trend of the treatment group after the introduction of the stress tests. Standard errors are clustered by BHC.

A potential threat to identification is that the selection of treatment and control groups could be correlated with other characteristics that could drive the results. To support the interpretation that the difference-in-differences coefficient reflects the effect of managerial protections, Table 2 compares the treatment and control groups with respect to the control variables for our sample of BHCs in a period before the introduction of the stress tests. It presents the mean for each variable and group in the period preceding the stress tests as well as the t-statistic on the coefficient η from estimating the regression

$$Y_{it} = \eta highG_i + \phi_t + \epsilon_{it}$$

where Y_{it} represents one of the control variables from equation (1). The two groups are similar with respect to most of the characteristics, and there is only one characteristic, the logarithm of assets, for which there is a statistically significant difference. The observable similarity between the treatment and control groups mitigates concerns that characteristics that are correlated with the extent of managerial protections would confound the results.

We provide additional evidence that the estimates are driven by managerial protections by interacting the specification in equation (1) with other firm characteristics. A detailed theoretical motivation is presented in Section 3. In the general setting of the model presented in Section 3, the basic intuition is that policies that constrain debt ratios can affect investment via two conflicting channels. In particular, they dampen the return on investment by distorting a firm's capital structure, but in the presence of managerial protections they can also increase the incentive to invest due to the fact that the level of investment partially substitutes for debt in disciplining managers. We show using the

Table 2: Comparison of observables for high G-index and low G-index bank holding companies. This table presents the means of bank characteristics within the subsample of bank holding companies (BHCs) with a high G-index (i.e. greater than the median among stress-tested BHCs) and banks with a low G-index (i.e. less than the median) for the period 2004Q1-2009Q4. It also presents the t-statistic for the coefficient η from estimating the regression $Y_{it} = \eta highG_i + \phi_t + \epsilon_{it}$ and computing bank-clustered standard errors.

	Low G	High G	T-statistic
Tier 1 capital/assets	6.326	7.547	1.587
NPLs/loans	0.874	0.589	-1.067
Non-interest expenses/assets	1.124	2.736	1.378
Net income/assets	0.995	0.741	-1.011
Liquid assets/assets	12.20	7.968	-.723
Sensitivity to market risk	3.963	4.784	.428
Log assets	20.46	18.85	-3.034

model that the dominant effect depends on other firm characteristics, such as the distribution of earnings. In particular, Proposition 5 shows that the pro-investment effect associated with managerial protections should be relatively strong in firms for which the earnings distribution cdf is concave. This determines a restriction for the joint hypothesis that the difference-in-differences coefficient reflects the effect of managerial protections and that the model's articulation of this effect is valid.

Accordingly, we present the results from estimating a similar specification as equation (1) after partitioning BHCs based on the concavity of the observed earnings distribution. Appendix A describes our methodology for determining the cocnavity of a BHC's earnings distribution cdf.

2.3 Results

The estimated coefficients from the regression are presented in Table 3. Column (1) presents the results from the baseline specification and indicates that that stress-tested BHCs with managerial protections increased the asset share of loans relative to other stress-tested BHCs by 16.331%, which is about 17% of the mean ratio of loans to assets and 30% of the standard deviation. The estimated coefficient is significant at the 5% level.

The remaining columns show that this effect depends on other firm characteristics in a way that is consistent with the model presented in Section 3. Column (2) indicates that the positive effect of stress testing on the lending for BHCs with a high degree of managerial protections is robust to restricting to BHCs with a dominantly concave return distribution, whereas column (3) indicates that this result is not present among BHCs with a dominantly convex return distribution.

In light of these empirical results, the next section of the paper develops the model to provide an explanation for why the stress tests were associated with a relative increase in lending for BHCs with relatively strong managerial productions, particularly among BHCs whose earnings distribution exhibited a relatively concave cdf.

3 Model

This section introduces a model in which managers take managerial protections into account when determining which projects to pursue. It then derives the capital structure and level of investment that maximize the value of the firm. Finally, it describes conditions under which policies that constrain debt ratios can lead to increased investment.[8]

3.1 Timeline

An entrepreneurial firm is run by a manager. The economy extends over three dates $t = 0, 1, 2$. As an overview, at date $t = 0$ the firm raises capital to invest in a risky project. At date $t = 1$ the manager observes a precise signal of the project's profitability and decides whether or not to liquidate it. If the project is continued to completion rather than liquidated, then at date $t = 2$ its cash flow is realized and distributed to the manager and external investors. Additionally, the manager may obtain a private benefit of control.

More specifically, at $t = 0$ the manager chooses to raise capital $K \geq 0$ to invest in a risky project. Investment that continues until $t = 2$ yields a cash flow of $\theta y(K)$, where

[8]The model relates to ideas introduced in the classical literature linking capital structure and investment. On the one hand, debt increases the probability of liquidation and dampens investment, which is analogous to the debt overhang effect described in Myers (1977). On the other hand, debt constrains managers from pursuing unprofitable projects, which is analogous to the disciplining effect of debt described in Jensen (1986). The model is also related to subsequent work in which these channels affect the determination of firm debt ratios (Morellec, Nikolov and Schürhoff (2012)). We specifically apply these ideas to consider how the level of investment varies with a firm's capital structure.

Table 3: Difference-in-differences regression of loans on G-index for stress-tested bank holding companies. This table presents results from estimating the regression $Loans/assets_{it} = \beta highG_i \times STactive_t + \mathbf{X}'_{it-1}\gamma + \psi_i + \phi_t + \epsilon_{it}$, where $Loans/assets_{it}$ is the ratio of loans and unused commitments to total assets for bank holding company (BHC) i at quarter t, $highG_i$ is an indicator for whether a BHC's 2006 G-index is greater than the median among the set of stress-tested BHCs, $STactive_t$ is an indicator for whether the quarter occurs on or after 2010Q1, ψ_i represents BHC fixed effects, ψ_t represents quarter fixed effects, and X_{it-1} is a set of control variables as described in Section 2.1. Standard errors are clustered by BHC. T-statistics computed using firm-clustered standard errors are reported in parentheses. * indicates statistical significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level. Column (1) runs the estimation on the full sample. Column (2) runs the estimation on the subsample of BHCs with a dominantly concave return distribution. Column (3) runs the estimation on the subsample of BHCs with a dominantly convex return distribution.

	(1) Full sample	(2) Concave	(3) Convex
High G-index x stress tests active	16.331**	16.309**	-3.163
	(2.56)	(4.70)	(-0.38)
Tier 1 capital/assets	5.555**	0.595	4.380
	(2.65)	(0.55)	(1.78)
NPLs/loans	2.030	-2.859	-0.551
	(0.92)	(-1.13)	(-0.22)
Non-interest expenses/assets	4.906*	3.383	4.612
	(1.79)	(1.11)	(1.39)
Net income/assets	0.476	-0.847	0.660
	(0.71)	(-0.59)	(0.84)
Liquid assets/assets	0.042	-0.627**	-0.314
	(0.07)	(-3.75)	(-0.62)
Sensitivity to market risk	-0.420	-0.033	-0.036
	(-0.92)	(-0.18)	(-0.12)
Log assets	3.394	8.449*	-15.514
	(0.50)	(2.88)	(-1.40)
Observations	645	220	500
R^2	0.966	0.953	0.965
BHC FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes

$\theta \sim H(\cdot)$ is a random variable with log-concave density $h(\theta)$ where $\theta \in [0, \infty)$.[9] We assume that $y(K)$ increases, is concave, and has convex marginal product $y'(K)$ and satisfies the boundary conditions $y(0) = 0$, $y'(0) = \infty$, and $y'(\infty) = 0$.

At $t = 1$, the manager observes θ and decides whether or not to liquidate the project. Liquidation incurs a fractional cost of $c \in (0, 1)$ and therefore yields a liquidation value of $(1 - c)K$.

The manager has sufficient funds to finance a fraction α_F of the investment and acquires external financing for the remaining fraction by issuing claims on the return realized at $t = 2$.[10] There are two types of external investors. *Informed investors* also observe θ and are offered equity shares. Denote the fraction of capital provided by informed investors by α_I. Informed investors are like passive investors who want to invest in the project without getting involved in controlling the firm. The simplest way for them to invest is to buy claims on the (residual values of) firm's cash flows in the form of equities. In particular, the informed investors are assumed to receive a fraction of equity shares corresponding to their initial investment. *Uninformed investors*, which correspond to creditors, do not observe θ. They have no bargaining power and their outside option is to forego investment and obtain zero net returns. If the project is liquidated, the creditors have a senior claim on the liquidation value of the firm. Denote the fraction of capital provided by uninformed investors by α_U. Note that the fraction of capital financed by equity can be written as $\alpha_I + \alpha_F = 1 - \alpha_U$.

If the project is continued until $t = 2$, the creditors are repaid and the equityholders, which include the manager and the informed investors, divide according to their shares the residual value of the firm.[11] Additionally, the manager receives a private benefit of control or rent

$$R(G, 1 - \alpha_U, \alpha_F) \equiv G \, \frac{\alpha_F}{1 - \alpha_U} \tag{2}$$

[9]Note that the family of log-concave distributions includes the Uniform, Normal, Exponential, and some classes of Beta and Gamma distributions. Moreover, it is easy to show that if the continuously differentiable p.d.f. $h(\cdot)$ is log-concave on $(\underline{\theta}, \bar{\theta})$, then the corresponding c.d.f. $H(\cdot)$ is also log-concave on $(\underline{\theta}, \bar{\theta})$. In addition, the corresponding hazard rate, i.e., $\frac{h(\theta)}{1-H(\theta)}$, is increasing in θ.

[10]Note that all of our results are robust to supposing that the manager invests a fixed volume of funds rather than a fixed share.

[11]In particular, since the total fraction of capital financed by equity is equal to $\alpha_F + \alpha_I$, informed investors receive a fraction $\frac{\alpha_I}{\alpha_I+\alpha_F}$ of shares of the residual value of the firm while the manager receives the remaining fraction of $\frac{\alpha_F}{\alpha_I+\alpha_F}$.

The private benefit of control increases in the exogenous degree of managerial protections G, which represents the contribution to managerial power of firm provisions or state laws that do not operate through voting power. For example, the empirical analog used to represent managerial protections in section 2 is the G-index introduced in Gompers, Ishii and Metrick (2003), which represents the number of policies that limit threats from shareholders and potential takeovers.[12]

For simplicity, it is convenient to allow the private benefit of control to also depend on the manager's share of the firm's equity in a multiplicative fashion. The results can be extended for more a general functional form $R(G, K, 1-\alpha_U, \alpha_F)$ as long as $\frac{\partial R}{\partial G} \geq 0$, $\frac{\partial R}{\partial K} \geq 0$, and $R\frac{\partial^2 R}{\partial K \partial(1-\alpha_U)} \leq \frac{\partial R}{\partial(1-\alpha_U)}\frac{\partial R}{\partial K}$.

Moreover, to justify this simple functional form intuitively, it is plausible to assume that the private benefit of control increases in the manager's share of the firm's equity. In particular, the manager's equity share represents the manager's voting power relative to the outside equityholders, which can in turn affect the ability of managers to improve their terms of employment (Morck, Shleifer and Vishny (1988)) or accrue nonpecuniary benefits from influencing the operations of the firm according to their personal preferences (Demsetz and Lehn (1985)).[13] Additionally, the choice to model managerial protections and the manager's equity share as contributing to the manager's private benefit of control in a complementary way is consistent with evidence showing that ownership concentration is positively associated with factors that increase a manager's potential private benefit of control (Dyck and Zingales (2004)).

3.2 Firm equity value

To determine the equity value of the firm, we first characterize the payment to the creditors as well as conditions under which the firm is liquidated or completed.

Since creditors are not informed, in order to participate in the investment they need to write an incentive compatible and individually rational contract with the manager. The following lemma characterizes the terms of the contract (i.e., the payments and termina-

[12] It is worth noting that the private benefit of control (see Eq. (2)) also specifies a conflict of interest between the manager and informed investors. This conflict decreases by reducing the degree of managerial protections G.

[13] Note that the fraction of equity held by managers is not infrequently substantial even in large publicly owned firms (La Porta et al. (1998)).

tion threshold of the project) and shows that the unique incentive compatible contract is debt.[14]

Lemma 1. *The unique incentive compatible contract between the manager and the uninformed investors is debt. Specifically, the uninformed investors are payed a fixed amount $p(K,\alpha_U)$ whenever the project is not liquidated at $t=1$. If the project does not generate a large enough return to repay the promised amount, then the project is liquidated at $t=1$ and the uninformed investors are paid the liquidation value up to the value of their investment $\alpha_U K$. Let $\theta^*(K,\alpha_U)$ denote the threshold for θ at which the project is liquidated. Then $p(K,\alpha_U)$ and $\theta^*(K,\alpha_U)$ satisfy one of the following:*

- *If $\alpha_U \leq 1-c$, then*

$$p(K,\alpha_U) = \alpha_U K \tag{3}$$

$$\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\} \tag{4}$$

- *If $\alpha_U \geq 1-c$, then*

$$p(K,\alpha_U) = \theta^*(K,\alpha_U)y(K) \tag{5}$$

$$\alpha_U K = H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\underbrace{\theta^*(K,\alpha_U)y(K)}_{=p(K,\alpha_U)} \tag{6}$$

Proof. See Appendix. □

The intuition for this result is as follows. First, consider the "high debt" case where $\alpha_U \geq 1-c$. If the project is liquidated, then the liquidation value $(1-c)K$ is insufficient to repay the value of the investment by the creditors $\alpha_U K$. Therefore the firm defaults and the creditors receive all of the liquidation value since they have a senior claim on it. Ex-ante, they charge a risk premium that is determined by an individual rationality condition that equates the creditors' expected payoff with their cost of investment. The risk premium and the default threshold are related by an incentive compatibility condition for the manager, which requires that, conditional on continuing the project to

[14] The contract follows similar features as in the costly state verification contracts developed by Townsend (1979).

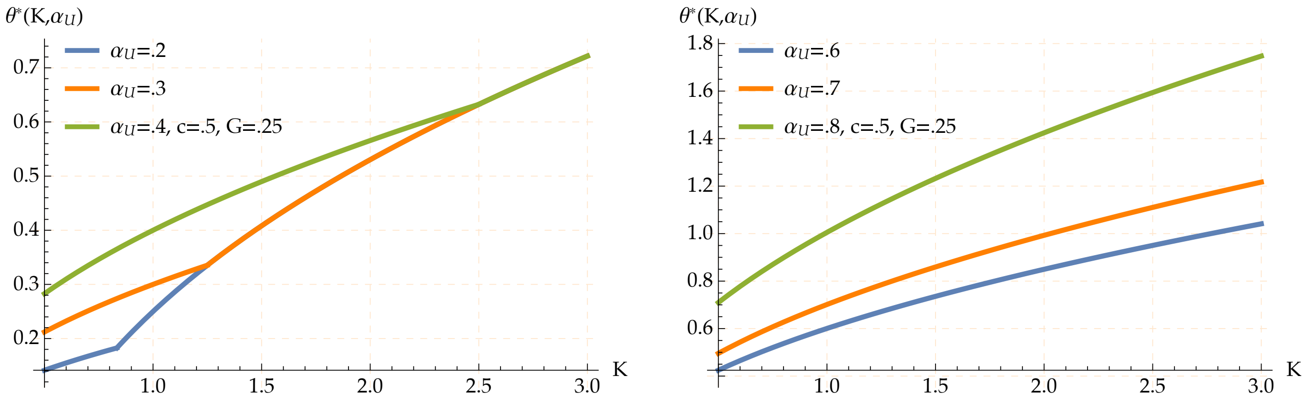


Figure 2: These panels plot $\theta^*(K,\alpha_U)$ for the high debt ($\alpha_U > 1-c$, the right panel) and low debt ($\alpha_U < 1-c$, the left panel) cases, as specified in Lemma 1. We assume $y(K) = \sqrt{K}$, $c = .5$ and the distribution is Uniform$[0,a]$, where a is large enough.

completion, the creditors receive the return of the firm at the marginal state at which it is continued, which is $\theta^*(K,\alpha_U)y(K)$.

Second, consider the "low debt" case where $\alpha_U \leq 1-c$. If the project is liquidated, the liquidation value $(1-c)K$ is sufficient to repay the creditors for their investment $\alpha_U K$. Therefore, by the individual rationality condition, the creditors always receive $\alpha_U K$. If the debt ratio is sufficiently small, then the manager is unconstrained by the debt contract and chooses the default threshold $\theta^*(K,\alpha_U)$ by equating his or her own payoffs from either liquidating or continuing the project:

$$\underbrace{\frac{\alpha_F}{1-\alpha_U}\left(\theta^*(K,\alpha_U)y(K) - \overbrace{\alpha_U K}^{=p(K,\alpha_U)} + G\right)}_{\text{continuation payoff}} = \underbrace{\frac{\alpha_F}{1-\alpha_U}((1-c)-\alpha_U)K}_{\text{liquidation payoff}} \tag{7}$$

As managerial protections increase, the manager has a stronger incentive to continue projects, which causes the liquidation threshold to decrease. However, feasibility requires the return at the marginal state of continuation to be large enough to pay the creditors the promised amount, which implies $\theta^*(K,\alpha_U)y(K) \geq \alpha_U K$. When the debt ratio is large relative to the degree of managerial protections, this constraint holds with equality.

After the creditors are repaid, the equity value of the firm is equal to

$$V\Big(K,\alpha_U\Big) = H\Big(\theta^*(K,\alpha_U)\Big)\Big[((1-c)-\alpha_U)K\Big]^+ + \int_{\theta^*(K,\alpha_U)}^{\infty}\Big(\theta y(K) - p(K,\alpha_U)\Big)dH(\theta) - (1-\alpha_U)K$$

where $[A]^+ = \max\{A, 0\}$. Note that because the creditors obtain zero expected profits, the value of the firm is also equal to the ex-ante welfare, which is defined as the sum of the net payoffs for the manager, outside equityholders, and creditors.

The manager then chooses to liquidate or continue projects to maximize his or her utility

$$u_m \equiv \frac{\alpha_F}{1-\alpha_U} V\Big(K, \alpha_U\Big) + G \frac{\alpha_F}{1-\alpha_U} \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big) \tag{8}$$

3.3 Efficient capital structure

To provide a benchmark for evaluating the effect of policies that constrain debt ratios, this subsection derives the efficient capital structure that maximizes the equity value of the firm.

To determine the efficient capital structure, we first derive the first-best liquidation rule that maximizes the value of the firm in the absence of external financing frictions and private benefits of control. In particular, eliminating these frictions is equivalent to maximizing the utility of manager (equation (8)) with $\alpha_F = 1$ and $G = 0$. At $t = 1$, it is straightforward to see that liquidating the project is efficient if and only if the return $\theta y(K)$ is less than the liquidation value $(1-c)K$, or equivalently

$$\theta \leq \theta^{opt}(K) \equiv \frac{(1-c)K}{y(K)}. \tag{9}$$

The efficient capital structure that induces the first-best liquidation rule is given by $\alpha_U = 1 - c$. Note that, for tractability, the model focuses on determinants of capital structure involving managerial discretion and omits potentially other relevant considerations. Accordingly, we interpret this result as a benchmark for a qualitative comparison with policies that constrain debt ratios, and we do not interpret it quantitatively.

Proposition 1. *If $\alpha_U = 1 - c$ then $\theta^*(K, \alpha_U) = \theta^{opt}(K)$. If $G > 0$ then the converse also holds.*

Proof. See Appendix. □

The intuition is as follows. On the one hand, in the "high debt" case where $\alpha_U > 1-c$, the creditors accrue losses when the project is liquidated. As a result, they require a greater repayment $\theta^*(K, \alpha_U)y(K)$ in states where the project is not liquidated. This implies an

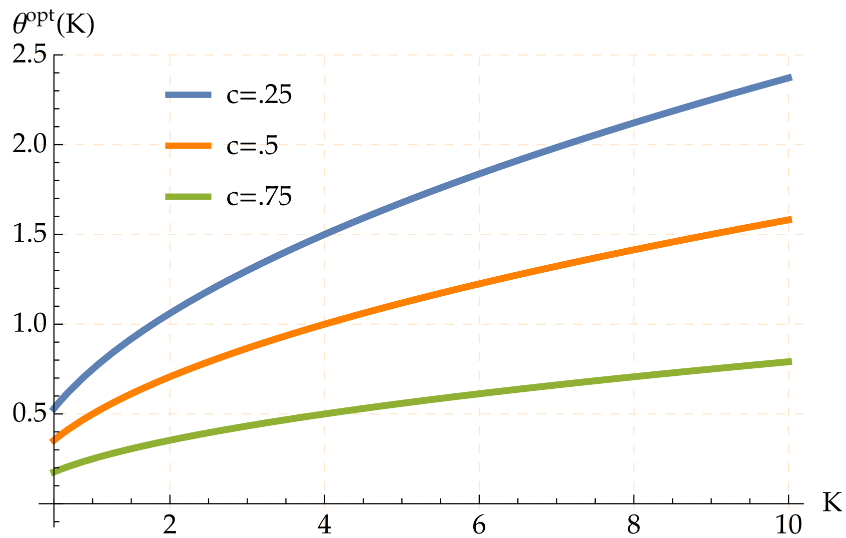


Figure 3: This panel shows that the efficient liquidation threshold $\theta^{opt}(K)$ is decreasing in the liquidation cost c and increasing in the invested capital K.

increase in the liquidation threshold $\theta^*(K, \alpha_U)$, which in turn increases the probability of having to liquidate profitable projects. On the other hand, in the "low debt" case where $\alpha_U < 1 - c$, the manager has an incentive to avoid liquidation in order to obtain the private benefit of control. This implies a decrease in the liquidation threshold $\theta^*(K, \alpha_U)$, which increases the probability of continuing unprofitable projects.[15] This tradeoff is summarized in the following proposition:

Proposition 2. *The liquidation threshold is increasing in the firm's debt ratio:*

$$\frac{\partial \theta^*(K, \alpha_U)}{\partial \alpha_U} \geq 0$$

Consequently, if $\alpha_U > 1 - c$, the manager excessively liquidates the project,

$$\theta^*(K, \alpha_U) > \theta^{opt}(K).$$

If $\alpha_U < 1 - c$ and $G > 0$, the manager excessively continues the project,

$$\theta^*(K, \alpha_U) < \theta^{opt}(K).$$

[15]Note that eliminating the private benefit of control by setting $G = 0$ restores the efficient liquidation threshold $\theta^*(K, \alpha_U) = \theta^{opt}(K)$.

Proof. See Appendix. □

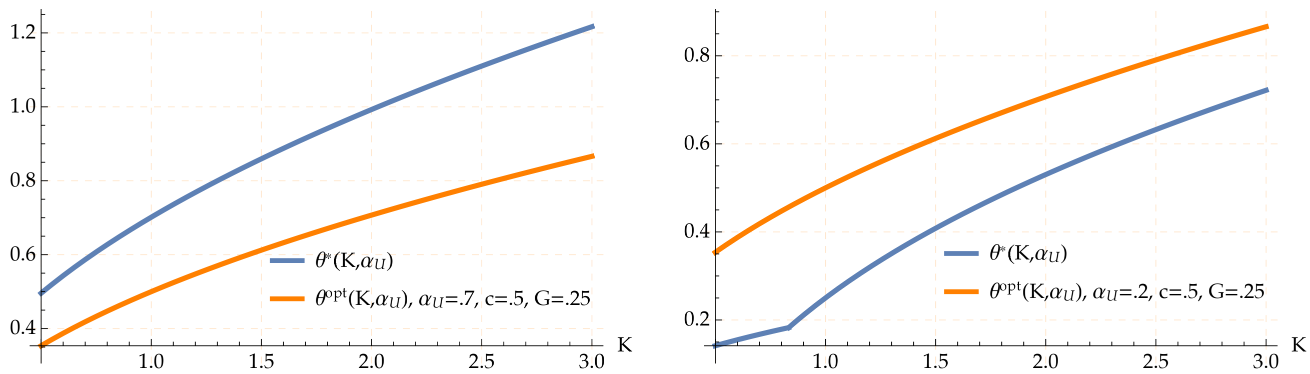


Figure 4: This panels show that when debt is high, i.e., $\alpha_U > 1-c$ (the left panel), the manager excessively liquidates the project $\theta^*(K,\alpha_U) > \theta^{opt}(K)$. When debt is low, i.e., $\alpha_U < 1-c$ and $G > 0$ (the right panel), the manager excessively continues the project, $\theta^*(K,\alpha_U) < \theta^{opt}(K)$.

3.4 Investment

This subsection first derives the first-best level of investment that maximizes the value of the firm in the absence of external financing frictions and managerial private benefits of control. It then illustrates channels by policies that constrain debt ratios can induce either an increase or a decrease in the constrained optimal level of investment.

3.4.1 Efficient level of investment

To illustrate how the firm's capital structure affects the level of investment, we first derive the first-best level of investment that maximizes the value of the firm in the absence of external financing frictions and managerial private benefits of control. Given that the first-best liquidation threshold is equal to $\theta^{opt}(K)$, the first-best level of investment solves:

$$\max_K \left\{ \int_{\theta^{opt}(K)}^{\infty} \theta\, y(K) dH(\theta) + \int_0^{\theta^{opt}(K)} (1-c) K\, dH(\theta) - K \right\} \tag{10}$$

Proposition 3. *There exists a unique interior capital level* $K^{opt} \in (0,\infty)$ *that solves* (10) *and* *satisfies the first order condition*

$$1 = (1-c)H\Big(\theta^{opt}(K^{opt})\Big) + y'(K^{opt})\int_{\theta^{opt}(K^{opt})}^{\infty}\theta\, dH(\theta). \tag{11}$$

Proof. See Appendix. □

Intuitively, the first-best level of investment is chosen to equate the marginal cost of investment with the marginal expected returns, which consists of the liquidation value from bad realizations plus the returns from good realizations.

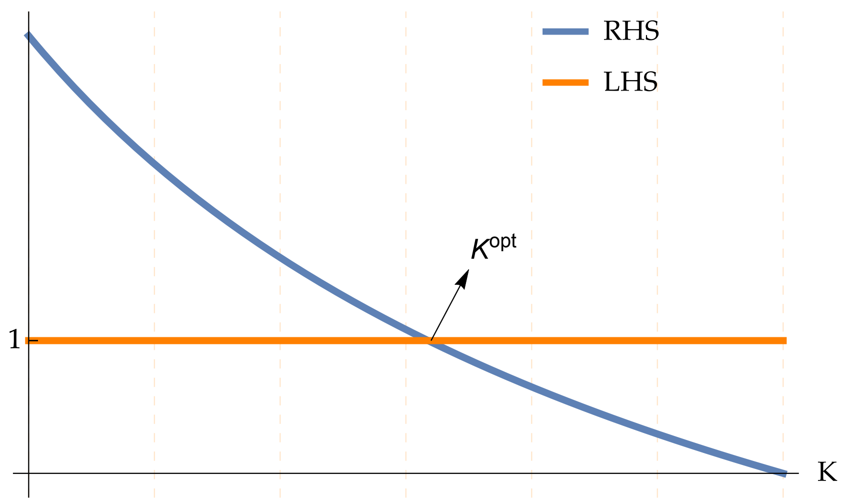


Figure 5: This panel shows K^{opt} is unique. The right hand side of equation (11) is decreasing and the left hand side (LHS) is constant. Proposition 3 proves that the intersection exists and is unique.

3.4.2 Effect of capital structure on the level of investment

To determine how capital structure affects the level of investment, denote the level of investment that maximizes the value of the firm for a given capital structure by $K^*(\alpha_U)$. Since the efficient capital structure $\alpha_U = 1-c$ induces the first-best liquidation threshold $\theta^{opt}(K)$ (see Proposition (1)), the corresponding conditionally efficient level of investment is equal to the first-best benchmark. The rest of this subsection illustrates how deviations from the efficient capital structure affect the conditionally efficient level of investment.

If the firm has greater than the efficient debt ratio, or $\alpha_U > 1-c$, then investment is distorted downwards relative to the first-best benchmark K^{opt}.

Proposition 4. *If $\alpha_U > 1-c$, then $K^*(\alpha_U) < K^{opt}$.*

Proof. See Appendix. □

To see this, recall that the cost of debt is increasing in the debt ratio in order to compensate creditors for losses in states where the project is liquidated. Because of the higher cost of debt, the firm sometimes liquidates profitable projects (see proposition (2)). This in turn reduces the marginal expected returns to investment.

If the firm has less than the efficient debt ratio, or $\alpha_U < 1 - c$, it is possible for the conditionally efficient level of investment to be distorted either downwards or upwards relative to the first-best benchmark K^{opt}. The direction of the distortion depends on the parameters, such as the concavity of the firm's earnings distribution cdf.

Proposition 5. *Suppose that the debt ratio is less than the efficient level, or $\alpha_U < 1-c$. If there are no managerial protections, or $G = 0$, then the level of investment is equal to the first-best benchmark and there is no distortion:*

$$K^*(\alpha_U) = K^{opt} = K^*(1-c)$$

If there are managerial protections, or $G > 0$, then the conditionally efficient level of investment can in general be greater or less than the first-best benchmark. If the debt ratio is low enough, or $\alpha_u \leq 1 - c - \frac{G}{K^{opt}}$, and H is strictly concave then investment is distorted upwards, or $K^*(\alpha_U) > K^{opt}$. If the cdf for the return distribution H is weakly convex, then the conditionally efficient level of investment is distorted downwards, or $K^*(\alpha_U) \leq K^{opt}$.

Proof. See Appendix. □

The intuition is that debt disciplines managers by requiring them to liquidate unprofitable projects. In particular, if the manager is not constrained by debt, he or she may continue some unprofitable projects in order to obtain the private benefit of control (see Proposition (2)). On the one hand, continuing unprofitable projects decreases the marginal expected return to investment, which in turn inhibits investment. On the other hand, increasing the level of investment increases the rate at which the firm liquidates unprofitable projects.

Lemma 2. *The termination threshold increases in the level of investment:* $\frac{\partial \theta^*(K,\alpha_U)}{\partial K} \geq 0$.

Proof. See Appendix. □

Intuitively, the increasing relationship between the level of investment and the probability of liquidation follows from the linear liquidation value and concave expected return. Therefore, increasing the level of investment partially corrects the excessive continuation of unprofitable projects by the manager. In this way, the level of investment can be interpreted as a partial substitute for debt in disciplining the manager. Due to these countervailing effects on the incentive to invest, the conditionally efficient level of investment can be either greater or less than the efficient level, depending on the parameters. Figure 8 illustrates a case in which requiring banks to have a lower than the optimal debt ratio can lead to higher investment.

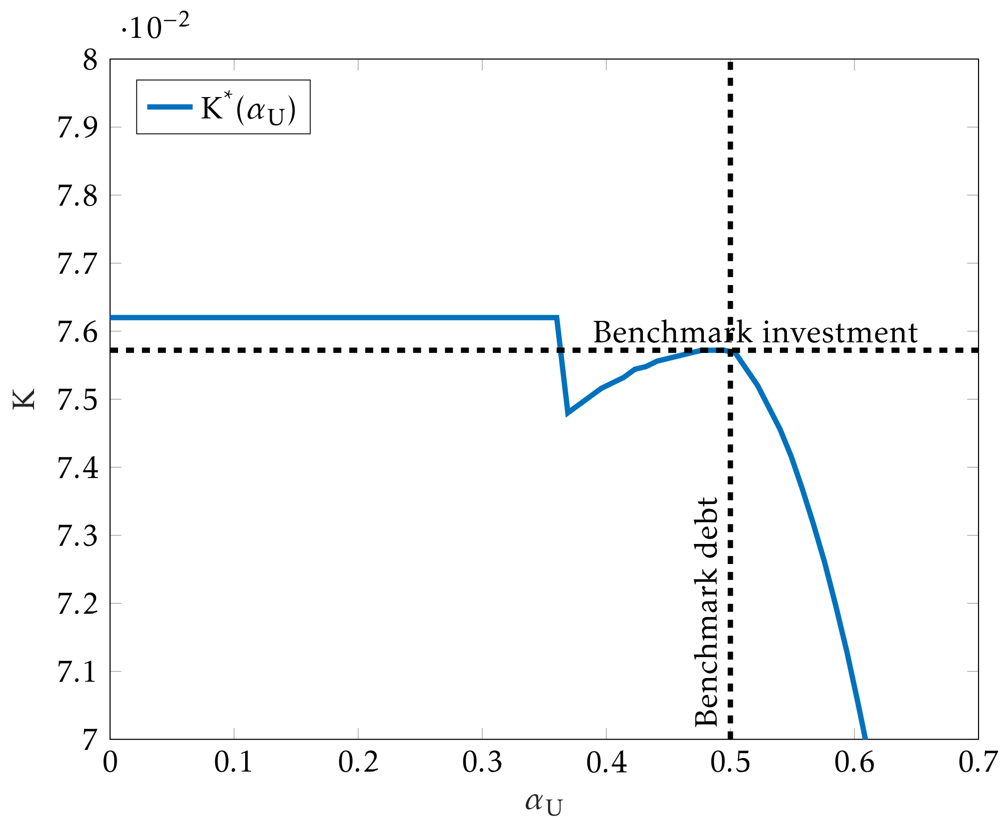


Figure 6: This plot shows the conditionally efficient level of investment and the manager's preferred level of investment as a function of the debt ratio when the return for the project follows an exponential distribution. The plot illustrates that requiring banks to have a lower than the optimal debt ratio can lead to higher investment, as shown in Proposition 5.

An implication of this result is that a firm that initially exhibits the efficient debt ratio $\alpha_U = 1 - c$ can have an incentive to increase investment in response to a policy that requires a reduction of its debt ratio. More specifically, it rationalizes the empirical finding from Section 2 that the stress tests led to a relative increase in lending for bank holding companies (BHCs) with relatively strong managerial protections. The conditions described in Proposition 5 are also consistent with the finding that this result is stronger for BHCs with a dominantly concave earnings distribution.

4 Extension: managerial selection of capital structure and investment

This section extends the model to speak to other facts in the corporate governance literature by allowing managers to choose the capital structure and level of investment of the firm. First, we illustrate channels by which managerial protections can cause managers to issue either too much or too little debt relative to the value-maximizing level. Second, we illustrate a novel channel by which managerial protections can cause managers to underinvest relative to the value-maximizing level. The model thereby provides new perspectives on existing evidence linking managerial protections with capital structure and investment.

4.1 Manager's preferred capital structure

This section describes channels by which managerial protections can cause a manager to choose an inefficient capital structure.

These channels correspond to terms in the derivative of the manager's utility evaluated at the efficient benchmark:

$$\begin{aligned}\frac{\partial u_m}{\partial \alpha_U}\Big|_{\alpha_U=1-c} &= \underbrace{\frac{\alpha_F}{(1-c)^2}V(K,1-c)}_{>0} + \frac{\alpha_F}{1-c}\underbrace{\frac{\partial V}{\partial \alpha_U}\Big|_{\alpha_U=1-c}}_{=0} \\ &+ \underbrace{G\frac{\alpha_F}{(1-c)^2}\Big(1-H\Big(\theta^*(K,1-c)\Big)\Big)}_{\geq 0} \underbrace{-G\frac{\alpha_F}{1-c}h\Big(\theta^*(K,1-c)\Big)\frac{\partial \theta^*(K,\alpha_U)}{\partial \alpha_U}\Big|_{\alpha_U=1-c}}_{\leq 0}\end{aligned} \tag{12}$$

The positive first term, which does not involve managerial protections, shows that managers have an incentive to issue excessive debt in order to increase their share of the value of the firm. The last two terms illustrate the effect of managerial protections, which can distort the manager's chosen level of debt either upwards or downwards. On the one hand, the positive term illustrates that protections can intensify the incentive for managers to increase their voting power by issuing debt rather than equity. This term is driven by the complementarity between the degree of managerial protections and the manager's equity share in determining managerial power and is consistent with evidence showing that ownership concentration is positively associated with estimated private benefits of control (Dyck and Zingales (2004)). On the other hand, the negative term illustrates that protections can lead managers to issue less debt in order to decrease the probability of liquidation.

This analysis provides a novel interpretation for the finding in John and Litov (2010) that managerial protections are positively associated with debt ratios. Figure 7 illustrates a case in which the manager's preferred debt ratio is increasing in the degree of managerial protections.

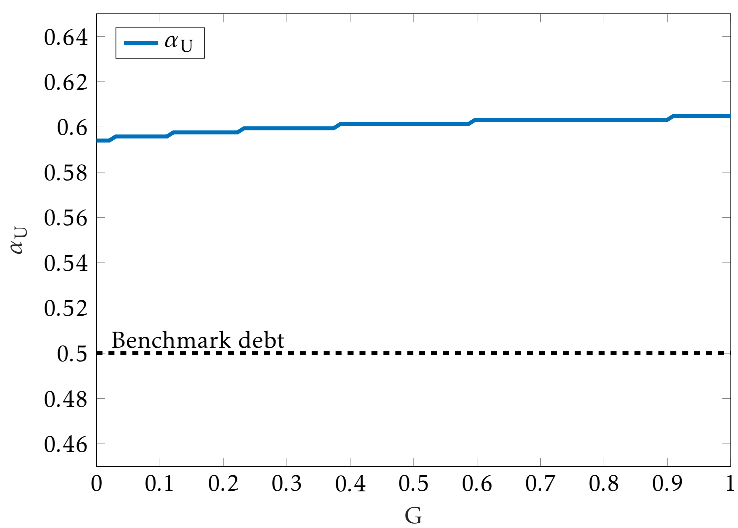


Figure 7: This figure shows the manager's preferred debt ratio as a function of managerial protections when $y(K) = \sqrt{K}$ and the return of the project has a uniform distribution. In this case, the debt ratio is increasing in the degree of managerial protections.

4.2 Manager's preferred level of investment

This section describes channels by which managerial protections can cause a manager to choose an inefficiently low level of investment.[16,17]

Denote the manager's preferred level of investment that maximizes his or her utility (equation (8)) for a given debt ratio α_U by $K_m(\alpha_U)$.

Proposition 6. *If there are no managerial protections, or $G = 0$, then the manager's preferred level of investment that maximizes his or her utility (equation (8)) for a given debt ratio α_U, $K_m(\alpha_U)$, is equal to the conditionally efficient level $K^*(\alpha_U)$. If there are managerial protections, or $G > 0$, then the manager's preferred level of investment is less than the conditionally efficient level, or $K_m(\alpha_U) < K^*(\alpha_U)$.*

Proof. See Appendix. □

The intuition is as follows. Because the manager can only obtain the private benefit of control while the firm is operational, the manager has a stronger incentive to avoid liquidating the project compared to other equityholders. Since investing less reduces the probability of liquidation due to the concave expected return (Lemma 2), the manager chooses a lower level of investment to increase the expected private benefit of control. Figure 8 illustrates this result by showing that the conditionally efficient level of investment is greater than manager's preferred level of investment for different levels of the debt ratio. Appendix Section C shows that this result is robust to alternatively assuming a competitive equity market in which the informed investors make zero rents, similar to the uninformed investors.

[16] Note that this result differs from the pro-investment effect of the stress tests in Proposition 5 in that it describes the effect allowing the manager to decide the investment level conditional on a given capital structure, whereas Proposition 5 analyzes the effect of varying the capital structure on the value-maximizing level of investment.

[17] Note that the model can also be extended to incorporate "empire building" preferences to produce a positive association between managerial protections and investment, similar to the empirical findings in Gompers, Ishii and Metrick (2003) and Giroud and Mueller (2011).

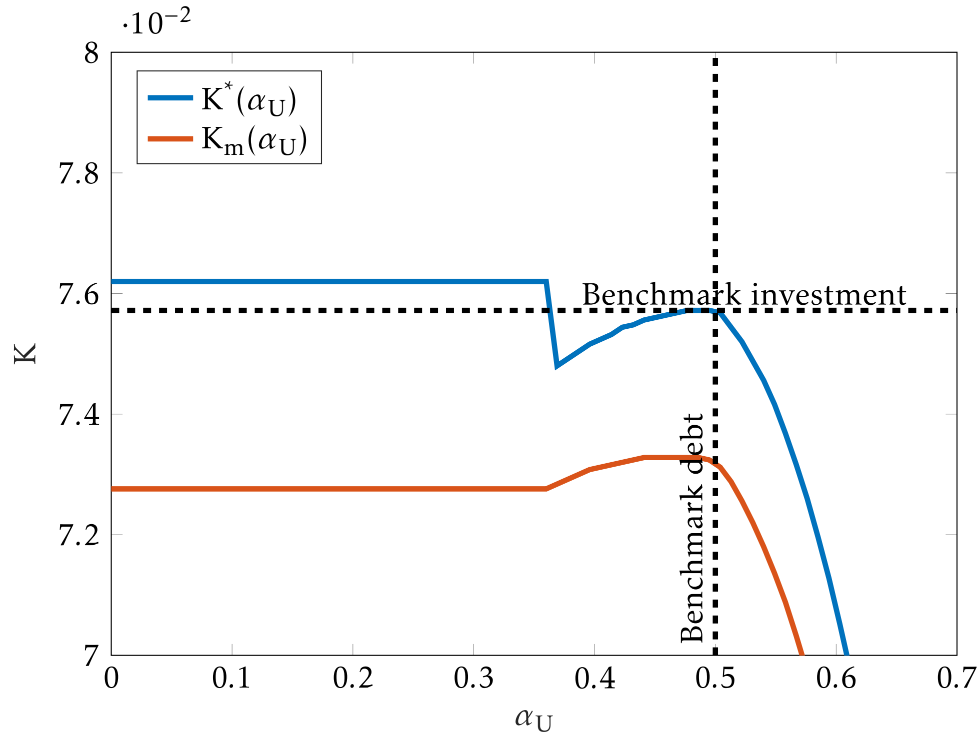


Figure 8: This plot shows the conditionally efficient level of investment and the manager's preferred level of investment as a function of the debt ratio when the return for the project follows an exponential distribution. The plot illustrates that the manager's preferred level of investment is less than the conditionally efficient level, as shown in Proposition 6.

This result provides a novel interpretation for the finding in Bertrand and Mullainathan (2003) that managerial protections are negatively associated with reduced investment.

5 Conclusion

This paper empirically motivates and theoretically illustrates channels by which managerial protections can affect the investment response of policies that constrain firm debt ratios.

We first empirically document the interaction between managerial protections and the U.S. bank stress tests in determining bank lending. Using a sample of bank holding companies (BHCs) that were included in the initial stress tests conducted by the Federal Reserve, we show that the introduction of the stress tests was associated with relatively greater lending for BHCs with strong managerial protections, as measured by the G-index constructed in Gompers, Ishii and Metrick (2003).

We then explain these results with a model in which managerial protections can distort a manager's decisions about which projects to pursue. The model demonstrates that, in the presence of managerial protections, policies that constrain debt ratios can motivate increased investment, which partially substitutes for the effect of debt in disciplining the manager from continuing unprofitable projects. Consistent with the empirical findings, the model shows that the pro-investment effect associated with managerial protections is stronger for firms with a relatively concave earnings distribution cdf.

Finally, we show that an extension of the model allowing the manager to choose the firm's capital structure and level of investment can speak to other facts from the literature. In particular, we show that the manager may have an excessively strong incentive to finance the firm with debt in order to maintain a high equity share, or it may have an excessively weak incentive to issue debt in order to reduce the probability of liquidation. We also show that managerial protections can lead managers to underinvest in order to decrease the probability of liquidation.

By analyzing the interactions between managerial protections, capital structure, and investment, this paper sheds light on channels by which policies that constrain debt ratios can affect investment and also illustrates mechanisms by which managerial protections can reduce firm performance.

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A Identifying firms with a dominantly concave return distribution

This section describes a methodology for distinguishing bank holding companies (BHCs) with a relatively concave return distribution cdf. This methodology is motivated by Proposition 5, which shows that managerial protections cause a firm's conditionally efficient level of investment to be greater than the first-best benchmark if the debt ratio is low enough, or $\alpha_U \leq 1 - c - \frac{G}{K^{opt}}$, and the return distribution cdf H is strictly concave

First, we compute the empirical cdf of each BHC's return distribution using observations of return on assets during from the start of the sample 2004Q1 until the approximate start of the financial crisis in 2007Q3 (Bernanke (2009)). We think of the manager as responding to the stress tests based on the typical return distribution of the firm, so we omit the crisis because it may have caused uncharacteristic returns.[18] Order the observed return on assets by θ_k.

Second, we compute a numerical approximation of the second derivative of the cdf at each point by the formula

$$\frac{\frac{H(\theta_{k+1})-H(\theta_k)}{\theta_{k+1}-\theta_k} - \frac{H(\theta_k)-H(\theta_{k-1})}{\theta_k-\theta_{k-1}}}{\frac{(\theta_{k+1}-\theta_k)+(\theta_k-\theta_{k-1})}{2}}$$

BHCs for which the mean second derivative is negative are identified as having a dominantly concave cdf, while BHCs for which the mean second derivative is postitive are identified as having a dominantly convex cdf.

Consistent with Proposition 5, Table 3 shows that, among the subset of BHCs with a dominantly concave return distribution, those with a higher degree of managerial protections experienced a relative increase in lending after the introduction of stress-testing.

B Model: omitted proofs

Lemma 1. *The unique incentive compatible contract between the manager and the uninformed investors is debt. Specifically, the uninformed investors are payed a fixed amount* $p(K, \alpha_U)$

[18] The results are qualitatively robust to minor variations in the timeframe for computing the return distribution.

whenever the project is not liquidated at $t = 1$. If the project does not generate a large enough return to repay the promised amount, then the project is liquidated at $t = 1$ and the uninformed investors are paid the liquidation value up to the value of their investment $\alpha_U K$. Let $\theta^*(K, \alpha_U)$ denote the threshold for θ at which the project is liquidated. Then $p(K, \alpha_U)$ and $\theta^*(K, \alpha_U)$ satisfy one of the following:

- *If $\alpha_U \leq 1 - c$, then*

$$p(K, \alpha_U) = \alpha_U K \tag{3}$$

$$\theta^*(K, \alpha_U) y(K) = \max\{(1-c)K - G, \alpha_U K\} \tag{4}$$

- *If $\alpha_U \geq 1 - c$, then*

$$p(K, \alpha_U) = \theta^*(K, \alpha_U) y(K) \tag{5}$$

$$\alpha_U K = H\Big(\theta^*(K, \alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\underbrace{\theta^*(K, \alpha_U) y(K)}_{=p(K, \alpha_U)} \tag{6}$$

Proof of Lemma 1. Fix K and α_F. Let $\xi(\theta; \alpha_U)$ denote the payment to the uninformed investors. Then, incentive compatibility requires that

$$\theta \in \arg\max_{\theta'} \left[\frac{\alpha_F}{1-\alpha_U} \max\Big\{ \theta y(K) - \xi(\theta'; \alpha_U) + G, ((1-c) - \alpha_U)K, 0 \Big\} \right] \tag{13}$$

where the first term is the manager's payoff if she does not terminate the project at $t = 1$ and the second term is her payoff when she terminates/liquidates the project at $t = 1$. Let $[0, \theta^*(K, \alpha_U))$ (i.e., the termination region) denote the region where in the manager terminates the project. And, when $\theta \in [\theta^*(K, \alpha_U), \infty)$ the manager continues the project at $t = 1$.

We need to consider several cases.

Case 1 (High debt). Suppose $1 - c - \alpha_U \leq 0$, equivalently, $(1 - c - \alpha_U)K \leq 0$. Then, due to Eq. (13), we only need to consider $\theta y(K) - \xi(\theta; \alpha_U) + G$. Since $\theta^*(K, \alpha_U)$ is the termination threshold, thus, following the above specifications, $\theta < \theta^*(K, \alpha_U)$ implies termination and $\theta > \theta^*(K, \alpha_U)$ leads to continuation of the project.

Clearly, when $\theta > \theta^*(K, \alpha_U)$, i.e., θ is in the continuation region, the manager has no incentive to instead misreport it by θ' where $\theta' < \theta^*(K, \alpha_U)$. Simply because if she does so, she misses out $G > 0$. So, for any $\theta', \theta'' \geq \theta^*(K, \alpha_U)$ we must have

$$\xi(\theta'; \alpha_U) = \xi(\theta''; \alpha_U) = p(K, \alpha_U) \leq \inf_{\theta > \theta^*(K, \alpha_U)} \theta y(K) = \theta^*(K, \alpha_U) y(K).$$

Moreover, when $\theta < \theta^*(K, \alpha_U)$, the manager, in order to accrue her private benefit $G > 0$, may want to misreport her true type θ to some higher type θ', for which $\theta' > \theta^*(K, \alpha_U)$. Therefore, for $\theta \leq \theta^*(K, \alpha_U)$, the manager should not have incentive to misreport it to $\theta' > \theta^*(K, \alpha_U)$. For this, we must have

$$p(K, \alpha_U) \geq \sup_{\theta \leq \theta^*(K, \alpha_U)} \theta y(K) = \theta^*(K, \alpha_U) y(K).$$

Therefore,

$$p(K, \alpha_U) = \theta^*(K, \alpha_U) y(K).$$

We further note that, when the manager does not liquidate project, she must be able to pay $p(K, \alpha_U)$ to the uninformed investors, which is enforced by the terms of the contract. Otherwise, she will be penalized by more than her private control rent G. This enforceability can be easily implemented by regulators observing firm's fundamental θ at $t = 2$.

Finally, the uninformed investors individual rationality (IR) must be satisfied. As a result, the termination threshold $\theta^*(K, \alpha_U)$ satisfies the following uninformed investor's IR constraint:

$$H\Big(\theta^*(K, \alpha_U)\Big)(1 - c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y(K) = \alpha_U K.$$

Case 2 (low debt). Suppose $1 - c - \alpha_U > 0$. Then, we need to consider how large the private reward of the manager is. In this case we need to consider when the following two payoffs (termination vs. continuation payoffs) meet

$$\underbrace{\theta^*(K, \alpha_U) y(K) - p(K, \alpha_U) + G}_{\text{continuation payoff}} = \underbrace{(1 - c - \alpha_U)K}_{\text{termination payoff}} \tag{14}$$

Suppose manager's protection G is sufficiently *large* so that

$$(1 - c - \alpha_U)K - G \leq 0.$$

In this case, then we are back to Case 1, as the above equality never holds.

Next, suppose that G is sufficiently *small* so that

$$(1-c-\alpha_U)K - G = ((1-c)K - G) - \alpha_U K > 0.$$

Thus, due to (14), we have $\theta^*(K,\alpha_U)y(K) - p(K,\alpha_U) > 0$. Moreover, the uninformed investors' individually rational (IR) constraint is also satisfied because

$$\underbrace{H\Big(\theta^*(K,\alpha_U)\Big)\alpha_U K}_{\substack{\text{ex-ante termination payoff}\\ \text{of uninformed investors}}} + \underbrace{\Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\theta^*(K,\alpha_U)y(K)}_{\substack{\text{ex-ante continuation payoff}\\ \text{of uninformed investors}}} \geq \alpha_U K.$$

and the above IR constraint binds when $p(K,\alpha_U) = \alpha_U K$. Hence,

$$\theta^*(K,\alpha_U)y(K) = (1-c)K - G.$$

Putting together when debt is small, ie. $\alpha_U < 1-c$, the termination rule $\theta^*(K,\alpha_U)$ is given by

$$\theta^*(\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\}.$$

Payment rule $p(K,\alpha_U)$: Finally, we note that the payment rule $p(K,\alpha_U)$ (both in high and low debt cases) is characterized by the uninformed investors' IR constraint. That is, in the high debt case (i.e., $\alpha_U \geq 1-c$) $p(K,\alpha_U)$ satisfies

$$H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)p(K,\alpha_U) = \alpha_U K,$$

and in the in the low debt case (i.e., $\alpha_U \leq 1-c$) $p(K,\alpha_U)$ satisfies

$$H\Big(\theta^*(K,\alpha_U)\Big)\alpha_U K + \Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)p(K,\alpha_U) = \alpha_U K.$$

□

Proposition 1. *If $\alpha_U = 1-c$ then $\theta^*(K,\alpha_U) = \theta^{opt}(K)$. If $G > 0$ then the converse also holds.*

Proof of Proposition 1. Fix K. Given that $\theta^{opt}(K) = \frac{(1-c)K}{y(K)}$, the proof immediately follows by Lemma 1. □

Proposition 2. *The liquidation threshold is increasing in the firm's debt ratio:*

$$\frac{\partial \theta^*(K,\alpha_U)}{\partial \alpha_U} \geq 0$$

Consequently, if $\alpha_U > 1-c$, the manager excessively liquidates the project,

$$\theta^*(K,\alpha_U) > \theta^{opt}(K).$$

If $\alpha_U < 1-c$ and $G > 0$, the manager excessively continues the project,

$$\theta^*(K,\alpha_U) < \theta^{opt}(K).$$

Proof of Proposition 2. Let $\alpha \equiv 1-\alpha_U$. Thus, to prove the claim it is sufficient to show $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} \le 0$. To do this, we need to consider two cases (high debt and low debt). Throughout fix K.

Case1. Low debt $(1-c > \alpha_U)$. In this case depending on the size of the manager's private rent (managerial protection), G, the termination threshold satisfies $\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K-G, \alpha_U K\} = \max\{(1-c)K-G,(1-\alpha)K\}$. Therefore, rearranging gives

$$\theta^*(K,\alpha_U) = \max\{\frac{(1-c)K-G}{y(K)}, \frac{(1-\alpha)K}{y(K)}\}.$$

Suppose $(1-c)K-G < (1-\alpha)K$. Then, using the implicit function theorem, we have $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} = \frac{-K}{y(K)} < 0$. And, when $(1-c)K-G > (1-\alpha)K$, then $\theta^*(K,\alpha_U) = \frac{(1-c)K-G}{y(K)}$ thus $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} = 0$. Putting together, $\frac{\partial\theta^*(K,\alpha_U)}{\partial\alpha} \le 0$.

Case2. High debt $(1-c < \alpha_U)$. For ease of notation let us denote the termination threshold by ℓ. In this case, the termination threshold satisfies the uninformed investor's IR constraint:

$$H(\ell)(1-c)K + (1-H(\ell))\ell y(K) = \alpha_U K = (1-\alpha)K.$$

We need to first show such threshold exists. Let us define for $\ell \ge \theta^{opt}(K) = \frac{(1-c)K}{y(K)}$,

$$\gamma(\ell) \equiv H(\ell)(1-c)K + \Big(1-H(\ell)\Big)\ell y(K) - \alpha_U K.$$

Thus, $\gamma(\frac{(1-c)K}{y(K)}) < 0$. As a result, the termination threshold is bigger that $\frac{(1-c)K}{y(K)}$. Taking derivative w.r.t. ℓ and substituting $(1-c)K$ by $\theta^{opt}(K)y(K)$ imply

$$\gamma'(\ell) = \left[y(K) + \Big(\theta^{opt}(K)-\ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\right]\Big(1-H(\ell)\Big).$$

Clearly, $\lim_{\ell \uparrow \theta^{opt}(K)} \gamma'(\ell) > 0$. However, since (by the monotone hazard rate assumption) $\frac{h(\ell)}{1-H(\ell)}$ is increasing ℓ, thus

$$\lim_{\ell \to \infty} \left[y(K) + \Big(\theta^{opt}(K) - \ell\Big) \frac{y(K)h(\ell)}{1 - H(\ell)} \right] = -\infty.$$

Moreover, $y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}$ is decreasing in ℓ, thus, there exists a unique $\tilde{\ell}$ so that

$$\gamma'(\tilde{\ell}) = 0.$$

Hence, $\tilde{\ell}$ is the unique maximizer of $\gamma(\ell)$. Clearly, if $\gamma(\tilde{\ell}) \leq 0$ then the IR constraint is always violated (i.e., issuing debt is impossible). So, $\gamma(\tilde{\ell}) > 0$ and there exists $\theta^*(K, \alpha_U)$ (where $\theta^{opt}(K) < \theta^*(K, \alpha_U) < \tilde{\ell}$) so that $\gamma(\theta^*(K, \alpha_U)) = 0$, that is

$$\begin{aligned}\gamma(\theta^*(K, \alpha_U)) &= H\Big(\theta^*(K, \alpha_U)\Big)(1 - c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y(K) - (1 - \alpha)K \\ &= 0\end{aligned}$$

Next, using the implicit function theorem, we have $\frac{\partial \theta^*(K,\alpha_U)}{\partial \alpha}\gamma'\Big(\theta^*(K, \alpha_U)\Big) + K = 0$, thus

$$\frac{\partial \theta^*(K, \alpha_U)}{\partial \alpha} = \frac{-K}{\gamma'(\theta^*(K, \alpha_U))} < 0,$$

where the last inequality follows because $\gamma'(\ell) > 0$ for $\theta^{opt}(K) < \ell < \tilde{\ell}$, finishing the proof. □

Proposition 3. *There exists a unique interior capital level $K^{opt} \in (0, \infty)$ that solves (10) and satisfies the first order condition*

$$1 = (1 - c)H\Big(\theta^{opt}(K^{opt})\Big) + y'(K^{opt}) \int_{\theta^{opt}(K^{opt})}^{\infty} \theta \, dH(\theta). \tag{11}$$

Proof of Proposition 3. We prove it in two steps. We first show that K^{opt} exists. Then, we show that it is unique.

Step 1. (Existence) Taking a derivate w.r.t. K from (10) and dividing by $1 - H\Big(\theta^{opt}(K)\Big)$

imply

$$y'(K)\mathbf{E}[\theta|\theta > \theta^{opt}(K)] - 1 - \frac{H\Big(\theta^{opt}(K)\Big)}{1 - H\Big(\theta^{opt}(K)\Big)}(1-c) \tag{15}$$

To show that the above equation has a solution we use continuity and the Rolle's theorem. We note that since $y(\cdot)$ (by assumption) is concave and increasing, thus $y'(K) \le \frac{y(K)}{K}$. Next, recall that $\theta^{opt}(K) = \frac{(1-c)K}{y(K)}$. Therefore, since $y'(K) \le \frac{y(K)}{K}$, thus $\theta^{opt}(K)$ is increasing in K. Moreover, since $y'(0) = \infty$, and $y'(\infty) = 0$, thus $\lim_{K\to 0}\theta^{opt}(K) = 0$ and $\lim_{K\to\infty}\theta^{opt}(K) = \infty$. Therefore,

$$\lim_{K\to 0} y'(K)\mathbf{E}\Big[\theta|\theta > \theta^{opt}(K)\Big] - 1 = \infty$$

and

$$\lim_{K\to 0} \frac{H\Big(\theta^{opt}(K)\Big)}{1 - H\Big(\theta^{opt}(K)\Big)}(1-c) = 0.$$

Thus,

$$\lim_{K\to 0}\left[y'(K)\mathbf{E}[\theta|\theta > \theta^{opt}(K)] - 1 - \frac{H\Big(\theta^{opt}(K)\Big)}{1 - H\Big(\theta^{opt}(K)\Big)}(1-c)\right] = \infty. \tag{16}$$

Next, we consider when $K \to \infty$. Recall that $y'(K) \le \frac{y(K)}{K}$, thus $\theta^{opt}(K)y'(K) < 1 - c < 1$. Moreover, one can show that (using the assumption that $\ln h(\theta)$ is concave):

$$\frac{H\Big(\theta^{opt}(K)\Big)}{1 - H\Big(\theta^{opt}(K)\Big)}\Big(\mathbf{E}\Big[\theta|\theta > \theta^{opt}(K)\Big] - \theta^{opt}(K)\Big) \le 1.$$

Thus, given that $\theta^{opt}(K) = \frac{(1-c)K}{y(K)}$, we must have

$$y'(K)\mathbf{E}\Big[\theta|\theta > \theta^{opt}(K)\Big] - 1 < \frac{y'(K)}{\frac{H\big(\theta^{opt}(K)\big)}{1-H\big(\theta^{opt}(K)\big)}}$$

Note that

$$\lim_{K\to\infty}\{\frac{y'(K)}{\frac{H\left(\theta^{opt}(K)\right)}{1-H\left(\theta^{opt}(K)\right)}}\}=0,$$

because $y'(\infty)=0$. Therefore, since $\lim_{K\to\infty}H\left(\theta^{opt}(K)\right)=1$, thus

$$\lim_{K\to\infty}\left[y'(K)\mathbf{E}\left[\theta|\theta>\theta^{opt}(K)\right]-1-\frac{H\left(\theta^{opt}(K)\right)}{1-H\left(\theta^{opt}(K)\right)}(1-c)\right]=-\infty. \tag{17}$$

Putting (16) and (17) together along with continuity of (15) imply that the solution K^{opt} exists.

Step 2. (Uniqueness) To prove the uniqueness we establish that

$$\frac{\partial}{\partial K}\left(y'(K)\mathbf{E}\left[\theta|\theta>\theta^{opt}(K)\right]-1-\frac{H\left(\theta^{opt}(K)\right)}{1-H\left(\theta^{opt}(K)\right)}(1-c)\right)<0.$$

Recall that $\frac{\partial\theta^{opt}(K)}{\partial K}>0$. Thus,

$$\frac{\partial}{\partial K}\left[\frac{H\left(\theta^{opt}(K)\right)}{1-H\left(\theta^{opt}(K)\right)}(1-c)\right]>0.$$

Therefore, to prove that above claim, we only need to show that

$$\frac{\partial\theta^{opt}(K)}{\partial K}\,y'(K)\frac{\partial\mathbf{E}[\theta|\theta>\theta^{opt}(K)]}{\partial\theta^{opt}(K)}+y''(K)\mathbf{E}\left[\theta|\theta>\theta^{opt}(K)\right]<0.$$

It is clear that

$$\frac{\partial\theta^{opt}(K)}{\partial K}\,y'(K)\frac{\partial\mathbf{E}[\theta|\theta>\theta^{opt}(K)]}{\partial\theta^{opt}(K)}>0$$

(because $\frac{\partial\theta^{opt}(K)}{\partial K}>0$, $y'(K)>0$, $\frac{\partial\mathbf{E}[\theta|\theta>\theta^{opt}(K)]}{\partial\theta^{opt}(K)}>0$) and

$$y''(K)\mathbf{E}\left[\theta|\theta>\theta^{opt}(K)\right]<0,$$

(because $y(\cdot)$ is concave).

Next, note that $\frac{\partial\theta^{opt}(K)}{\partial K} = \frac{1-c-\theta^{opt}(K)y'(K)}{y(K)} > 0$ and $0 \le \frac{\partial \mathbf{E}[\theta|\theta>\theta^{opt}(K)]}{\partial\theta^{opt}(K)} \le 1$, thus

$$\begin{aligned}
&\frac{\partial\theta^{opt}(K)}{\partial K}\, y'(K)\frac{\partial \mathbf{E}[\theta|\theta > \theta^{opt}(K)]}{\partial\theta^{opt}(K)} + y''(K)\mathbf{E}[\theta|\theta > \theta^{opt}(K)] \\
&< y''(K)\mathbf{E}[\theta|\theta > \theta^{opt}(K)] + (1-c)\frac{y'(K)}{y(K)} - \theta^{opt}(K)\frac{y'(K)^2}{y(K)} \\
&< (1-c)\frac{y'(K)}{y(K)} + \theta^{opt}(K)\left(y''(K) - \frac{y'(K)^2}{y(K)}\right) \\
&= (1-c)\left(\frac{y'(K)}{y(K)} + K[\frac{d}{dK}\left(\frac{y'(K)}{y(K)}\right)]\right)
\end{aligned}$$

Now, recall that $y'(\cdot)$ and $\frac{1}{y(K)}$ are convex and decreasing, and thus $\frac{y'(K)}{y(K)}$ is convex and decreasing.[19] As a result,

$$\frac{y'(K)}{y(K)} + K[\frac{d}{dK}\left(\frac{y'(K)}{y(K)}\right)] < 0$$

so the upper bound is negative, finishing the proof. □

Proposition 4. *If $\alpha_U > 1-c$, then $K^*(\alpha_U) < K^{opt}$.*

Proof of Proposition 4. When debt is high, due to Proposition 2, we have $\theta^*(K,\alpha_U) > \theta^{opt}(K)$. Thus, $K^*(\alpha_U)$ solves

$$K^*(\alpha_U) \in \arg\max_K \left\{H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K)dH(\theta) - K - \mathsf{C}_{h.\ debt}\right\} \tag{18}$$

where the high debt **cost term** is

$$\mathsf{C}_{h.\ debt} \equiv \int_{\theta^{opt}(K)}^{\theta^*(K,\alpha_U)} K(\frac{\theta y(K)}{K} - (1-c))dH(\theta).$$

Clearly, the derivative of

$$H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K)dH(\theta) - K$$

evaluated at $K = K^{opt}$ is zero (by (11)). Hence, to prove the claim we only need to show

[19] Note that for any convex and decreasing function $\phi(x)$ we have $|\phi'(x)| > \frac{\phi(x)}{x}$.

that the derivative of the high debt **cost term** $\mathsf{C}_{h.\ debt}$ evaluated at $K = K^{opt}$ is positive.[20]

We prove this statement in several steps.

It observes that

$$\frac{d\mathsf{C}_{h.\ debt}}{dK} = \int_{\theta^{opt}(K)}^{\theta^*(K,\alpha_U)} \Big(\theta y'(K) - (1-c)\Big) dH(\theta) + \frac{\partial\theta^*(K,\alpha_U)}{\partial K} h\Big(\theta^*(K,\alpha_U)\Big)\Big(\theta^*(K,\alpha_U)y(K) - (1-c)K\Big).$$

Note that

$$(1-c)K = \theta^{opt}(K)y(K)$$

and

$$(1-c) = \frac{\partial\theta^{opt}(K)}{\partial K}y(K) + \theta^{opt}(K)y'(K).$$

Therefore,

$$\begin{aligned}\frac{d\mathsf{C}_{h.\ debt}}{dK} &\geq \frac{\partial\theta^{opt}(K)}{\partial K}y(K)\Big(H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))\Big) + \frac{\partial\theta^*(K,\alpha_U)}{\partial K}\Big(\theta^*(K,\alpha_U) - \theta^{opt}(K)\Big)\\ &= y(K)\left[-\frac{\partial\theta^{opt}(K)}{\partial K}\Big(\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))\Big) + \frac{\partial\theta^*(K,\alpha_U)}{\partial K}\Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big)\right],\end{aligned}$$

where the last equality follows from differentiating (5) with respect to K.

Next to finish the proof, we show that

$$-\frac{\partial\theta^{opt}(K)}{\partial K}\Big(\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))\Big) + \frac{\partial\theta^*(K,\alpha_U)}{\partial K}\Big(1 - H\Big(\theta^*(K,\alpha_U)\Big)\Big) > 0. \tag{20}$$

Note that differentiating (5) with respect to K implies that

$$\begin{aligned}&\frac{\partial\theta^*(K,\alpha_U)}{\partial K}\left[1 - H\Big(\theta^*(K,\alpha_U)\Big) - \Big(\theta^*(K,\alpha_U) - \theta^{opt}(K)\Big)h\Big(\theta^*(K,\alpha_U)\Big)\right]\\ &= \frac{\partial\theta^{opt}(K)}{\partial K}\left[\frac{\alpha_U}{1-c} - H(\theta^{opt}(K)) - \Big(H\Big(\theta^*(K,\alpha_U)\Big) - H(\theta^{opt}(K))\Big)\right]\end{aligned}$$

[20] In more details, consider the F.O.C. of (18) w.r.t. K implying

$$0 = \Big(H(\theta^{opt}(K))(1-c) - 1 + \int_{\theta^{opt}(K)}^{\infty} \theta y'(K)dH(\theta)\Big) - \frac{d\mathsf{C}_{h.\ debt}}{dK}. \tag{19}$$

When K goes to zero, then $H(\theta^{opt}(K))(1-c) - 1 + \int_{\theta^{opt}(K)}^{\infty}\theta y'(K)dH(\theta)$ goes to infinity and $\frac{d\mathsf{C}_{h.\ debt}}{dK}$ goes to zero. Hence, the R.H.S. in (19) goes to infinity. Moreover, when $K = K^{opt}$ (using Proposition 3), then $H(\theta^{opt}(K))(1-c) - 1 + \int_{\theta^{opt}(K)}^{\infty}\theta y'(K)dH(\theta) = 0$. Hence, to prove the main result, we only need to show that $\frac{d\mathsf{C}_{h.\ debt}}{dK}|_{K=K^{opt}} > 0$, implying $K^*(\alpha_U) < K^{opt}$.

therefore, showing (20) is equivalent to show

$$\left(\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))\right)\left(\theta^*(K,\alpha_U) - \theta^{opt}(K)\right)h\left(\theta^*(K,\alpha_U)\right)$$
$$> \left(1 - H\left(\theta^*(K,\alpha_U)\right)\right)\left(H\left(\theta^*(K,\alpha_U)\right) - H(\theta^{opt}(K))\right) \tag{21}$$

To show (21) we first note that since $\alpha_U > 1-c$ and $\theta^{opt}(K) < \theta^*(K,\alpha_U)$ thus

$$\frac{\alpha_U}{1-c} - H(\theta^{opt}(K)) > 1 - H(\theta^{opt}(K)) > 1 - H\left(\theta^*(K,\alpha_U)\right) \tag{22}$$

In addition, when $H(\cdot)$ is convex in the set $[\theta^{opt}(K), \theta^*(K,\alpha_U)]$ then

$$h\left(\theta^*(K,\alpha_U)\right) > \frac{H\left(\theta^*(K,\alpha_U)\right) - H(\theta^{opt}(K))}{\theta^*(K,\alpha_U) - \theta^{opt}(K)} \tag{23}$$

and (22) and (23) together show (21).

Similarly, when $G(\cdot)$ is concave in the set $[\theta^{opt}(K), \theta^*(K,\alpha_U)]$ then since $\frac{h(z)}{1-H(z)}$ is increasing in z (the monotone hazard rate assumption) and $\theta^{opt}(K) < \theta^*(K,\alpha_U)$ thus

$$\frac{1 - H\left(\theta^*(K,\alpha_U)\right)}{1 - H(\theta^{opt}(K))} \leq \frac{h\left(\theta^*(K,\alpha_U)\right)}{h(\theta^*)} \leq \left(\frac{\theta^*(K,\alpha_U) - \theta^{opt}(K)}{H\left(\theta^*(K,\alpha_U)\right) - H(\theta^{opt}(K))}\right)h\left(\theta^*(K,\alpha_U)\right)$$

and as a consequence,

$$\left(1 - H(\theta^{opt}(K))\right)\left(\theta^*(K,\alpha_U) - \theta^{opt}(K)\right)h\left(\theta^*(K,\alpha_U)\right)$$
$$\geq \left(1 - H\left(\theta^*(K,\alpha_U)\right)\right)\left(H\left(\theta^*(K,\alpha_U)\right) - H(\theta^{opt}(K))\right) \tag{24}$$

Replacing $1 - H(\theta^{opt}(K))$ with $\frac{\alpha_U}{1-c} - H(\theta^{opt}(K))$ in (24) and noting $\frac{\alpha_U}{1-c} - H(\theta^{opt}(K)) > 1 - H(\theta^{opt}(K))$ finish the proof.[21]

□

Lemma 2. *The termination threshold increases in the level of investment:* $\frac{\partial \theta^*(K,\alpha_U)}{\partial K} \geq 0$.

Proof of Lemma 2. To prove the claim, we need to consider two cases (high debt and low debt).

[21] Notice that in the case where $H(\cdot)$ is convex in $\theta^{opt}(K)$ and concave in $\theta^*(K,\alpha_U)$ similar analysis hold. In this case (due to the Rolle's theorem) there exists $\tilde{\theta} \in [\theta^{opt}(K), \theta^*(K,\alpha_U)]$ so that $H'(\tilde{\theta}) = \frac{H(\theta^*(K,\alpha_U)) - H(\theta^{opt}(K))}{\theta^*(K,\alpha_U) - \theta^{opt}(K)}$.

Case1. Low debt $(1-c > \alpha_U)$. In this case depending on the size of the manager's private rent (managerial protection), G, the termination threshold satisfies $\theta^*(K,\alpha_U)y(K) = \max\{(1-c)K - G, \alpha_U K\} = \max\{(1-c)K - G, \alpha_U K\}$. Therefore, rearranging gives

$$\theta^*(K,\alpha_U) = \max\{\frac{(1-c)K-G}{y(K)}, \frac{\alpha_U K}{y(K)}\}.$$

Suppose $(1-c)K - G > \alpha_U K$. Then, $\theta^*(K,\alpha_U) = \frac{(1-c)K-G}{y(K)}$. Since $y(K)$ is increasing and concave thus $\frac{-1}{y(K)}$ and $\frac{K}{y(K)}$ are both increasing, thus $\theta^*(K,\alpha_U)$ is increasing in K. A similar argument holds when $\theta^*(K,\alpha_U) = \frac{(1-\alpha)K}{y(K)}$. Therefore, $\theta^*(K,\alpha_U)$ is increasing in K, i.e., $\frac{\partial\theta^*(K,\alpha_U)}{\partial K} \geq 0$.

Case2. High debt $(1-c < \alpha_U)$. For ease of notation let us denote the termination threshold by ℓ. In this case, the termination threshold satisfies the uninformed investor's IR constraint:

$$H(\ell)(1-c)K + (1-H(\ell))\ell y(K) = (1-\alpha)K = \alpha_U K.$$

We need to first show such threshold exists. Let us define for $\ell \geq \theta^{opt}(K) = \frac{(1-c)K}{y(K)}$,

$$\gamma(\ell) \equiv H(\ell)(1-c)K + \Big(1-H(\ell)\Big)\ell y(K) - \alpha_U K.$$

Thus, $\gamma(\frac{(1-c)K}{y(K)}) < 0$. As a result, the termination threshold is bigger that $\frac{(1-c)K}{y(K)}$. Taking derivative w.r.t. ℓ and substituting $(1-c)K$ by $\theta^{opt}(K)y(K)$ imply

$$\gamma'(\ell) = \Bigg[y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\Bigg]\Big(1-H(\ell)\Big).$$

Clearly, $\lim_{\ell\uparrow\theta^{opt}(K)}\gamma'(\ell) > 0$. However, since (by the monotone hazard rate assumption) $\frac{h(\ell)}{1-H(\ell)}$ is increasing ℓ, thus

$$\lim_{\ell\to\infty}\Bigg[y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}\Bigg] = -\infty.$$

Moreover,

$$y(K) + \Big(\theta^{opt}(K) - \ell\Big)\frac{y(K)h(\ell)}{1-H(\ell)}$$

is decreasing in ℓ, thus, there exists a unique $\tilde{\ell}$ so that

$$\gamma'(\tilde{\ell}) = 0.$$

Hence, $\tilde{\ell}$ is the unique maximizer of $\gamma(\ell)$. Clearly, if $\gamma(\tilde{\ell}) \leq 0$ then the IR constraint is always violated (i.e., issuing debt is impossible). So, $\gamma(\tilde{\ell}) > 0$ and there exists $\theta^*(K, \alpha_U)$ (where $\theta^{opt}(K) < \theta^*(K, \alpha_U) < \tilde{\ell}$) so that $\gamma(\theta^*(K, \alpha_U)) = 0$, that is

$$\begin{aligned}\gamma(\theta^*(K, \alpha_U)) &= H\Big(\theta^*(K, \alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y(K) - \alpha_U K \\ &= 0,\end{aligned}$$

and $\gamma'(\ell) > 0$ for $\theta^{opt}(K) < \ell < \tilde{\ell}$.

Next, by the implicit function theorem, we have

$$\frac{\partial \theta^*(K, \alpha_U)}{\partial K} \gamma'(\theta^*(K, \alpha_U)) + H\Big(\theta^*(K, \alpha_U)\Big)(1-c) + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y'(K) - \alpha_U = 0.$$

Therefore,

$$\begin{aligned}\frac{\partial \theta^*(K, \alpha_U)}{\partial K} &= \frac{-1}{\gamma'\Big(\theta^*(K, \alpha_U)\Big)}\left[H\Big(\theta^*(K, \alpha_U)\Big)(1-c) + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y'(K) - \alpha_U\right] \\ &> 0.\end{aligned}$$

The last equality follows, because $\gamma'\Big(\theta^*(K, \alpha_U)\Big) > 0$ and

$$H\Big(\theta^*(K, \alpha_U)\Big)(1-c) + \Big(1 - H(\theta^*(K, \alpha_U))\Big)\theta^*(K, \alpha_U) y'(K) - \alpha_U < 0,$$

because $y'(K) < \frac{y(K)}{K}$ and

$$\gamma\Big(\theta^*(K, \alpha_U)\Big) = H\Big(\theta^*(K, \alpha_U)\Big)(1-c)K + \Big(1 - H\Big(\theta^*(K, \alpha_U)\Big)\Big)\theta^*(K, \alpha_U) y(K) - \alpha_U K = 0,$$

finishing the proof. □

Proposition 5. *Suppose that the debt ratio is less than the efficient level, or $\alpha_U < 1-c$. If there are no managerial protections, or $G = 0$, then the level of investment is equal to the first-best benchmark and there is no distortion:*

$$K^*(\alpha_U) = K^{opt} = K^*(1-c)$$

If there are managerial protections, or $G > 0$, then the conditionally efficient level of investment can in general be greater or less than the first-best benchmark. If the debt ratio is low enough, or $\alpha_u \leq 1 - c - \frac{G}{K^{opt}}$, and H is strictly concave then investment is distorted upwards, or $K^*(\alpha_U) >$

K^{opt}. *If the cdf for the return distribution H is weakly convex, then the conditionally efficient level of investment is distorted downwards, or $K^*(\alpha_U) \leq K^{opt}$.*

Proof of Proposition 5. We prove the proposition in the following two parts.

Concave $H(\cdot)$ with low debt ratio capital structure. When $\alpha_U < 1 - c$ (i.e., low debt), due to Proposition 2, we have $\theta^*(K, \alpha_U) < \theta^{opt}(K)$. Thus, $K^*(\alpha_U)$ solves

$$K^*(\alpha_U) \in \arg\max_K \left\{ H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K - \mathsf{C}_{l.\ debt} \right\}$$

where the low debt **cost term** is

$$\mathsf{C}_{l.\ debt} \equiv \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}} K\left(1 - c - \frac{\theta y(K)}{K}\right) dH(\theta).$$

Clearly, the derivative of

$$H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K) dH(\theta) - K$$

evaluated at $K = K^{opt}$ is zero (by (11)). Hence, to prove the claim we only need to show that the derivative of the low debt **cost term** $\mathsf{C}_{l.\ debt}$ evaluated at $K = K^{opt}$ is negative.

Moreover, when $\alpha_U < 1 - c$ then (due to (3))

$$\theta^*(K, \alpha_U) y(K) = \max\{(1-c)K - G, \alpha_U K\}.$$

Suppose that at $K = K^{opt}$ we have

$$\max\{(1-c)K^{opt} - G, \alpha_U K^{opt}\} = (1-c)K^{opt} - G$$

(which is ensured when $K^{opt} > \frac{G}{1-c-\alpha_U}$). Therefore,

$$\theta^*(K^{opt}, \alpha_U) y(K^{opt}) = (1-c)K^{opt} - G$$

and taking a derivative w.r.t. K implies

$$\frac{\partial \theta^*(K^{opt}, \alpha_U)}{\partial K} y(K^{opt}) + \theta^*(K^{opt}, \alpha_U) y'(K^{opt}) = 1 - c. \tag{25}$$

Next, to finish the proof, using (25), we show that the margin of the low debt **cost term** $\mathsf{C}_{l.\ debt}$ evaluated at $K = K^{opt}$ is negative.

$$\begin{aligned}
\frac{d\mathsf{C}_{l.\ debt}}{dK} &= -\frac{\partial\theta^*(K,\alpha_U)}{\partial K}h(\theta^*(K,\alpha_U))\Big((1-c)K-\theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}(K)}(1-c-\theta y'(K))dH(\theta) \\
&\leq -\frac{\partial\theta^*(K,\alpha_U)}{\partial K}h(\theta^*(K,\alpha_U))\Big((1-c)K-\theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \Big(H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U)\Big)\Big(1-c-\theta^*(K,\alpha_U)y'(K)\Big) \\
&= -\frac{\partial\theta^*(K,\alpha_U)}{\partial K}h(\theta^*(K,\alpha_U))\Big((1-c)K-\theta^*(K,\alpha_U)y(K)\Big)+ \\
&\qquad \Big(H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U)\Big)\frac{\partial\theta^*(K,\alpha_U)}{\partial K}y(K) \\
&= \frac{\partial\theta^*(K,\alpha_U)}{\partial K}y(K)\Bigg(H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U))-\Big(\theta^{opt}(K)-\theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U))\Bigg) \\
&< 0,
\end{aligned}$$

where the last inequality follows because $\frac{\partial\theta^*(K,\alpha_U)}{\partial K} \geq 0$ and

$$H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U))-\Big(\theta^{opt}(K)-\theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U))<0,$$

because $H(\cdot)$ is concave (using the Taylor expansions). As a consequence, the margin of the low debt cost term is negative at K^{opt}, implying that $K^*(\alpha_U) > K^{opt}$.

Convex and weakly convex (e.g., Uniform distribution) $H(\cdot)$ with low debt ratio capital structure. It is also possible to show that managerial protections can be associated with less investment for firms with low debt ratios, or $\alpha_U < 1-c$. In this section we show if the cdf for the return distribution H is weakly convex, then the conditionally efficient level of investment is less than the first-best benchmark, or $K^*(\alpha_U) \leq K^{opt}$.

Here we show when debt ratio is low and the distribution of θ is uniform (i.e., $\alpha_U < 1-c$ and $H(\cdot) \sim \text{Uniform}[a,b]$) then (similar to the high debt capital structure environment) we obtain $K^*(\alpha_U) \leq K^{opt}$. A similar result identically holds when the cdf $H(\cdot)$ is weakly convex on its compact support.

Recall that when $\alpha_U < 1-c$ (i.e., low debt), due to Proposition 2, we have $\theta^*(K,\alpha_U) < \theta^{opt}(K)$. Thus, then $K^*(\alpha_U)$ solves

$$K^*(\alpha_U) \in \arg\max_K \left\{H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty}\theta y(K)dH(\theta) - K - \mathsf{C}_{l.\ debt}\right\}$$

where the low debt **cost term** is

$$\mathsf{C}_{l.\ debt} \equiv \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}} K\left(1-c-\frac{\theta y(K)}{K}\right)dH(\theta).$$

Clearly, the derivative of

$$H\Big(\theta^{opt}(K)\Big)(1-c)K + \int_{\theta^{opt}(K)}^{\infty} \theta y(K)dH(\theta) - K$$

evaluated at $K = K^{opt}$ is zero (by (11)). Hence, to prove the claim that $K^*(\alpha_U) \le K^{opt}$ we only need to show that the derivative of the low debt **cost term** $\mathsf{C}_{l.\ debt}$ evaluated at $K = K^{opt}$ is positive.

This claim follows because:

$$\begin{aligned}
\frac{d\mathsf{C}_{l.\ debt}}{dK} &= -\frac{\partial\theta^*(K,\alpha_U)}{\partial K}h(\theta^*(K,\alpha_U))\Big((1-c)K-\theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \int_{\theta^*(K,\alpha_U)}^{\theta^{opt}(K)}(1-c-\theta y'(K))dH(\theta) \\
&\ge -\frac{\partial\theta^*(K,\alpha_U)}{\partial K}h(\theta^*(K,\alpha_U))\Big((1-c)K-\theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \Big(H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U))\Big)\Big(1-c-\theta^{opt}(K)y'(K)\Big) \\
&= -\frac{\partial\theta^*(K,\alpha_U)}{\partial K}h(\theta^*(K,\alpha_U))\Big((1-c)K-\theta^*(K,\alpha_U)y(K)\Big) \\
&\qquad + \Big(H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U))\Big)\frac{\partial\theta^{opt}(K)}{\partial K}y(K) \\
&\ge \frac{\partial\theta^*(K,\alpha_U)}{\partial K}y(K)\Big(H(\theta^{opt}(K))-H(\theta^*(K,\alpha_U))-\Big(\theta^{opt}(K)-\theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U))\Big) \\
&= 0,
\end{aligned} \tag{26}$$

where the third relation follows because $(1-c)K = y(K)\theta^{opt}(K)$ and thus

$$1-c = y'(K)\theta^{opt}(K) + \frac{\partial\theta^{opt}(K))}{\partial K}y(K);$$

the fourth relation follows because (due to supermodularity of the threshold $\theta^*(K,\alpha_U)$ in α_U and K) when $\theta^*(K,\alpha_U) < \theta^{opt}(K)$ we have

$$\frac{\partial\theta^*(K,\alpha_U)}{\partial K} < \frac{\partial\theta^*(K,1-c)}{\partial K} = \frac{\partial\theta^{opt}(K)}{\partial K}.$$

Finally, since the derivative of the low debt **cost term** $\mathsf{C}_{l.\ debt}$ evaluated at $K = K^{opt}$ is positive, thus $K^*(\alpha_U) \leq K^{opt}$. Note that when $H(\cdot)$ is weakly convex on its compact support, then the last equality in (26) will be replaced by " $\geq$ " because for the convex C.D.F. $H(\cdot)$ we have

$$H(\theta^{opt}(K)) - H(\theta^*(K,\alpha_U)) - \Big(\theta^{opt}(K) - \theta^*(K,\alpha_U)\Big)h(\theta^*(K,\alpha_U)) \geq 0.$$

□

Proposition 6. *If there are no managerial protections, or $G = 0$, then the manager's preferred level of investment that maximizes his or her utility (equation (8)) for a given debt ratio α_U, $K_m(\alpha_U)$, is equal to the conditionally efficient level $K^*(\alpha_U)$. If there are managerial protections, or $G > 0$, then the manager's preferred level of investment is less than the conditionally efficient level, or $K_m(\alpha_U) < K^*(\alpha_U)$.*

Proof of Proposition 6. Suppose

$$K_m(\alpha_U) = \arg\max_K u_m = \arg\max_K \frac{\alpha_F}{1-\alpha_U} V\Big(K,\alpha_U\Big) + G\frac{\alpha_F}{1-\alpha_U}\Big(1 - H(\theta^*(K,\alpha_U))\Big).$$

Then, by the corresponding F.O.C. condition,

$$\frac{\partial u_m}{\partial K}|_{K=K_m(\alpha_U)} = 0. \tag{27}$$

Moreover, as shown in section 3.4.2, we have

$$\frac{\partial V(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)} = 0. \tag{28}$$

Then, evaluating $\frac{\partial u_m}{\partial K}$ at $K = K^*(\alpha_U)$ implies

$$\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} = \frac{\alpha_F}{1-\alpha_U}\left(\underbrace{\frac{\partial V(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)}}_{=0,\text{ by (28)}} - G\ h\Big(\theta^*(K,\alpha_U)\Big)\underbrace{\frac{\partial \theta^*(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)}}_{>0,\text{ by Lemma 2}}\right) < 0$$

the above inequality then shows that $\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} < 0$, as a consequence, $K_m(\alpha_U) < K^*(\alpha_U)$ for all level of debt α_U. □

C Competitive equity market

The baseline model introduced in Section 3.1 assumes that informed investors obtain a fraction $\frac{\alpha_I}{1-\alpha_U}$ of the realized equity value. This section shows that Proposition 6, which broadly says that managerial protections can reduce a manager's preferred level of investment, is robust to alternatively assuming a competitive equity market in which the informed investors make zero rents, similar to the uninformed investors.

In that case, the individual rationality constraint for informed investors holds with equality, which implies

$$\lambda_I(K,\alpha_U) = \frac{\alpha_I K}{\int_{\theta^*(K,\alpha_U)}^{\infty} \theta y(K) dH(\theta) + H\Big(\theta^*(K,\alpha_U)\Big)(1-c)K - \alpha_U K} \tag{29}$$

where $\lambda_I(K,\alpha_U)$ is the *endogenous* shares from the realized equity value that goes to informed investors, contributing $\alpha_I K$ in investment. Note that $\lambda_I(K,\alpha_U)$ implies that the ex-ante individual rationality constraint for informed investors bind. Hence, informed investors and debt holders are ex-ante identical. Importantly, this complex object $\lambda_I(K,\alpha_U)$ does depend on K.

Given the competitive market for informed investors, the manager's problem, fixing α_F, is to solve

$$\max_{K_m,\alpha_U} u_m = \max_{K_m,\alpha_U} \underbrace{V(K,\alpha_U)}_{\text{ex-ante realized equity value}} + \underbrace{\Big(1-\lambda_I(K,\alpha_U)\Big) G \Big(1-H(\theta^*(K,\alpha_U))\Big)}_{\text{private benefit of control}} \tag{30}$$

where

$$V\Big(K,\alpha_U\Big) = H\Big(\theta^*(K,\alpha_U)\Big)\Big[((1-c)-\alpha_U)K\Big]^+ + \int_{\theta^*(K,\alpha_U)}^{\infty} \Big(\theta y(K) - p(K,\alpha_U)\Big) dH(\theta) - (1-\alpha_U)K$$

(see subsection 3.1) and $\Big(1-\lambda_I(K,\alpha_U)\Big) G \Big(1-H(\theta^*(K,\alpha_U))\Big)$ is the manager's private benefit of control given that the project is not terminated at $t=1$, which is proportional to multiplication of the manager's protection G and the manager's equity share $\Big(1-\lambda_I(K,\alpha_U)\Big)$, consistent with the logic in (2).

While $\lambda_I(K,\alpha_U)$ is endogenous and complex, the following proposition shows that we still have $K_m(\alpha_U) < K^*(\alpha_U)$. That is, even when informed investors, like debt-holders, are ex-ante in zero gain, i.e., their individual rationality constraint binds, the under-investment result in Proposition 6 still holds.

Proposition C.1. *Fix $\alpha_F < c$. Suppose the market for informed investors is competitive, i.e., informed investors' share from the realized equity value is*

$$\lambda_I(K, \alpha_U) = \frac{\alpha_I K}{\int_{\theta^*(K,\alpha_U)}^{\infty} \theta y(K) dH(\theta) + H\Big(\theta^*(K, \alpha_U)\Big)(1-c)K - \alpha_U K}.$$

Then, for all level of debt α_U, we have $K_m(\alpha_U) < K^*(\alpha_U)$.

Proof of Proposition C.1. The proof follows similar steps as in the proof of Proposition 6. To show that for all α_U, $K_m(\alpha_U) < K^*(\alpha_U)$, it is enough to prove that $\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} < 0$. Note that since $\lambda_I(K, \alpha_U)$ depends on K, we first note that

$$\begin{aligned}
\frac{\partial \lambda_I(K, \alpha_U)}{\partial K}|_{K=K^*(\alpha_U)} &= \frac{\partial}{\partial K}\left\{\frac{\alpha_I K}{V(K, \alpha_U) + (1-\alpha_U)K}\right\}|_{K=K^*(\alpha_U)} \\
&= \left(\frac{V(K, \alpha_U) + (1-\alpha_U)K - K\Big(\frac{\partial V(K,\alpha_U)}{\partial K} + 1 - \alpha_U\Big)}{V(K, \alpha_U) + (1-\alpha_U)K}\right)\left(\frac{\alpha_I}{V(K, \alpha_U) + (1-\alpha_U)K}\right)|_{K=K^*(\alpha_U)} \\
&> 0 \qquad (31)
\end{aligned}$$

where that inequality follows by $\frac{\partial V(K,\alpha_U)}{\partial K}|_{K=K^*(\alpha_U)} = 0$, (as shown in section 3.4.2, see also (28). Using this result, next we have

$$\frac{\partial u_m}{\partial K}|_{K=K^*(\alpha_U)} = \underbrace{\frac{\partial V(K, \alpha_U)}{\partial K}|_{K=K^*(\alpha_U)}}_{=0, \text{ by } (28)} + \underbrace{\frac{\partial}{\partial K}\left(\Big(1 - \lambda_I(K, \alpha_U)\Big)\, G\, \Big(1 - H(\theta^*(K, \alpha_U))\Big)\right)|_{K=K^*(\alpha_U)}}_{<0, \text{ by } (31) \text{ and Lemma } 2} < 0,$$

(note that $\Big(1 - \lambda_I(K, \alpha_U)\Big)$ is decreasing in K, by (31), and $\Big(1 - H(\theta^*(K, \alpha_U))\Big)$ is decreasing in K by Lemma 2) finishing the proof. □