Types of Conflict in Strategic Communication*

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Abstract

We study communication in coordination games on networks with either conflicting beliefs or conflicting preferences. We show these two ways of modeling conflict lead to opposite conclusions. While for any type of conflict truthful communication is always beneficial from the ex-ante perspective, there is however a credibility issue at the interim stage, i.e., information transmission pattern crucially depends on the nature of disagreement. Under conflicting preferences information transmission exhibits *negative externality effect*: greater information obtained by some agent discourages further information accumulation by harming the credibility of other agents. In contrast, under conflicting beliefs information transmission exhibits *positive externality effect*: greater information obtained by some agent encourages further information accumulation by improving the credibility of other agents.

By applying our findings to several frameworks, we show how strategic motives, depending on the types of conflict, crucially affect decision making, information aggregation in polls, communication network formations, information flow direction in networks, and segregations and aggregations in communications. We finally extend our analysis to the general case where the types of conflict are both simultaneously present.

Keywords: Types of conflict. communication. heterogeneous beliefs. heterogeneous preferences. networks. corporate governance.

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1 Introduction

Decision-making under imperfect information is a challenge that is faced in the context of many economic activities. In such environments, oftentimes, each agent needs to take an action that matches his preferences given the state of the world and that action affects the payoffs of other agents.[1,2] Moreover, agents base their strategic decisions not only on their private information, but also on the information obtained from other agents during preliminary discussions. However, even when agents get to know the same information about the state of the world, they might still disagree on the proper actions, either because they have *conflicting preferences* or because they have *conflicting beliefs*. Under conflicting preferences, the agents agree on the fundamentals of the uncertain economic environment but simply like different courses of action.[3] Under conflicting beliefs, the agents would prefer the same actions, if they had known the exact state of the world; however, under incomplete information, they diverge in their beliefs (opinions) about the uncertain environment and, thus, deem different actions optimal.[4]

In this study we ask: Are economic insights sensitive to the *types of conflict*? How does type of conflict between individuals affect decision making in strategic communications? And, how does information transmission pattern change in networks depending on the nature of disagreements? We argue that types of conflict between individuals can lead to distinctly different insights in strategic communications. To achieve this, we develop a multiplayer cheap talk model in networks that addresses how strategic motives, de-

[1]For example, managers of a corporation must choose marketing strategies, the effectiveness of which depends on tomorrow's market condition. Managers are mainly responsible for their respective divisions and might differ in their goals and in their perceptions of the uncertain economic environment. Nevertheless, the divisions are parts of one corporation, and the strategy chosen by some manager has an effect on other divisions' payoffs.

[2]In particular, in this paper, we formalize the interdependence of individuals' payoffs by assuming that each agent faces a loss when his action and the actions of other agents differ from his own "ideal action"—an action that matches the individual's preferences given the state of the world.

[3]For example, some managers advocate rather aggressive expansion strategies because they have a taste for empire building. On possible reasons for the existence and persistence of different preferences see, e.g., Austen-Smith [1993], Caillaud and Tirole [2007], Morgan and Stocken [2008] and Galeotti et al. [2013] for a discussion of the assumption of different preferences in the economics literature.

[4]For example, some managers prefer a more aggressive expansion strategy, because they hold a more optimistic belief (opinion) about the corporation's prospects in tomorrow's market. On possible reasons for the existence of different priors see, e.g., Tversky and Kahneman [1974], Aumann [1976], Banerjee and Somanathan [2001], Acemoglu and Ozdaglar [2011], Sethi and Yildiz [2012], Acemoglu et al. [2016] and Sethi and Yildiz [2016]. See also Morris [1995] for a discussion of the assumption of different priors in the economics literature.

pending on the types of conflict, affect decision making, information aggregation, segregations and aggregations in communications, communication network formations and direction of information flow in communities.

Our model includes n individuals with preference biases $b_1, \cdots, b_n$, who face an uncertain economic environment summarized by θ. Agents might hold different beliefs (opinions) about the economic environment: agent i's prior of θ is characterized by Beta distribution with parameters (α_i, β_i). The state of the world relevant to the decision-making process, Ω, is composed of D different aspects, $\Omega = \sum_{d=1}^{D} \omega_d$. Conditional on the underlying economic environment θ, the aspects are independent and identically distributed, with each ω_d taking a value of 1 with probability θ and 0 with complementary probability $1 - \theta$. The total number of aspects is greater than the number of agents, $D \geq n$, and each agent i is privately informed of the respective aspect ω_i.

Agents can simultaneously communicate their private information to each other according to a communication network, which is set prior to the aspects' realization. The network is described by the directed graph G, such that individual i can send cheap talk messages to other agents that he has links to.[5] After the communication takes place, individuals simultaneously choose actions $a_1^{\mathsf{G}}, \cdots, a_n^{\mathsf{G}}$ that influence each other's payoffs, such that every individual i obtains $u_i(a^{\mathsf{G}}|\Omega) = -\sum_{j=1}^{n}(a_j^{\mathsf{G}} - \Omega - b_i)^2$. That is, individual i's payoff depends on how close his action, as well as the actions of other agents, are to individual i's ideal action, $\Omega + b_i$. Under conflicting preferences, individuals have different biases $b_1, ..., b_n$, but share the common prior belief, $(\alpha_i, \beta_i) = (\alpha, \beta)\ \forall i$. Under conflicting beliefs, individuals diverge in their prior beliefs about θ, but have fully aligned preferences, $b_i = 0\ \forall i$.

1.1 Main Results

Assume that individual i has a link to individual j and, apart from i's message, individual j gets to know k_j aspects from other agents. Under either type of disagreement between the agents, the two parties, i and j, would ex-ante prefer truthful revelation of ω_i to agent j. However, there is a credibility issue at the interim stage. Importantly, the credibility of communication and the way it is influenced by k_j crucially depends on the nature of disagreement between the individuals.

[5]We note that binary structure of signals let us think of equilibrium as having a network structure where some pairs communicate perfectly and some pairs do not communicate at all.

Our main result is to show than when the two parties diverge in their beliefs, the communication pattern exhibits **positive externality effect**: greater number of aspects obtained by agent j—higher k_j—*improves the credibility* of agent i. Intuitively, as k_j increases, two things happen. First, agent i expects the ex-post belief of agent j to become more congruent. Indeed, agent i deems other signals revealed to individual j distributed according to i's belief, hence, he expects individual j to be persuaded and adjust his ex-post belief in the "right" direction (from i's point of view). Second, the magnitude of the effect of an additional signal on j's action decreases with k_j. However, the additive nature of the state Ω in aspects ω_d insures that the rate of decrease is sufficiently low, so that the effect of i's message on j's action remains significant enough to prevent individual i from misreporting to individual j (given that i expects j to become more congruent).[6]

In sharp contrast, we show when the parties disagree in their preferences, the communication pattern exhibits **negative externality effect**: greater information available to individual j—higher k_j—*harms the credibility* of individual i. Intuitively, the magnitude of the effect from an additional signal on j's action decreases with the informativeness of agent j. Thus, for sufficiently high k_j, the expected effect might become so small that i would prefer to lie in order to shift j's action closer towards i's preferred one. Moreover, the credibility of individual i deteriorates with his preference divergence from individual j. This insight from the negative externality effect in communication under conflicting preferences (i.e. biases) has also been identified in other literature (e.g., Austen-Smith [1993], Morgan and Stocken [2008] and Galeotti et al. [2013]). However, in contrast to them, we obtain this negative externality effect when the state of the world is additive. As mentioned in the above, the additive nature of the state Ω in aspects is sufficient to insure that the positive externality effect exists under conflicting prior beliefs.

By studying these two externality effects together, we also show that these two ways of modeling conflict (i.e., belief v.s. preference) lead to opposite conclusions. To achieve this, we next apply our findings to answer several questions (some inspired by previous works) with many-to-one and many-to-many (with arbitrary) communication structures.

[6] Another natural way to think about the positive externality effect is to view the privately observed aspects as **complements**. Indeed, if individual i reports to j truthfully, then other individual l might be credible in truthful reporting to j as well. On the other hand, if individual i does not report to individual j, then it might not be credible for l to transmit his private information truthfully. Thus, private signals of i and l act as complements.

1.1.1 Applications

Information Aggregation. Often, policymakers (managers) who have to make a policy choice have inadequate knowledge of the costs and benefits of the choice they are about to make. Therefore, they usually use polls for determining policy outcomes. Polls are used for various reasons. One important reason is that polls can aggregate information dispersed among strategic constituents. As an application of our findings, inspired by the important paper by Morgan and Stocken [2008], we consider how poll size affects information aggregation in polls. In this framework, a benevolent policymaker (hereafter, PM) polls constituents to obtain information about a payoff relevant state variable so as to find a policy that affects the welfare of all the constituents in the polity. In this setting we ask: How does type of conflict between agents affect information aggregation via truth-telling strategies in polls? Does information aggregation occur in large polls or small polls? The answer crucially depends on the nature of disagreement. Under conflicting preferences, similarly to Morgan and Stocken [2008], we find information aggregation via truth-telling strategies is *impossible in large polls*. This is because, due to the negative externality effect, the magnitude of the effect from an additional signal on the PM's action decreases with the informativeness of the PM. Thus, for sufficiently large poll size, the expected effect might become so small that any polled constituent i would prefer to lie in order to shift the PM's action closer towards i's preferred bias. In fact, we show there exists a finite upper bound on the poll size where truthful information revelation is an equilibrium.

In contrast, however, we find under conflicting beliefs information aggregation is *impossible in small polls*. In fact, on the one hand, due to the positive externality effect, for sufficiently large poll size each polled constituent i expects the ex-post belief of the PM to become more congruent. On the other hand, the magnitude of the effect of an additional signal on the PM's action reduces with the poll size. However, due to the additive nature of the state Ω in any aspect ω_d the rate of decrease becomes sufficiently low, so that the effect of i's message on the PM's action remains significant enough to prevent polled constituent i from misreporting to the PM. In fact, we show there exists a finite lower bound on the poll size where truthful information revelation is an equilibrium.

Information Flow in Networks. As another application of the main findings, in a framework, inspired by the new paper by Galeotti et al. [2013], we study the direction of information flow (transmission) in networks between two homogenous communities (groups). Here, the question is: How does type of conflict between individuals affect information transmission across the groups? The answer crucially depends on the type of conflict. Similarly to Galeotti et al. [2013], we find that under conflicting preferences, the information is more likely to flow from a larger community to a smaller one: either only members of the large community truthfully report to members of the small community, or members from the opposite communities report to each other (with more intensive information flow from the large community to the small one). Intuitively, members of the large community accumulate many aspects from their like-minded colleagues, which, by the negative externality effect, curbs truthful reporting from the members of the small community.

Under conflicting beliefs, however, we show the opposite communication pattern is more natural to appear, where members of the small community are more likely to report truthfully to members of the large community. The reason being that greater information accumulated inside the large community, by the positive externality effect, improves the credibility of the small community members.

Communication Network Formations. Allowing for arbitrary many-to-many communication structures, we further fully characterize truthful communication network structures arising in equilibria. We show under conflicting preferences, any pairwise stable equilibrium[7] generates the same set of in-degrees. These in-degrees are necessarily maximal and, by the negative externality effect, are neither too low nor too large. On the contrary, under conflicting beliefs, individuals can have different in-degrees in different pairwise stable equilibria. Moreover, by the positive externality effect, these in-degrees are either sufficiently low or quite large (i.e., it is easy to be credible to an individual with a high in-degree, and it is difficult to be truthful to a low in-degree individual).

In a particular case in which the agents cumulatively possess all the relevant information, $D = n$, under conflicting beliefs, the unique maximal equilibrium has a complete communication network. However, aside from the maximal equilibrium, there may also

[7] An equilibrium is *pairwise stable*, if no pair of individuals can improve the communication pattern between them and increase their ex-ante expected utilities, while satisfying the truth-telling constraints and keeping other strategies fixed. For more details, see Appendix J.

exist pairwise stable equilibria that fail to aggregate all of the information (i.e., some individuals in the communication networks have quite low in-degrees).

Replicas: Segregation vs. Aggregation. As another application of the main findings, we study how types of conflict lead to a segregation or aggregation in communications due to adding new informed agents. In particular we ask: Does introduction of new agents who are privately informed about other aspects help agents to accumulate greater information about the state of the world? The answer crucially depends on the nature of disagreement. We show increasing the number of agents of the same type *localizes* communication under conflicting preferences; and *intensifies cross-type* communication under conflicting beliefs. Therefore, surprisingly, adding new informed agents does *not* necessarily improve the aggregation of the available information. In fact, we show under conflicting preferences, replicating the set of individuals, by the negative externality effect, curbs cross-type communication and can lead to a segregation of communication according to the agents' types. Under conflicting beliefs, on the contrary, adding new informed agents, by the positive externality effect, boosts cross-type communication, which necessarily leads to greater information aggregation.

1.1.2 Extensions

Finally, we extend our benchmark model to consider strategic communications when the two sources of conflict are both simultaneously present. To present the results, consider again individual i who has a link to individual j and, apart from i's message, individual j gets to know k_j aspects from other agents. We obtain three results in this general setting.

First: We find a *necessary* and *sufficient* condition ensuring truthful communication of i to j.

Second: In the interim stage, in contrast to the benchmark case, the effect of increasing k_j on i's truthful communication to j is, in general, ambiguous. That is, expectedly, depending on the interplay between the positive and negative externality effects, impact of increasing k_j on the credibility of communication, can be *positive*, *negative* or even *neutral*. In particular, neutrality arises when these two externality effects fully offset one another.

Third: More interestingly, in the ex-ante stage the above ambiguous effect of increasing k_j disappears. That is, similar to the benchmark case, individual i *before* learning his private signal (i.e. ex-ante stage), would always gain benefit when j obtains more signals with increasing k_j. However, the margin of this gain gets *smaller* with increasing k_j. This is because, in individual i's view, with increasing k_j agent j obtains more information about the state of the world which is ex-ante beneficial for i. Thus, this ex-ante monotonic but concave behavior of benefit due to increasing k_j always holds and it is *independent* of the types of conflict between individuals.

2 Literature

This paper contributes to strand of literature on strategic information transmission which builds on the classic model of cheap talk communication by Crawford and Sobel [1982] and Green and Stokey [2007].[8]

This work is the first that analyzes strategic information transmission in networks under conflicting belief. Our new insight to the literature is to introduce the positive externality effect in communication under conflicting prior beliefs. Also, one important implication of this paper is to introduce a framework that shows these two ways of modeling conflict (i.e., belief v.s. preference) leads to opposite conclusions in communication.

Our insight from the negative externality effect in communication under conflicting preferences (also called congestion effect) has also been identified in other literature (e.g. Austen-Smith [1993], Morgan and Stocken [2008] and Galeotti et al. [2013]). However, in contrast to all of those works, we obtain this negative externality effect when the state of the world, that is appeared in players' payoffs, is additive.

The closest work to our paper is the recent paper by Galeotti et al. [2013] who focus on preference conflict to study strategic communication in networks. Our work features similar type of payoff interdependence however differs from their formulation because we consider: (i) additive state of the world, and (ii) allow for belief conflict. Naturally, because the marginal value of learning of an additional signal decreases with the in-

[8]This literature has been extended in several directions. For example, Farrell and Gibbons [1989] consider multiple receivers, while Austen-Smith [1993] consider multiple senders of information. Battaglini [2002] considers multiple senders combined with a multi-dimensional state space, while Aumann and Hart [2003] consider multiple rounds of cheap talk. This literature is very rich, See also Gibbons et al. [2013] for an excellent review of the literature on decisions in organizations.

formativeness of the individual, we obtain similar results under conflicting preferences, namely, the negative externality result that replicates the congestion effect of Galeotti et al. [2013]. However, in their work the state of the world, appeared in a player's payoff function, is singleton (denoted by θ), whereas in our work it is additive (denoted by Ω), which is the aggregate of D different aspects. It is important to note that when state of the world is singleton (like their model) the positive externality effect under conflicting belief does not exist. However, the positive externality effect is present in other settings where the marginal effect of an additional signal is sufficiently low, including our additive/aggregate state of the world. In terms of applications, inspired from their work, we also analyze information transmission in networks. We, however, show that role of type of conflict is actually crucial, as it may lead to opposite direction of information transmission between two homogenous groups in networks.

The polling application of our model is inspired by the important paper by Morgan and Stocken [2008] who focus on preference conflict to study the informative substance of straw polls that are held prior to elections. Similar to them we also assume that the prior beliefs about the fundamental has a Beta distribution. Our set up, however, differs from their many-to-one formulation because we consider: (i) additive state of the world, (ii) finite number of individuals (constituents), (iii) many-to-many with arbitrary network structure, and also (iv) allow for belief conflict. Given these differences, however, similar to them we show due to the negative externality effect, under conflicting preferences information aggregation is impossible in large polls. On the contrary, this policy insight may depend on the types of conflict between constituents in a polity (corporation), because under conflicting beliefs information aggregation can only be aggregated in large polls. Moreover, we identify that when the sources of conflict are both present, impact of increasing the poll size on information aggregation can be positive, negative or even neutral, depending on the interplay between the positive and negative externality effects.

Following the common trend in the literature with network frictions (e.g. Galeotti et al. [2013], Calvó-Armengol and de Martí [2007], Hagenbach and Koessler [2010], Galeotti et al. [2010]), and the literature on polling (e.g. Austen-Smith [1993], Morgan and Stocken [2008]), we assume the payoff function of individuals takes a form of a quadratic function. Quadratic payoff has attractive properties for performing our analysis of the network formation and information transmission in network while at the same time offering considerable modeling flexibility.

Similar to ours, a small number of papers also identified that types conflicts can lead to different outcomes. However, they are not about strategic information transmission and their frameworks are distinctly different from ours. Che and Kartik [2009] show that difference of belief between a decision maker and an advisor can increase the advisor's incentive to exert effort to persuade the decision maker; at the same time, differences in preferences cannot induce such a persuasion motive. Hirsch [2011], Van den Steen [2006] and Van den Steen [2009] also analyze this mechanism. Hirsch [2011] illustrates how open disagreement in beliefs between a principal and an agent creates a persuasion-based rationale for deference: the principal can allow the agent to implement the agent's preferred policy in the first period in order to let the agent learn from his own mistakes and increase his effort on the principal's preferred policy in the second period. Van den Steen [2009] shows that a principal will rely on persuasion—costly alteration of the agent's beliefs—for projects that need high agent's effort. Relatedly, Van den Steen [2006] shows that a principal may delegate decision rights to the agent in order to increase his efforts, which is crucial for the project's success.[9]

[9] Also related are studies that focus on questions of coordination and adaptation with verifiable information transmission in communication networks (Calvó-Armengol and de Martí [2007], Calvó-Armengol and de Martí [2009], Calvó-Armengol et al. [2011]). Relatedly, in a coordination framework Hagenbach and Koessler [2010] focus on preference conflict and study strategic cheap talk communication in networks. However, their communication model distinctly differs from ours in two ways: First, the state of the world is a sum of aspects that are sampled from some *known* distributions. Hence, knowing one aspect does not help in predicting some other unknown aspect (while in our model, in which all aspects come from the same distribution, knowing some aspect allows one to make an inference about an unknown one). This assumption implies a constant marginal value of every additional signal; hence, the negative externality result is absent from their model. Second, they consider a different type of coordination, i.e., each individual not only wants others' actions to be closer to his action (as is the case in our model), but also also wants to match his action with the actions of others. Loginova [2012] also characterizes the pure cheap talk in communication networks with the setting in which costly verifiable information transmission is feasible. She finds that the availability of verifiable communication allows each agent to get a weakly greater number of truthful messages compared to the pure cheap talk setting, but nevertheless can decrease the total welfare if both parties bear the cost of a hard link. Finally, the analysis of the communication patterns arising in equilibria contributes to the literature of strategic network formation, which includes Jackson and Wolinsky [1996], Bala and Goyal [2000], Goyal [2007] and Jackson [2008]. Our paper also contributes to the theoretical literature on communication in corporate boards. Members of corporate boards have usually diverse expertise in various aspects of the company's business. Therefore, decision making, voting and communication between the board members and the company's CEO is critical to company functioning and performance. Harris and Raviv [2008], Raheja [2005], Hagenbach and Koessler [2010] and Malenko [2014] consider communication within the board members, while Hirshleifer and Thakor [1994], Hermalin and Weisbach [1998], Song and Thakor [2006], Morgan and Stocken [2008], Kumar and Sivaramakrishnan [2008] and Adams and Ferreira [2007] focus on the board-CEO interaction and consider the board as a single agent. See Adams et al. [2010] for a comprehensive review of the literature.

Outline. The rest of the paper is organized as follows. Section 3 presents the model. Section 4 solves the model. Section 5 focuses on the applications: Section 5.1 considers information flow in networks. Section 5.2 characterizes communication network structures arising in equilibria. Section 5.3 considers segregations and aggregations in communications due to adding new informed agents. Section 6 considers information aggregation in polls and studies extensions of the benchmark model when the types of conflict are both simultaneously present. Finally, Section 7 concludes. Proofs are relegated to the Appendix.

3 Model

Let the set of agents/individuals be $N = \{1, 2, \cdots, n\}$ with $n \geq 2$, where each agent i has the preference bias b_i. The underlying *economic environment* is summarized by θ that is unknown to the individuals. Following Morgan and Stocken [2008], each agent i's prior of θ is characterized by Beta distribution with parameters (α_i, β_i) and density of

$$f_i(\theta) = \frac{1}{\mathbf{B}(\alpha_i, \beta_i)} \theta^{\alpha_i - 1} (1 - \theta)^{\beta_i - 1}. \tag{1}$$

The preference profile $\{b_1, ..., b_n\}$ and the individuals' priors are publicly known. There are D different *aspects* $\omega_1, ..., \omega_D$ that determine the *state of the world* as

$$\Omega = \sum_{d=1}^{D} \omega_d. \tag{2}$$

Conditional on the underlying economic environment θ, the aspects $\{\omega_d\}_{d=1}^{D}$ are independent and identically distributed, and $\omega_d = 1$ with probability θ and $\omega_d = 0$ with complementary probability $1 - \theta$. The total number of aspects is greater than the number of individuals, $D \geq n$, and each individual i is privately informed of the aspect ω_i (i.e., individual i receives the private signal ω_i). Thus, all individuals cumulatively get to know the first n of D aspects. Note that if $D = n$, then the individuals jointly hold all the relevant information regarding the state of the world. The communication network is set prior to the aspects' realization and is described by a directed graph $\mathsf{G} \in \{0, 1\}^{n \times n}$, where individual i communicates a cheap talk message about his signal ω_i to individual

j if and only if $g_{ij} = 1$.[10] It is assumed that the communication links are cheap to sustain: each involved party, either sending the message or receiving it, bears just an infinitesimal cost $\varepsilon > 0$. While the individuals are aware of each other's existence and each other's preferences and priors, the communication network G is not commonly known. Rather, each individual i knows only the structure of his respective neighborhood: the set of individuals to whom individual i has links, $N_i(\mathsf{G}) = \{j \in N : g_{ij} = 1\}$, and the set of individuals who have links directed to individual i, $N_i^{-1}(\mathsf{G}) = \{j \in N : g_{ji} = 1\}$.

Communication network. After the signals are realized, individuals communicate their privately observed aspects of the state of the world. Each individual i sends private message $m_{ij}^{\mathsf{G}} \in \{0, 1\}$ to every individual j that he has a link to in the communication network G. It is assumed that communication takes the form of cheap talk, and that messages are sent simultaneously and are observed only by the sending and the receiving parties. A *communication strategy* of individual i with the private signal ω_i defines a vector

$$\mu_i^{\mathsf{G}}(\omega_i) = \{\mu_{ij}^{\mathsf{G}}(\omega_i)\}_{j \in N_i(\mathsf{G})} \in \{0, 1\}^{|N_i(\mathsf{G})|}.$$

A communication strategy profile is denoted by $\mu^{\mathsf{G}} = \{\mu_1^{\mathsf{G}}, \ldots, \mu_n^{\mathsf{G}}\}$. The messages actually sent by individual i are denoted by vector $\widehat{m}_i^{\mathsf{G}}$; the profile of all sent messages is $\widehat{m}^{\mathsf{G}} = \{\widehat{m}_1^{\mathsf{G}}, \ldots, \widehat{m}_n^{\mathsf{G}}\}$.[11]

Decision making. After the communication has taken place, each individual i chooses an action $a_i^{\mathsf{G}} \in \mathbb{R}$. The *action strategy* of individual i is a function of his information set that consists of his own signal, ω_i, and the messages he gets from $N_i^{-1}(\mathsf{G})$, $\widehat{m}_{N_i^{-1}(\mathsf{G}),i}^{\mathsf{G}}$:

$$a_i^{\mathsf{G}} : \{0, 1\} \times \{0, 1\}^{|N_i^{-1}(\mathsf{G})|} \to \mathbb{R}.$$

Let $a^{\mathsf{G}} = \{a_1^{\mathsf{G}}, \cdots, a_n^{\mathsf{G}}\}$ denote the action strategy profile. Conditional on the state of the world Ω, if the chosen action profile is $\hat{a}^{\mathsf{G}} = \{\hat{a}_1^{\mathsf{G}}, ..., \hat{a}_n^{\mathsf{G}}\}$ then the realized payoff (utility)

[10] The assumption that the communication network is set up before realization of the signals can, for example, be justified by the necessity of forming a communication schedule beforehand.

[11] The superscript G for communication strategies highlights the dependence on the network structure. For the sake of simplicity, we use the same superscript G for all individuals; note, however, that every individual i conditions his communication strategy only on the structure of his respective neighborhood.

of individual i is

$$u_i(\hat{a}^{\mathsf{G}}|\Omega) = -\sum_{j=1}^{n}(\hat{a}_j^{\mathsf{G}} - \Omega - b_i)^2.$$

The payoff of individual i increases as his own action and the actions of other individuals get closer to individual i's ideal action, $\Omega + b_i$.

Remark 1. *We introduce the following time notation in order to distinguish between periods with different scopes of information available to the agents: "ex-ante" to denote the stage prior to when the signals are realized, "interim" for the time period after the signals' realization but prior to communication, and finally, "ex-post" to represent the period after communication has occurred but before the actions are taken.*

3.1 Type of conflict

We consider the analysis under two different assumptions about the nature of disagreement between the individuals:

Conflicting beliefs. Under conflicting beliefs, the individuals have fully aligned preferences, i.e., $b_i = 0$ for all $i \in N$; however, at least some individuals hold different prior beliefs about θ, i.e., $(\alpha_i, \beta_i) \neq (\alpha_j, \beta_j)$ for some i and j. In the main body of the paper, we assume that the sum of parameters of Beta distribution is the same across all individuals: $\alpha_i + \beta_i = \gamma$, $i \in N$.[12]

Conflicting preferences. Under conflicting preferences, at least some individuals diverge in their preference biases, i.e., $b_i \neq b_j$ for some i and j. However, all agents share the common prior belief about the underlying economic environment θ that has Beta distribution with parameters (α, β), i.e., $(\alpha_i, \beta_i) = (\alpha, \beta)$ for every $i \in N$.

[12]Condition $\alpha_i + \beta_i = \gamma$, $i \in N$, simplifies derivations and presentation of the results. Importantly, it ensures that the monotone likelihood ratio for prior distributions holds, which, in particular, means that the individuals interpret the signals and update their beliefs consistently with each other. Note, however, that the main results hold in a general case as well, even though the intuition behind them in cases in which the monotone likelihood property fails is somewhat more subtle.

3.2 Solution concept

To solve the model, we use the concept of pure strategies Perfect Bayesian Equilibrium (PBE). The restriction to pure strategies simplifies the analysis and implies that cheap talk communication can be either *truthful* (the message reflects the signal perfectly), or *uninformative* (for any signal ω_i individual i sends the same message, either 0 or 1). In the case of uninformative communication, we assume that an off-equilibrium-path message is ignored by the receiving party. This simplification implies that the equilibrium beliefs are defined as follows: any message received in truthful communication induces perfect knowledge about the underlying signal, while any message received in uninformative communication leaves the prior belief about the underlying signal unchanged.

For any given communication network G, it is natural to determine the communication and action strategy profile $(\mu^{\mathsf{G}}, a^{\mathsf{G}}) = (\{\mu_i^{\mathsf{G}}\}_{i\in N}, \{a_i^{\mathsf{G}}\}_{i\in N})$ by using the standard PBE solution concept. Note, however, conditional on a particular choice of G and equilibrium $(\mu^{\mathsf{G}}, a^{\mathsf{G}})$, some links might be ex-ante undesired by at least one involved party. In particular, all links with uninformative communication through them are ex-ante unprofitable to both parties. To account for this, we define an equilibrium as a triple $\left\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\right\}$ such that (1) the pair $(\mu^{\mathsf{G}}, a^{\mathsf{G}})$ forms a PBE given the communication network G, and (2) no individual would prefer to break some incoming or outgoing link at the ex-ante stage.

To formally state the equilibrium definition, let $\mathbb{E}u_l\left(\mathsf{G}, \mu^{\mathsf{G}}, a^{\mathsf{G}}\right)$ be the ex-ante expected utility of agent l that takes into account all link costs that accrue to agent l. Let $\mathsf{G}(g_{ij} = 0)$ be the communication network with the same set of links as in G, except that there is no link from i to j; let $\mu^{\mathsf{G}(g_{ij}=0)}$ be the profile of communication strategies that coincides with μ^{G} everywhere, except that now there is no communication from i to j; and let $a^{\mathsf{G}(g_{ij}=0)}$ be the same action profile as a^{G} for all individuals but j, while individual j's action is now optimally defined conditional on the lower number of truthful messages. Below is the formal definition, in which, under conflicting preferences, all expectations are evaluated using the common prior, while under conflicting beliefs, each individual uses his own prior when choosing communication and action strategies.

Definition 1. *Equilibrium* $\left\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\right\}$ *consists of a communication network* G *and a strategy profile* $(\mu^{\mathsf{G}}, a^{\mathsf{G}}) = (\{\mu_i^{\mathsf{G}}\}_{i\in N}, \{a_i^{\mathsf{G}}\}_{i\in N})$*, such that the following properties hold:*

- *The pair* $(\mu^{\mathsf{G}}, a^{\mathsf{G}})$ *forms a PBE given the communication network* G*.*

– *Given the action strategies profile a^{G}, every agent $i \in N$ for any $\omega_i \in \{0,1\}$ and every individual j to whom i has a link, $j \in \mathrm{N}_i(\mathsf{G})$, chooses a message $\mu_{ij}^{\mathsf{G}}(\omega_i) \in \{0,1\}$ in order to maximize his interim expected utility.*

– *Every individual $i \in N$, for any private signal, $\omega_i \in \{0,1\}$, and any set of received messages, $\widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i} \in \{0,1\}^{|N_i^{-1}(\mathsf{G})|}$, chooses an action $a_i^{\mathsf{G}}\left(\omega_i, \widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i}\right)$ to maximize his ex-post expected utility.*

– *The beliefs are consistent with the communication strategies.*

- *For any i and j such that $g_{ij} = 1$:*

$$\mathbb{E}u_l\left(\mathsf{G}, \mu^{\mathsf{G}}, a^{\mathsf{G}}\right) \geq \mathbb{E}u_l\left(\mathsf{G}(g_{ij}=0), \mu^{\mathsf{G}(g_{ij}=0)}, a^{\mathsf{G}(g_{ij}=0)}\right), \text{ for } l = i, j.$$

Remark 2. *In any equilibrium $\left\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\right\}$, communication network G is truthful, i.e., all links of G represent truthful revelation of private signals. Different equilibria correspond to different communication networks.*

4 Solving the model

4.1 Choice of action

To derive the optimal choice of action, fix the communication network G and consider agent i who learned his private signal ω_i and received the messages $\widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i}$ from his neighbors $N_i^{-1}(\mathsf{G})$. Individual i then chooses an action $a_i^{\mathsf{G}}(\omega_i, \widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i})$ to maximize his ex-post expected payoff,

$$\mathbb{E}_i\left(-\sum_{j=1}^{n}(a_j^{\mathsf{G}} - \Omega - b_i)^2 \,\middle|\, \omega_i, \widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i}\right),$$

where the subscript i in the expectation operator $\mathbb{E}_i$ signifies that individual i uses his prior of Beta distribution with parameters (α_i, β_i). This means that the agent optimally picks

$$
\begin{aligned}
a_i^{\mathsf{G}}(\omega_i, \widehat{m}_{N_i^{-1}(\mathsf{G}),i}) &= \arg\max_{a_i^{\mathsf{G}}} \left\{ \mathbb{E}_i \left(-(a_i^{\mathsf{G}} - \Omega - b_i)^2 \,\middle|\, \omega_i, \widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i} \right) \right\} \\
&= b_i + \mathbb{E}_i \left(\Omega \,|\, \omega_i, \widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i} \right).
\end{aligned}
$$

Because the communication is assumed to be either truthful or uninformative, the information set $\left(\omega_i, \widehat{m}^{\mathsf{G}}_{N_i^{-1}(\mathsf{G}),i}\right)$ can be equivalently represented as the set of revealed signals. Specifically, assume that individual i gets to know k signals summarized in a set ω_R. The unknown $D-k$ signals are denoted as a set ω_{-R}. Using this notation, i's optimal action can be written as

$$
\begin{aligned}
a_i^{\mathsf{G}}(\omega_R) &= b_i + \sum_{\omega_d \in \omega_R} \omega_d + \mathbb{E}_i \left(\sum_{\omega_d \in \omega_{-R}} \omega_d \,\middle|\, \omega_R \right) \\
&= b_i + \sum_{\omega_d \in \omega_R} \omega_d + (D-k)\mathbb{E}_i \left(\omega \,|\, \omega_R \right),
\end{aligned} \tag{3}
$$

where the first equality follows from extracting known part of the sum out of conditional expectation, and the second equality holds because the aspects are identically distributed. Thus, the optimal action is the sum of the preference bias, the known aspects, and the prediction of the unknown part of the state.

The binary nature of the signal ensures that $\mathbb{E}_i(\omega \,|\, \omega_R) = \mathbb{E}_i(\theta \,|\, \omega_R)$. Assume that l signals in ω_R are 1. The conditional probability that l signals out of k are 1 given θ is

$$
f(l|\theta,k) = \frac{k!}{l!(k-l)!}\theta^l(1-\theta)^{k-l}.
$$

The posterior of θ, $f_i(\theta|l,k)$, is proportional to the product of the prior and the conditional probability, $f_i(\theta)f(l|\theta,k)$. A straightforward algebra implies that individual i's posterior is Beta distribution with parameters $(\alpha_i + l, \beta_i + k - l)$:

$$
f_i(\theta|l,k) = \frac{1}{\mathbf{B}(\alpha_i + l, \beta_i + k - l)}\theta^{\alpha_i+l-1}(1-\theta)^{\beta_i+k-l-1}.
$$

The expected value of θ then is $\mathbb{E}_i(\theta|l,k) = \frac{\alpha_i+l}{\alpha_i+\beta_i+k} = \frac{\alpha_i+l}{\gamma+k}$ and the optimal action becomes

$$a_i^{\mathsf{G}}(\omega_R) = b_i + \sum_{\omega_d \in \omega_R} \omega_d + (D-k)\frac{\alpha_i + l}{\gamma + k}. \tag{4}$$

Assume now that individual i gets to know an additional signal ω. The net effect on the optimal action $a_i^{\mathsf{G}}(\omega_R)$ consists of two parts: the first is transferring the aspect from the predicted to the known part of the state; the second is improving the prediction of the unknown part with the additional information. The following proposition states that for any $j \in N$ the magnitude of the effect from individual j's ex-ante perspective is lower the more aspects that are revealed to individual i, but remains bounded away from 0.

Proposition 1. *Consider individual i who obtains k aspects ω_R. Then the magnitude of the effect from an additional signal ω on i's action from individual j ex-ante perspective $\mathbb{E}_j(a_i^{\mathsf{G}}(\omega_R) - a_i^{\mathsf{G}}(\omega_R, \omega))^2$ decreases with k, but remains greater than $\frac{\alpha_j \beta_j}{(\gamma+1)^2}$ for all k and D.*

Proof. See Appendix A. □

Intuitively, the common underlying economic environment insures that the expected magnitude of the impact from an extra signal decreases, while the additive nature of the state of the world guarantees that it remains bounded from 0. Indeed, learning an additional signal improves the information about the underlying economic environment θ by less and less the more signals are already known. At the same time, the magnitude of the effect from an additional signal remains strictly bounded away from zero because the state of the world is additive in aspects that have a non-zero variance for any $\theta \in (0, 1)$. Importantly, it is this sufficiently slow rate at which the impact of an additional signal decreases—and not the particular assumptions behind the nature of the state of the world—that is one of the crucial elements for the main results.

4.2 Equilibrium networks

Assume that the strategy profile $(\mu^{\mathsf{G}}, a^{\mathsf{G}})$ is such that the communication network G is truthful: communication through each link of G leads to a perfect signal revelation. Define $k_j(\mathsf{G})$ to be the number of other individuals who report truthfully to j via either channel, and refer to it as the *in-degree* of individual j.

In order to characterize the structure of equilibrium networks, we study how the ex-ante expected benefits from signal revelation and the interim credibility of individual i in communicating to individual j depend on the number of signals revealed to j by other individuals. Assume that agent i has a link to agent j and, apart from individual i's message, individual j gets to know k_j truthful aspects: 1 aspect individual j obtains himself and $k_j - 1$ aspects he infers from other individuals' messages, excluding i. Denote the set of known k_j aspects as $\omega_R \in \{0,1\}^{k_j}$ and the set of other aspects, excluding ω_i, as $\omega_{-R} \in \{0,1\}^{D-k_j-1}$.

We show that under either type of disagreement between the individuals, the two parties, i and j, would ex-ante prefer truthful revelation of ω_i to individual j. However, the credibility of communication and the way it is influenced by k_j crucially depend on the nature of disagreement between the individuals. In what follows, we consider each case of disagreement in turn.

4.3 Conflicting beliefs

Benefits of signal revelation. Start from an uninformative communication from individual i to individual j, in which case individual j optimally chooses

$$a_j^G(\omega_R) = \sum_{\omega_d \in \omega_R} \omega_d + (D - k_j)\frac{\alpha_i + l}{\gamma + k_j}.$$

The ex-ante expected input from individual j into i's utility (from individual i's perspective) is

$$\begin{aligned}
&-\mathbb{E}_i\left[\mathbb{E}_j\left(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\right) - \mathbb{E}_i\left(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\right)\right]^2 \\
&-\mathbb{E}_i\left[\mathrm{Var}_i\left(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\right)\right] \\
&= -A_{ij}(k_j) - B_i(k_j).
\end{aligned}$$

Here the first term depends on the ex-post belief divergence between individuals i and j given the information ω_R, and the second term is the expected residual variance. The subscripts ij and i for A and B, respectively, signify the priors used in evaluating (parts of) these expressions.

When individual j learns an additional signal ω_i, both terms are reduced. Indeed, the greater information and the smaller unknown part of the state Ω imply that the expected residual variance $B_i(k_j)$ decreases. Regarding the first term $A_{ij}(k_j)$, the monotone likelihood ratio guarantees that individuals are "consistent" with each other in updating their priors. Hence, the term $A_{ij}(k_j)$ decreases, because individual i expects the additional signal to "persuade" individual j, such that they have a smaller expected divergence in their ex-post beliefs. On the whole, individual i expects to derive a benefit of $A_{ij}(k_j) + B_i(k_j) - A_{ij}(k_j+1) - B_i(k_j+1) > 0$, which is a decreasing function of k_j.

Analogously, some individual $l \in N$ enjoys an expected benefit of $A_{lj}(k_j) + B_l(k_j) - A_{lj}(k_j+1) - B_l(k_j+1)$. In particular, individual j himself expects to get the benefit only from reducing the residual variance (evaluated using individual j's prior), $B_j(k_j) - B_j(k_j+1)$. The exact expressions for the terms are given in the following proposition.

Proposition 2. *Fix a truthful network* G *and consider individual* j *with the in-degree* $k_j - 1 < n-1$*. If* j *learns one extra signal* ω_i*, then individual* $l \in N$ *from the ex-ante perspective expects to receive a benefit of* $A_{lj}(k_j) + B_l(k_j) - A_{lj}(k_j+1) - B_l(k_j+1) > 0$*, where*

$$A_{lj}(k_j) = \frac{(\alpha_l - \alpha_j)^2 (D - k_j)^2}{(\gamma + k_j)^2} \text{ and } B_l(k_j) = \frac{\alpha_l \beta_l (D+\gamma)(D-k_j)}{\gamma(\gamma+1)(\gamma+k_j)}.$$

Proof. See Appendix B. □

Positive externality effect. To study the credibility of communication, assume that individual j believes i's message. If individual i reports truthfully, individual j chooses

$$a_j^{\mathsf{G}}(\omega_R, \omega_i) = \sum_{\omega_d \in \omega_R} \omega_d + \omega_i + (D - k_j - 1)\frac{\alpha_i + l + \omega_i}{\gamma + k_j + 1}$$

if individual i lies, then individual j picks

$$a_j^{\mathsf{G}}(\omega_R, 1-\omega_i) = \sum_{\omega_d \in \omega_R} \omega_d + 1 - \omega_i + (D - k_j - 1)\frac{\alpha_i + l + 1 - \omega_i}{\gamma + k_j + 1}.$$

Individual i will choose truthful reporting if and only if this induces a greater interim expected payoff:

$$\sum_{\substack{\omega_R \in \{0,1\}^{k_j}, \\ \omega_{-R} \in \{0,1\}^{D-k_j-1}}} - \left[(a_j^{\mathsf{G}}(\omega_R, \omega_i) - \Omega)^2 - (a_j^{\mathsf{G}}(\omega_R, 1-\omega_i) - \Omega)^2 \right] P_i(\omega_R, \omega_{-R} | \omega_i) \geq 0.$$

Note that individual j uses his prior in determining his actions $a_j^{\mathsf{G}}(\omega_R, \omega_i)$ and $a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i)$, while individual i uses his own prior in probability assessments $P_i(\omega_R, \omega_{-R}|\omega_i)$ when evaluating the expected payoffs.

Clearly, if $k_j = n - 1 = D - 1$, then reporting the last remaining signal ω_i truthfully is incentive compatible because it eliminates any uncertainty about the state of the world. As a result, individual j is able to match his action to the state Ω—an ideal outcome for each individual $i \in N$. Consider now $k_j \leq n - 1 < D - 1$ and the baseline case where $\alpha_i + \beta_i = \gamma$, $i \in N$. The incentive compatibility constraint of truth-telling takes a simple form of:[13]

$$|\alpha_j - \alpha_i| \leq \frac{\gamma + D}{2(D - k_j - 1)}, \qquad k_j < D - 1. \tag{5}$$

Under conflicting beliefs, greater k_j relaxes the incentive compatibility constraint, so that it becomes easier for individual i to report to individual j truthfully the more signals individual j gets to know from other individuals. The intuition behind this result is the following. As the number of signals that j gets to know increases, two things happen. First, individual i expects the ex-post belief of individual j to become more congruent (hence, the optimal action of individual j to become closer to i's preferred one). Indeed, because individual i considers other signals revealed to individual j to be distributed according to i's interim belief, he expects individual j to be "persuaded" and to adjust his ex-post belief in the "right" direction from i's point of view. Second, as Proposition 1 demonstrates, the effect of an additional signal on j's action decreases with k_j. However, the rate of decrease is sufficiently low, so that the effect of i's message on j's action remains significant enough to prevent individual i from misreporting to individual j, given that i expects j to become more congruent.

Thus, individual i might be non-credible when individual j doesn't have much infor-

[13]The derivation of this condition is contained in the proof of Proposition 3 presented in the Appendix.

mation because individual i is concerned that if individual j has an extreme belief, he will misuse the reported signal. On the contrary, individual i can become credible when individual j's information improves, because individual i expects individual j to be persuaded and to become able to properly take into account i's signal when choosing his action. We refer to this effect as the *positive externality effect* of information transmission, because greater information has a positive effect on further information accumulation by encouraging other individuals to report truthfully as well.

Remark 3. *In case of positive externality, it is natural to view the private aspects as* **complements***. Indeed, if individual i reports to j truthfully, then other individual l might be credible in truthful reporting to j as well. On the other hand, if individual i does not report to individual j, then it might not be incentive compatible for l to transmit his private information truthfully. Thus, private signals of i and l act as complements.*

In any equilibrium network, for individual j to have the in-degree of $k_j(\mathsf{G}) > 0$, it must be the case that the incentive compatibility constraint (5) is satisfied for every individual i reporting truthfully to j. The formal equilibrium characterization is provided on the proposition below.

Proposition 3. *Consider a triple $\left\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\right\}$ and assume that the individual action strategies satisfy the optimality condition (4). Then $\left\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\right\}$ forms an equilibrium if and only if the communication network G is truthful, and for any individual j with the in-degree $k_j = k_j(\mathsf{G}) < D - 1$*

$$|\alpha_j - \alpha_i| \leq \frac{\gamma + D}{2(D - k_j - 1)} \text{ for all } i \in N_j^{-1}(\mathsf{G}) = \{i \in N : g_{ij} = 1\}. \tag{6}$$

Proof. See Appendix C. □

Remark 4. *It is worthwhile to mention that the positive externality effect is not an artifact of a particular assumption that $\alpha_i + \beta_i = \gamma$ for all $i \in N$. Rather, the positive externality effect is present in a general case as well. The corresponding incentive compatibility constraint has a more complicated form, which is presented and derived in the proof of Proposition 3 (see Appendix C).*

Remark 5. *In a particular case when the individuals collectively hold all the information about the state of the world, i.e., $n = D$, reporting the last missing signal truthfully is incentive compatible because it eliminates any uncertainty and disagreement. Thus, there always exists an equilibrium in which each individual gets to know all aspects of the state, i.e., the communication network is complete.*

4.4 Conflicting preferences

Benefits of signal revelation. First, assume uninformative communication from individual i to individual j, which implies that individual j conditions his action only on ω_R and optimally chooses

$$a_j^{\mathsf{G}}(\omega_R) = b_j + \sum_{\omega_d \in \omega_R} \omega_d + (D - k_j)\frac{\alpha + l}{\gamma + k_j}.$$

Consider the ex-ante expected input from individual j into i's utility. Given the quadratic loss utility function, the input is comprised of an expected residual variance of the unknown part of the state Ω and a square of the preference divergence:

$$-\mathbb{E}\left[\mathrm{Var}\left(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\right)\right] - (b_j - b_i)^2 = -h(k_j) - (b_j - b_i)^2.$$

Next, if individual i truthfully communicates his signal to individual j, then j conditions his action on $k_j + 1$ signals (ω_R, ω_i), and the ex-ante expected input becomes $-h(k_j + 1) - (b_j - b_i)^2$. Thus, the expected benefit of learning an additional signal ω_i is in reducing the expected residual variance, $h(k_j) - h(k_j + 1) > 0$ (because of the smaller unknown part of the state Ω and greater information). Clearly, individual j himself—as well as any other individual from N—enjoys the same ex-ante benefit of $h(k_j) - h(k_j + 1)$ independently of the bias difference.[14] The following proposition presents the exact expression for this benefit and establishes that it is a decreasing function of k_j.

Proposition 4. *Fix a truthful network* G *and consider individual* j *with the in-degree* $k_j - 1 < n - 1$. *If* j *learns one extra signal* ω_i, *then each individual* $l \in N$ *derives the ex-ante expected benefit of* $h(k_j) - h(k_j + 1) > 0$ *that decreases with* k_j, *where*

$$h(k_j) = \frac{\alpha\beta(\gamma + D)(D - k_j)}{\gamma(\gamma + 1)(\gamma + k_j)}.$$

Proof. See Appendix D. □

[14] That all individuals derive the same benefit from individual j learning an additional signal is due to the assumption that the inputs from all individuals are equally weighted in the payoff function.

Negative externality effect. While Proposition 4 states that the truthful reporting of ω_i is desired by both parties i and j from an ex-ante perspective, it might not be interim credible. To study the incentive to report truthfully, assume that individual j believes i's message, i.e., j assigns the probability 1 that $\omega_i = m_{ij}$. If i reveals his private signal, $m_{ij} = \omega_i$, then individual j optimally picks an action

$$a_j^{\mathsf{G}}(\omega_R, \omega_i) = b_j + \sum_{\omega_d \in \omega_R} \omega_d + \omega_i + (D - k_j - 1)\frac{\alpha + l + \omega_i}{\gamma + k_j + 1};$$

if i misreports and sends $m_{ij} = 1 - \omega_i$, then j chooses

$$a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i) = b_j + \sum_{\omega_d \in \omega_R} \omega_d + 1 - \omega_i + (D - k_j - 1)\frac{\alpha + l + 1 - \omega_i}{\gamma + k_j + 1}.$$

Individual i reveals his signal whenever it results in a greater interim expected payoff:

$$\sum_{\substack{\omega_R \in \{0,1\}^{k_j}, \\ \omega_{-R} \in \{0,1\}^{D-k_j-1}}} - \left[(a_j^{\mathsf{G}}(\omega_R, \omega_i) - \Omega - b_i)^2 - (a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i) - \Omega - b_i)^2 \right] P(\omega_R, \omega_{-R} | \omega_i) \geq 0.$$

As we show in Proposition 5, this incentive compatibility constraint can be rewritten as

$$|b_j - b_i| \leq \frac{\gamma + D}{2(\gamma + k_j + 1)}. \tag{7}$$

While this constraint (7) is always satisfied when $|b_j - b_i| \leq 1/2$, it might fail to hold when the preference divergence is significant, $|b_j - b_i| > 1/2$. In the latter case, contrary to the conflicting beliefs case, individual i can be credible so long as individual j doesn't get to know too many signals relative to the divergence in their preferences. To see the intuition behind this, recall that, by Proposition 1, the magnitude of the effect from an additional signal on j's action decreases with k_j. Thus, for sufficiently high k_j, the expected effect might become so small that i would prefer to lie in order to shift j's action closer towards i's preferred one. On the other hand, when k_j is quite low, the expected effect of an additional signal is quite big, in which case misreporting can change j's action by too much, making it undesirable.

In this paper, we refer to this effect as the *negative externality effect* of information transmission—greater information has a negative effect on further information accumulation by discouraging other individuals to report truthfully.

It is worth noting that the incentive compatibility constraint (7) is related to the congestion effect of Galeotti et al. [2013]. However, in their work the state of the world, that is appeared in the agents' payoffs, is singleton (denoted by θ), but here it is additive (denoted by Ω) which is aggregate of D different aspects (see Eq. (2)).[15]

Remark 6. *Another natural way to interpret the negative externality effect is to view the privately observed aspects as* **substitutes**. *Indeed, if individual i reports to j truthfully, then some other individual l might not be credible in communicating to individual j. If, on the other hand, individual i does not report to j, then individual l might be able to transmit his private information truthfully. This means that in such a communication process, private signals of i and l act as substitutes.*

Clearly, in order for individual j to receive truthful messages from k_j individuals in equilibrium, the incentive compatibility constraint (7) must be satisfied for each of those individuals. The following proposition provides the formal equilibrium characterization.

Proposition 5. *Consider a triple $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ and assume that each element of a^{G} satisfies the optimality condition (4). Then $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ forms an equilibrium if and only if the communication network G is truthful, and for any individual j with the in-degree $k_j = k_j(\mathsf{G})$*

$$|b_j - b_i| \leq \frac{\gamma + D}{2(\gamma + k_j + 1)} \quad \text{for all } i \in N_j^{-1}(\mathsf{G}) = \{i \in N : g_{ij} = 1\}. \tag{8}$$

Proof. See Appendix E. □

5 Applications

In this section we present some network applications of the the main findings. In particular, we analyze the direction of information flow among communities (Section 5.1),

[15] This difference proves to be crucial to obtain our novel positive externality effect (see Section 4.3). Precisely, to obtain the positive externality effect under conflicting beliefs it is necessary that the marginal effect of an additional signal does not decrease too fast. Importantly, this means that when the state of the world is singleton (as in their work) the positive externality effect is *absent*. But positive externality effect is present in other settings where the marginal effect of an additional signal is sufficiently low, including our additive/aggregate state of the world.

study communication network formations (Section 5.2), and show how strategic motives, depending on the types of conflict, crucially affect segregations and aggregations in communications due to adding new informed agents (Section 5.3). We consider the information aggregation in polls in the extension section (Section 6).

5.1 Information flow in networks

In this section we study the direction of information flow (transmission) in networks between two homogenous communities (groups), related to Galeotti et al. [2013]. In particular we ask: How does type of conflict between individuals affect information transmission across groups? The answer crucially depends on the type of conflict.

Two communities. A set of individuals consists of two homogenous communities (or groups), N_1 and N_2, with sizes n_1 and n_2, respectively, where $1 \leq n_1 < n_2$ and the total number of individuals is $n = n_1 + n_2$. Under conflicting preferences, group N_1 members have preference biases normalized to 0, while group N_2 members have biases of b. Under conflicting beliefs, all individuals in N_1 agree on the same prior with parameters (α_1, β_1); all individuals in N_2 have the prior with (α_2, β_2), and we still maintain the assumption that $\alpha_i + \beta_i = \gamma,\ i = 1, 2$.

Conflicting preferences. Clearly, in any pairwise stable equilibrium, there is complete communication inside each group. In addition, members of the same group have equal in-degrees, because exactly the same individuals can report truthfully to them. Introduce notation similar to Galeotti et al. [2013]: denote by k_i the in-degree of an arbitrary individual in group N_i. Further, $k_i = k_{ii} + k_{ij}$, where $k_{ii} = n_i - 1$ reflects the level of *intra-group communication*—the number of signals revealed by N_i members, and k_{ij} stands for the level of *cross-group communication*—the number of truthful messages received from members of the opposite community N_j.

By the negative externality effect, if members of a smaller group N_1 report truthfully to some members of N_2, then the pairwise stability implies that members of a larger group N_2 reveal their signals to some members of N_1. Thus, depending on the parameters, cross-group communication can take one of the following 3 forms (see cases 1-3 in Figure 1):

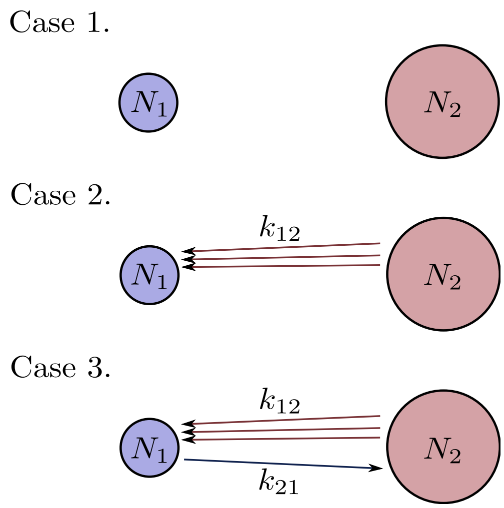

Figure 1: Communication networks of pairwise stable equilibria under conflicting preferences.

1. No cross-group communication, i.e., $k_{21} = k_{12} = 0$, whenever $\frac{\gamma+D}{2(\gamma+n_1+1)} < b$.
2. Communication from group N_2 to group N_1, i.e., $k_{12} > 0$ and $k_{21} = 0$, whenever $\frac{\gamma+D}{2(\gamma+n_2+1)} < b \leq \frac{\gamma+D}{2(\gamma+n_1+1)}$.
3. Cross-group communication, i.e., $k_{12} > 0$ and $k_{21} > 0$, whenever $b \leq \frac{\gamma+D}{2(\gamma+n_2+1)}$. In particular, when $b \leq \frac{\gamma+D}{2(\gamma+n)}$ the network is complete, i.e., $k_{12} = n_2$ and $k_{21} = n_1$.

Conflicting beliefs. In any pairwise stable equilibrium, communication inside each group is necessarily complete, because members of the same community agree in their beliefs. Regarding cross-group communication, note that, by the positive externality effect, if one individual from N_j reports truthfully to a particular individual in the opposite community N_i, then so do all of the other individuals in N_j. These observations immediately imply that the in-degree of a individual in N_i can be either $n_i - 1$ or $n - 1$.

To simplify the characterization, we focus only on symmetric pairwise stable equilibria, i.e., equilibria in which individuals with the same priors have the same in-degrees. This implies that all individuals in group N_i have the same in-degree $k_i = k_{ii} + k_{ij}$, where k_{ij} is the level of cross-group communication and $k_{ii} = n_i - 1$ is the level of intra-group communication. Assuming that $D > n$, cross-group communication can take the following 4 forms (see cases 1-4 in Figure 2):[16]

[16] The case of $D = n$ differs in that the pairwise stable equilibrium with the complete communication

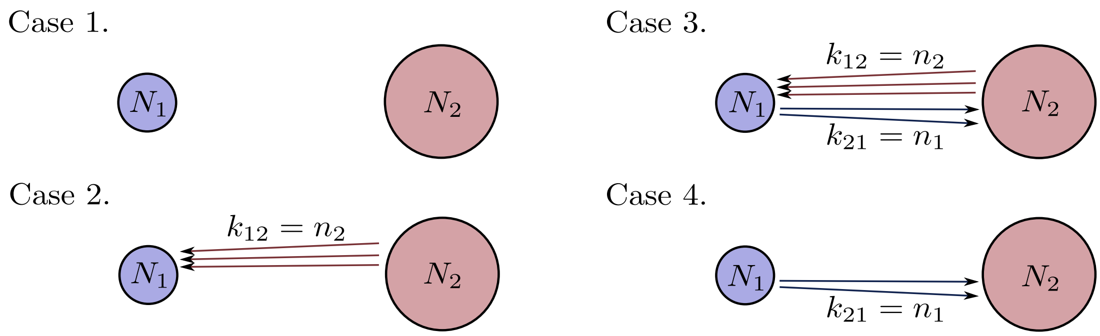

Figure 2: Communication networks of symmetric pairwise stable equilibria under conflicting beliefs.

1. No cross-group communication, i.e., $k_{21} = k_{12} = 0$, if $\frac{\gamma+D}{2(D-n_2-1)} < |\alpha_1 - \alpha_2|$.

2. Communication from group N_2 to group N_1, i.e., $k_{12} = n_2$ and $k_{21} = 0$, if $\frac{\gamma+D}{2(D-n_2-1)} < |\alpha_1 - \alpha_2| \leq \frac{\gamma+D}{2(D-n)}$.

3. Cross-group communication (complete truthful network), i.e., $k_{12} = n_2$ and $k_{21} = n_1$, if $|\alpha_1 - \alpha_2| \leq \frac{\gamma+D}{2(D-n)}$.

4. Communication from group N_1 to group N_2, i.e., $k_{12} = 0$ and $k_{21} = n_1$, if $\frac{\gamma+D}{2(D-n_1-1)} < |\alpha_1 - \alpha_2| \leq \frac{\gamma+D}{2(D-n)}$.

Discussion. Comparing the communication patterns under conflicting preferences and under conflicting beliefs, two things should be noted. First, under conflicting preferences, the preference divergence uniquely pins down the vector of in-degrees in a pairwise stable equilibrium. Under conflicting beliefs, for some parameters, multiple in-degrees can be realized in different pairwise stable equilibria. Specifically, a member of N_i can receive truthful messages from either only N_i members (low in-degree of $n_i - 1$), or from all individuals (high in-degree of $n - 1$). Second, under conflicting preferences, the information is more likely to flow from a larger group to a smaller one: either only members of a large group report to members of a small group, or members from opposite groups report to each other (with more information flow from the large group to the small one). Under conflicting beliefs, the opposite communication pattern appears: for some parameters, there exists a pairwise stable equilibrium in which only members of a small group report to members of a large group, and not the other way around.

network exists for all parameters.

5.2 Communication network formations

In this section we establish that communication network structures arising in equilibria crucially depends on the types of conflict.

Pairwise stable equilibria. As shown in Section 4, under both types of conflict, each individual i ex-ante prefers that he and any other individual $j \in N$ learn more aspects of the state, but the respective communication may fail to be credible from the interim perspective. There may be multiple equilibria that induce different truthful networks, and so, as a natural refinement, we adapt the notion of pairwise stability commonly used in the networks literature (e.g., Jackson and Wolinsky [1996], Bala and Goyal [2000], Goyal [2007], Jackson [2008]). Specifically, call an equilibrium $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ *pairwise stable*, if no two individuals can improve the communication pattern between them, while satisfying the interim truth-telling incentives and holding other strategies fixed.

Definition 2. *An equilibrium $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ is pairwise stable if for any $i, j \in N$, $g_{ij} = 0$ only if, holding other strategies fixed, i cannot be credible in reporting to j, assuming that j believes i's message.*

To prove the existence and determine the characteristics of pairwise stable equilibria, define the *maximal equilibrium* as an equilibrium in which the truthful network has the maximal in-degrees across all equilibrium networks.

Definition 3. *Equilibrium $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ with the in-degrees $k_1 = k_1(\mathsf{G}), \cdots, k_n = k_n(\mathsf{G})$ is maximal if for any other equilibrium network with in-degrees $k'_1, ..., k'_n$:*

$$k_i \geq k'_i, \ i = 1, ..., n.$$

In turn, the in-degrees $k_1, ..., k_n$ are called maximal in-degrees.

The following proposition states that the set of pairwise stable equilibria is non-empty and, under conflicting preferences, is a subset of maximal equilibria. Under conflicting beliefs, the maximal equilibrium is unique and is necessarily pairwise stable.

Proposition 6. *There exist a maximal and a pairwise stable equilibrium. Under conflicting preferences, any pairwise stable equilibrium is maximal. Under conflicting beliefs, the unique maximal equilibrium is pairwise stable.*

Proof. See Appendix F. □

Since each individual benefits when any other individual receives more signals, the ex-ante expected payoff of every individual is the largest in the maximal equilibria. Clearly, the individuals are indifferent between the maximal equilibria, because the ex-ante expected payoffs depend only on the vector of in-degrees and not on the particular network structure (see Propositions 4 and 2). Assume that the maximal in-degrees are $k_1, ..., k_n$. Then, the maximal equilibrium that is pairwise stable can, for example, be constructed in the following way. Let every individual i receive truthful messages from k_i individuals who are the closest to i in their preference divergence $|b_j - b_i|$ (or beliefs divergence $|\alpha_j - \alpha_i|$). Because closer preferences (or beliefs) relax the truth-telling incentives, these individuals are credible. At the same time, due to the maximality of k_i, other individuals cannot report truthfully to individual i.

Proposition 6 implies that, under conflicting preferences, any pairwise stable equilibrium generates the same set of in-degrees. These in-degrees are necessarily maximal and, by the negative externality effect, are neither too low nor too large. On the contrary, under conflicting beliefs, individuals can have different in-degrees in different pairwise stable equilibria. Moreover, by the positive externality effect, these in-degrees are either sufficiently low or quite large (it is easy to be credible to a individual with a high in-degree, and it is difficult to be truthful to a low in-degree individual).[17]

Interested readers are referred to Appendix J for an additional discussion of the pairwise stable equilibria.

5.3 Replicas: Segregation vs. Aggregation

This section studies how types of conflict lead to a segregation or aggregation in communications due to adding new informed individuals. We show increasing the number of individuals of the same type localizes communication under conflicting preferences; and intensifies cross-type communication under conflicting beliefs. In fact, under conflicting preferences, replicating the set of individuals curbs cross-type communication and can

[17]In a particular case in which the individuals cumulatively possess all the relevant information, $D = n$, under conflicting beliefs, the unique maximal equilibrium has a complete communication network. However, aside from the maximal equilibrium, there may also exist pairwise stable equilibria that fail to aggregate all of the information (some individuals in the communication networks have quite low in-degrees).

lead to a *segregation* of communication according to the individuals' types. Therefore, surprisingly, it does not necessarily improve the aggregation of the available information. Under conflicting beliefs, on the contrary, adding new informed individuals amplifies cross-type communication, which necessarily leads to greater information *aggregation*.

Consider a set of $m \geq 2$ individuals and let the total number of aspects be D such that $Rm \leq D < (R+1)m$ for some $R \geq 1$. Under conflicting preferences, individuals are characterized by the preference biases $b_1, \cdots, b_m$ with the minimum difference of at least b: $\min_{i,j} |b_i - b_j| = b > 0$. Under conflicting beliefs, individuals' prior beliefs are described by $(\alpha_1, \beta_1), \cdots, (\alpha_m, \beta_m)$ where $\min_{i,j} |\alpha_i - \alpha_j| = \alpha > 0$ and $\alpha_i + \beta_i = \gamma$ for all $i = 1, \cdots, m$. Call by *r-replica* the setting where there are rm individuals, r of each preference type b_i (respectively, belief type (α_i, β_i)), $i = 1, \cdots, m$. Assume that $r \leq R$, the total number of aspects in r-replica remains D and each individual becomes privately informed of the respective aspect, so collectively all individuals in r-replica learn rm aspects. In what follows we focus on pairwise stable equilibria.

Conflicting preferences. Consider some r_1-replica, $1 \leq r_1 < R$. In any pairwise stable equilibrium, individuals with the same preference bias communicate truthfully with each other (meaning that the in-degree of each individual is at least $r_1 - 1$). Now consider r_2-replica, where $r_1 < r_2 \leq R$. Clearly, in r_2-replica, intra-type communication is more intense than in r_1-replica: each individual necessarily gets to know $r_2 - 1 > r_1 - 1$ aspects from individuals of the same type. By the negative externality effect, this threatens the credibility of communication between individuals of different types. The fact that some individual j gains $r_2 - r_1$ new same-type communication links in r_2-replica might crowd out up to $r_2 - r_1$ cross-type communication links directed to j. As a result, in r_2-replica individuals have (weakly) greater in-degrees caused by more intra-type communication and (weakly) less cross-type communication.

Interestingly, regardless of the fact that in a higher-order r_2-replica the individuals collectively get to know more aspects than in r_1-replica, the individual awareness might fail to improve—the individuals' in-degrees in r_2-replica might remain the same as in r_1-replica. The following example illustrates this point.

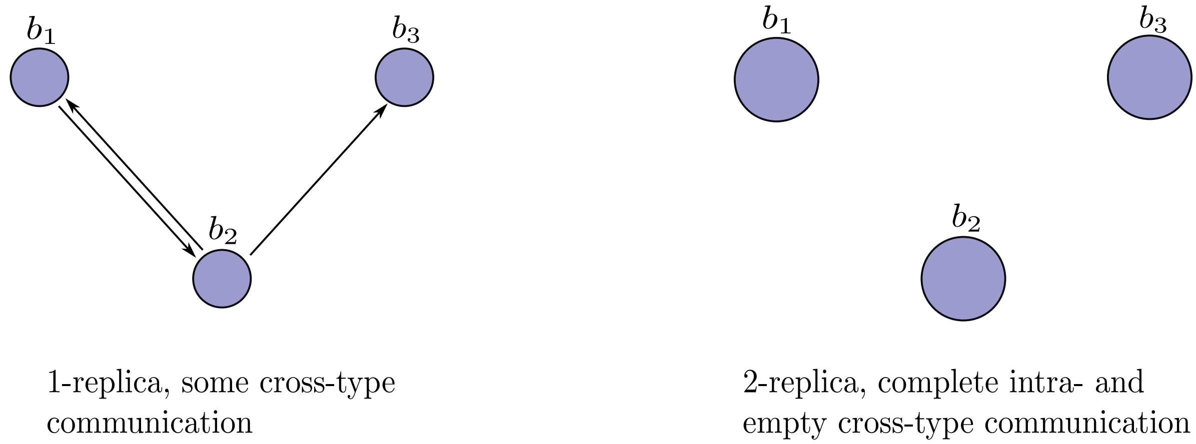

Figure 3: Communication networks in 1- and 2-replicas, conflicting preferences. Each circle corresponds to a group of individuals of particular type.

Example. Let the total number of aspects $D = 6$ and the prior distribution of θ be uniform on the interval $[0, 1]$ that corresponds to Beta distribution with parameters $(1, 1)$ and $\gamma = 2$. Consider $m = 3$ preference types $b_1 = 0$, $b_2 = b$, $b_3 = 2b$, where $\frac{4}{5} < b \leq 1$. Such preference structure implies that truthful reporting to a individual with the bias b_j is incentive compatible for a individual with the bias $b_i \neq b_j$ if and only if $|b_i - b_j| = b$ and nobody else reports truthfully. Thus, in 1-replica, each individual receives exactly one truthful message from a individual with an adjacent bias (see Figure 3 for a possible communication network). In 2-replica, the individuals collectively hold all of the relevant information about the state of the world, but fail to aggregate it: each individual communicates truthfully only with other same-type individual; cross-type communication is necessarily empty (see Figure 3). This example also demonstrates how increasing the number of same-type individuals can wipe out all cross-type communication. More generally, whenever $b > \frac{\gamma+D}{2(\gamma+r+1)} \geq \frac{1}{2}$, $D \geq rm$, then in any pairwise stable equilibrium of r-replica, there is a complete segregation of communication with respect to the types of preferences.

Conflicting beliefs. Consider r_1- and r_2-replicas with $1 \leq r_1 < r_2 \leq R$. Because the intra-type communication is necessarily complete, individuals in r_2-replica receive $r_2 - r_1$ more truthful messages from the same-type individuals. The positive externality effect implies that some different-type individuals that could not be credible in r_1-replica might find truthful communication incentive compatible in r_2-replica. Thus, individuals have strictly greater maximal in-degrees in r_2-replica than in r_1-replica, due to (strictly) more

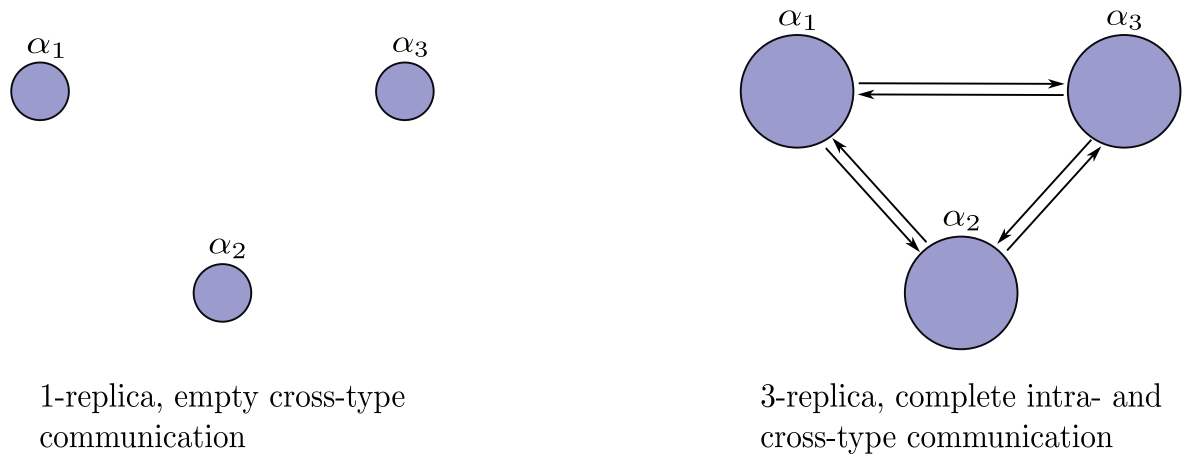

Figure 4: Communication networks in 1- and 2-replicas, conflicting beliefs. Each circle corresponds to a group of individuals of particular type.

intensive intra-type communication and (weakly) more intensive cross-type communication.

As an illustration of how individuals' awareness improves in a higher-order replica, consider the following example.

Example. Assume that the total number of aspects $D = 10$ and let $m = 3$ prior belief types be such that $\gamma = 2$, $\alpha_1 = 1$, $\alpha_2 = 1 + \alpha$ and $\alpha_3 = 1 + 2\alpha$, where $\frac{6}{7} < \alpha \leq 1$. Condition $\frac{6}{7} < \alpha$ insures that, in 1-replica, the only pairwise stable equilibrium has an empty communication network. At the same time, condition $\alpha \leq 1$ implies that, in 3-replica, the unique pairwise stable equilibrium has a complete communication network—within and across different types of individuals (see Figure 4). This example shows how increasing the number of same-type individuals can induce complete available information aggregation. More generally, when $\max_{i,j} |\alpha_i - \alpha_j| \leq \frac{\gamma+D}{2(\gamma+rm)}$, $D \geq rm$, then the maximal pairwise stable equilibrium of r-replica has a complete communication network.

Discussion. Does introduction of new individuals who are privately informed about other aspects help agents to accumulate greater information about the state of the world? As the above analysis illustrates, the answer crucially depends on the nature of disagreement. Under conflicting preferences, replicating the set of individuals curbs cross-type communication and can lead to a segregation of communication according to the individuals' types. As a result, it does not necessarily improve the aggregation of the available information. Under conflicting beliefs, on the contrary, adding new informed individuals boosts cross-type communication, which necessarily leads to greater information aggregation.

6 Extensions: Information aggregation in polls

Policymakers who have to make a policy choice have usually insufficient information of the costs and benefits of the choice they are about to make. Therefore, they usually use polls to determine policy outcomes. Polls are used for various reasons. One important reason is that polls can aggregate information dispersed among constituents. However, oftentimes constituents have disagreements (in opinions or preferences) about policies proposed by policymakers. Therefore, as polls affect polices, polled constituent respond strategically to the polls in order to shape the policies.

In this section, using our above findings about the externality effects, we study the implications of the types of conflict between constituents on information aggregation in polls. Similarly to Morgan and Stocken [2008], we consider how *poll size* affects information aggregation in polls.[18] In particular we ask: How does type of conflict between agents affect information aggregation via truth-telling strategies in polls? Does information aggregation occur in large polls or small polls? We show the answers crucially depend on the nature of disagreements. The main derivers of the result are qualitative features of the IC constraints.

We also extend our benchmark model presented in Section 3. We consider the same formulation as in Section 3 but now individuals disagree, *simultaneously*, not only in their opinions (conflicting beliefs) but also in their biases (conflicting preferences). That is, individuals not only have different biases $b_1, ..., b_n$ but also, simultaneously, diverge in their prior beliefs about the state. Along with this extension in the model formulation, we analyze when information can be aggregated in polling.

To change the model to a polling environment, we simplify the general network G to a many-to-one network as follows.

[18] The argument also holds when communication is many-to-many with arbitrary network structure.

Many-to-one communication network. Suppose individual j is the policymaker (PM) who dose not observe the state and nor receive a signal about it.[19] Therefore, he should rely on a poll to obtain information about the state. Hence, before characterizing the welfare maximizing policy, he conducts a poll consisting of a (commonly known) **poll size** of k_j (where $k_j \leq n$) constituents numbered by $i = 1, 2, \cdots, k_j$. Without loss of generality, normalize $b_j = 0$.

Given the PM's policy a_j, each constituent i's payoff is quadratic given by[20]

$$u_i(a_j, \Omega, \alpha_i, b_i) = -(a_j - \Omega - b_i)^2.$$

After all polled constituents $i = 1, 2, \cdots, k_j$ send their messages to the PM j, the PM finds a policy that maximizes the welfare in the entire polity, i.e. where welfare is

$$W \equiv -\sum_{i=1}^{n} u_i(a_j, \Omega, \alpha_i, b_i) = -\sum_{i=1}^{n} (a_j - \Omega - b_i)^2.$$

We first have the following *necessary* and *sufficient* result for truth communication of constituents i to the PM j in the interim stage.

Proposition 7. *In equilibrium, in the interim stage, constituents i reports truthfully to the PM j if and only if:*

$$\left| [\gamma + k_j]\ b_i + [D - k_j]\ (\alpha_i - \alpha_j) \right| \leq \frac{1}{2} (\gamma + D)\,. \tag{9}$$

Moreover, information aggregation in polls *occurs, i.e. all constituents in the poll report truthfully to to the PM j, if and only if*

$$\max_{i \in N_j(\mathsf{G})} \left| [\gamma + k_j]\ b_i + [D - k_j]\ (\alpha_i - \alpha_j) \right| \leq \frac{1}{2} (\gamma + D)\,. \tag{10}$$

Proof. See Appendix G. □

The left hand side of the above inequality (that is appropriately weighted by the

[19] This holds without loss of generality, i.e. we could also assume that the PM privately observes one aspect of Ω.

[20] We make a simple change in the payoff structure of individuals in the polity so as to make it consistent with the polling frameworks. These changes has no effects on our previous findings.

poll size k_j) measures how far consistent i's preference, captured by b_i, and his belief, captured by α_i, differ from the PM j's bias[21] and his prior belief, respectively. However, the right hand side is *fixed* for all the individuals.

Remark 7. *The both types of conflict are simultaneously present in the above result. It is worth commenting about the generality of* (9)*. In fact, if $b_i = 0 = b_j$, then there is only conflicting beliefs between the individuals, and, thus,* (9) *reduces to* (5)*. Similarly, when $\alpha_i = \alpha_j$, then there is only conflicting preferences and* (9) *simplifies to* (8)*.*[22]

Using the above proposition, the following result identifies how increasing the poll size k_j can play positive, negative or even ambiguous roles on information aggregation[23] in polls, depending on the types of conflict between the constituents.

Proposition 8 (Information aggregation in polls)**.** *Let $b \equiv \max_{i=1,2,\cdots,k_j} 2|b_i|$ and $\alpha \equiv \max_{i=1,2,\cdots,k_j} 2|\alpha_i - \alpha_j|$ then the followings hold.*

i. Under conflicting preferences, *i.e. $\alpha = 0$, information aggregation occurs if and only if poll size (i.e. k_j) is sufficiently* small*; Precisely:*

$$k_j \leq \frac{1}{b}(D + \gamma(1 - b)). \tag{11}$$

ii. Under conflicting beliefs, *i.e. $b = 0$, information aggregation occurs if and only if poll size is sufficiently* large*; Precisely:*

$$k_j \geq \frac{1}{\alpha}(D(\alpha - 1) - \gamma). \tag{12}$$

iii. Under conflicting both beliefs and preferences, *i.e. $\alpha \neq 0$ and $b \neq 0$, the impact of poll size on information revelation is* ambiguous.

Proof. See Appendix H. □

Proposition 8 determines that the credibility of communication and the way it is influenced by the extent of information obtained by the PM j, which is captured by the

[21] Recall that j's bias is normalized to zero.

[22] Recall that j does not obtain a private signal, otherwise we would have $k_j + 1$ instead of k_j in (9).

[23] Information aggregation means all the polled constituents truthfully communicate/report their private information to the PM, who is maximizing welfare in the polity.

poll size k_j, crucially depends on the nature of disagreement between the individuals. In fact, under conflicting preferences, due to the negative externality effect, greater information available to the PM—larger k_j—harms the credibility of any polled constituent $i, i = 1, 2, , \cdots, k_j$. Intuitively, the magnitude of the effect from an additional signal on the PM's action decreases with the informativeness of the j. Thus, for sufficiently large poll size k_j, the expected effect might become so small that i would prefer to lie in order to shift the PM's action closer towards i's preferred bias. As a consequence, according to (11), there exists a finite upper bound on the poll size k_j where truthful information revelation is an equilibrium. That is, similarly to Morgan and Stocken [2008], under conflicting preferences, information aggregation in networks via truth-telling strategies is *impossible when poll size are large*.

However, the above intuition holds in the opposite direction under conflicting beliefs. In fact, when parties disagree in their beliefs, because of the positive externality effect, greater number of aspects obtained by the PM j—higher k_j—improves the credibility of any polled constituent i. Intuitively, as the poll size k_j increases, two things happen. First, the polled constituent i expects the ex-post belief of the PM to become more congruent. Indeed, i deems other signals revealed to j distributed according to i's belief, hence, he expects PM to be persuaded and adjust his ex-post belief in the "right" direction (from i's point of view). Second, the magnitude of the effect of an additional signal on PM' action decreases with the poll size k_j. However, the additive nature of the state Ω in aspects ω_d, $d = 1, \cdots, D$, guarantees that the rate of decrease is sufficiently low, so that the effect of i's message on the PM's action remains significant enough to prevent i from misreporting to j (given that i expects PM to become more congruent). In effect, by the positive externality effect, with increasing the poll size k_j each polled constituent $i, i = 1, 2, \cdots, k_j$, has more incentive to communicate truthfully with the PM j. Moreover, according to (12), there exists a finite lower bound on the poll size k_j where truthful information revelation is an equilibrium. That is, under conflicting beliefs, information aggregation via truth-telling strategies is *impossible when in-degrees are small*.

Finally, in contrast to the above cases, when the two sources of conflict are both simultaneously present, depending on the extent of discrepancies in beliefs and preferences, the impact of poll size, k_j, on information aggregation is ambiguous. To see this, let $\Delta_b \equiv b_i - b_j \geq 0$ and $\Delta_\alpha \equiv \alpha_i - \alpha_j \geq 0$ capture the extent of preference and belief discrepancies, respectively. Then, according (9), three cases are possible. Case 1: If $\Delta_b > \Delta_\alpha$, then negative externality effect dominates positive externality effect and, therefore, increasing the poll size k_j has a negative impact for information aggregation. Case 2: If $\Delta_b < \Delta_\alpha$, then positive externality effect dominates negative externality effect and, thus, increasing k_j helps information aggregation. Case 3: If $\Delta_b = \Delta_\alpha$, then these externality effects fully offset one another, *neutralizing* the role of poll size for information aggregation, i.e. poll size has *no* impact for strategic truthful communication and, thus, information aggregation in polls.

The above results are for the *interim* period, i.e. after the signals' realization but prior to communication. How does poll size affect agent's payoff in the *ex-ante* period, i.e. prior to when the signals are realized? Is the impact of poll size on information aggregation again sensitive to the types of conflict between constituents? We next answer to these questions.

Role of poll size in the *ex-ante* equilibrium payoff. In the following, we first explicitly characterize the impact of PM's action on the ex ante (equilibrium) payoff of any constituent i. Then, we show each constituent i before learning his private signal, would always gain benefit when PM obtains more signals, i.e. increasing the poll size k_j. However, the margin of this gain gets smaller with increasing the poll size k_j. Importantly, this monotonicity and concavity of the ex-ante benefit of increasing the poll size is *independent* of the types of conflict between the individuals.

Proposition 9 (Ex ante (equilibrium) payoff of constituent i)**.** *Given the poll size k_j, the ex ante (equilibrium) payoff of any constituent i is given by*

$$\begin{aligned}\mathbb{E}_i[u_i|k_j] = -b_i^2 - 2b_i\Lambda(k_j)(\alpha_i-\alpha_j) - \big(\Lambda(k_j)(\alpha_i-\alpha_j)\big)^2 \\ - \Lambda(k_j)\frac{\alpha_i(\gamma-\alpha_i)(D+\gamma)}{\gamma(\gamma+1)},\end{aligned} \tag{13}$$

where $\Lambda(k_j) = \frac{D-k_j}{\gamma+k_j}$.

Proof. See Appendix I. □

The above proposition immediately implies the following result.

Proposition 10 (Role of poll size *before* constituents observe their signals)**.** *Let $b_i(\alpha_i - \alpha_j) \geq 0$. Then, ex ante (equilibrium) payoff of any constituent i is* increasing *and* concave *in* *the poll size k_j, that is:*

$$\frac{\partial \mathbb{E}_i[u_i|k_j]}{\partial k_j} > 0, \qquad \textit{(monotonically increasing (ex ante) payoff in poll size)} \tag{14}$$

$$\frac{\partial^2 \mathbb{E}_i[u_i|k_j]}{\partial k_j^2} < 0 \qquad \textit{(diminishing return in poll size)} \tag{15}$$

Proof. See Appendix I. □

Proposition 10 shows the ex-ante benefit of increasing k_j is always certain, i.e. there is no ambiguous effect (as in part iii of Proposition 8). That is, similar to the benchmark case, individual *i before* learning his private signal (i.e. ex-ante stage) would always gain benefit when the PM j obtains more signals, i.e. increasing the poll size k_j is always ex-ante beneficial. However, this monotone benefit is concave, i.e. the margin of this gain gets *smaller* with increasing k_j. This is because, in individual i's view with increasing k_j, the PM j obtains more information about the state of the world which is ex-ante beneficial, but it has a diminishing return. Importantly, this ex-ante monotone but concave behavior of benefit due to increasing k_j always holds, and it is independent of the types of conflict between individuals.

7 Conclusion

We study strategic information transmission in networks with either conflicting preferences or conflicting beliefs. Our new insight to the literature is to introduce the positive externality effect in communication under conflicting prior beliefs. We also obtain the negative externality effect when the state of the world is additive. Studying these two externality effects together enables us to show that these two ways of modeling conflict (i.e., belief v.s. preference) lead to opposite conclusions. We next apply this framework to answer several questions (some inspired by previous works) with many-to-one and many-to-many (with arbitrary network) communication structures.

As one application, we study how poll size affects policy outcomes via information aggregation in polls. We show that impact of increasing poll size on information aggregation, depending on the interplay between positive and negative externality effects, is ambiguous when the sources of conflict are both present. Particularly, under conflicting preferences, due to the negative externality effect, information aggregation is impossible in large polls. In fact, there exists a finite upper bound on the poll size where truthful information revelation is an equilibrium. In sharp contrast, under conflicting beliefs information aggregation is impossible in small polls. That is, there exists a finite lower bound on the poll size where truthful information revelation is an equilibrium. Although in the interim stage information aggregation crucially depends on the types of conflict, each polled constituent before learning his private signal, independent of the types of conflict, would always gain benefit from increasing the poll size. However, the margin of this gain gets smaller as the poll size increases.

As another application, we study communication network formations. We establish that network structures arising in equilibria crucially depends on the types of conflict. Under conflicting preferences, any pairwise stable equilibrium generates the same set of in-degrees. These in-degrees are necessarily maximal and, by the negative externality effect, are neither too low nor too large. On the contrary, under conflicting beliefs, individuals can have different in-degrees in different pairwise stable equilibria. Moreover, by the positive externality effect, these in-degrees are either sufficiently low or quite large.

We further analyze the direction of information flow between communities. Under conflicting preferences information is likely to flow from a larger community to a smaller one; while, under conflicting beliefs, a smaller community is more likely to report to a larger one. In fact, under conflicting preferences, members of the large community accumulate many aspects from their like-minded colleagues, which, by the negative externality effect, curbs truthful reporting from the members of the small community. Under conflicting beliefs, however, the opposite communication pattern is more natural to appear, where members of the small community are more likely to report truthfully to members of the large community.

Finally, we study how types of conflict lead to a segregation or aggregation in communications due to adding new informed agents. We show that increasing the number of individuals of the same type localizes communication under conflicting preferences and intensifies cross-type communication under conflicting beliefs. Thus, under conflicting preferences, replicating the set of individuals, by the negative externality effect, curbs cross-type communication and can lead to a segregation of communication according to the individuals' types. Under conflicting beliefs, on the contrary, adding new informed individuals, due to the positive externality effect, boosts cross-type communication, which necessarily leads to greater information aggregation.

In a nutshell, our findings shed light on the importance of taking into account the types of conflict between individuals in strategic communication models.

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Appendix: Omitted proofs

A Proof of Proposition 1

Proof. Let l denote the number of signals 1 in ω_R and consider the effect on i's action of an additional signal ω. After some algebra, this effect can be expressed as

$$\begin{aligned}
a_i^{\mathsf{G}}(\omega_R) - a_i^{\mathsf{G}}(\omega_R, \omega) &= -\omega + (D-k)\mathbb{E}_i(\theta|\omega_R) - (D-k-1)\mathbb{E}_i(\theta|\omega_R, \omega) \\
&= -\omega + (D-k)\frac{\alpha_i + l}{\gamma + k} - (D-k-1)\frac{\alpha_i + l + \omega}{\gamma + k + 1} \\
&= \frac{\gamma + D}{\gamma + k + 1}\left[-\omega + \frac{\alpha_i + l}{\gamma + k}\right].
\end{aligned}$$

Thus, the expected magnitude of the effect from an additional signal is

$$\begin{aligned}
\mathbb{E}_j(a_i^{\mathsf{G}}(\omega_R) - a_i^{\mathsf{G}}(\omega_R, \omega))^2 &= \frac{(\gamma + D^2)}{(\gamma + k + 1)^2}\mathbb{E}_j\left(\omega - \mathbb{E}_i(\theta|\omega_R)\right)^2 \\
&= \frac{(\gamma + D^2)}{(\gamma + k + 1)^2}\left[\mathbb{E}_j\left(\omega - \mathbb{E}_j(\theta|\omega_R)\right)^2 + \mathbb{E}_j\left(\mathbb{E}_j(\theta|\omega_R) - \mathbb{E}_i(\theta|\omega_R)\right)^2\right].
\end{aligned}$$

In this expression,

$$\begin{aligned}
\mathbb{E}_j\left(\omega - \mathbb{E}_j(\theta|\omega_R)\right)^2 &= \mathbb{E}_j[\omega^2] - \mathbb{E}_j[\mathbb{E}_j(\theta|\omega_R)^2] \\
&= \frac{\alpha_j}{\gamma} - \frac{1}{(\gamma + k)^2}\mathbb{E}_j(\alpha_j + l)^2.
\end{aligned}$$

From the proof of Proposition 4 below, $\mathbb{E}_j(\alpha_j + l)^2 = \frac{\alpha_j(\gamma + k)}{\gamma(\gamma + 1)}[k(\alpha_j + 1) + \alpha_j(\gamma + 1)]$. This implies that

$$\begin{aligned}
\mathbb{E}_j\left(\omega - \mathbb{E}_j(\theta|\omega_R)\right)^2 &= \frac{\alpha_j}{\gamma} - \frac{\alpha_j}{(\gamma + k)\gamma(\gamma + 1)}[k(\alpha_j + 1) + \alpha_j(\gamma + 1)] \\
&= \frac{\alpha_j \beta_j(\gamma + k + 1)}{(\gamma + k)\gamma(\gamma + 1)}.
\end{aligned}$$

Consider now the term $\mathbb{E}_j\left(\mathbb{E}_j(\theta|\omega_R) - \mathbb{E}_i(\theta|\omega_R)\right)^2$:

$$\mathbb{E}_j\left(\mathbb{E}_j(\theta|\omega_R) - \mathbb{E}_i(\theta|\omega_R)\right)^2 = \mathbb{E}_j\left(\frac{\alpha_j + l}{\gamma + k} - \frac{\alpha_i + l}{\gamma + k}\right)^2 = \frac{(\alpha_j - \alpha_i)^2}{(\gamma + k)^2}.$$

As a result, the expected magnitude is

$$\mathbb{E}_j(a_i^{\mathsf{G}}(\omega_R) - a_i^{\mathsf{G}}(\omega_R, \omega))^2 = \frac{(\gamma + D^2)}{(\gamma + k + 1)^2}\left[\frac{\alpha_j\beta_j(\gamma + k + 1)}{(\gamma + k)\gamma(\gamma + 1)} + \frac{(\alpha_j - \alpha_i)^2}{(\gamma + k)^2}\right].$$

It is straightforward to see that the magnitude of the effect from an additional signal on i's action is a decreasing function of k. For the last aspect, i.e., $k = D - 1$, it boils down to

$$\mathbb{E}_j(a_i^{\mathsf{G}}(\omega_R) - a_i^{\mathsf{G}}(\omega_R, \omega))^2 = \frac{\alpha_j\beta_j(\gamma + D)}{(\gamma + D - 1)\gamma(\gamma + 1)} + \frac{(\alpha_j - \alpha_i)^2}{(\gamma + D - 1)^2},$$

which exceeds $\frac{\alpha_j\beta_j}{\gamma(\gamma+1)}$ for any D.

Note that under conflicting preferences, the individuals agree on the common prior and the expected magnitude is just

$$\mathbb{E}_j(a_i^{\mathsf{G}}(\omega_R) - a_i^{\mathsf{G}}(\omega_R, \omega))^2 = \frac{(\gamma + D)^2}{(\gamma + k + 1)^2} \cdot \frac{\alpha_j\beta_j(\gamma + k + 1)}{(\gamma + k)\gamma(\gamma + 1)}.$$

In case of conflicting beliefs, there is an additional decreasing in k term $\frac{(\gamma+D)^2}{(\gamma+k+1)^2} \cdot \frac{(\alpha_j-\alpha_i)^2}{(\gamma+k)^2}$, which corresponds to the fact that the additional signal in expectation makes the posteriors of individuals i and j closer to each other. Indeed, because the condition $\alpha_i + \beta_i = \gamma$, for all $i \in N$, implies the monotone likelihood ratio for prior distributions, the individuals interpret the signals and update their beliefs consistently with each other. □

B Proof of Proposition 2

Proof. Assume that $\alpha_i + \beta_i = \alpha_j + \beta_j = \gamma$. If individual j learns signals $\omega_R \in \{0,1\}^{k_j}$, he optimally chooses the action

$$a_j^{\mathsf{G}}(\omega_R) = \sum_{\omega_d \in \omega_R} \omega_d + \mathbb{E}_j\Big(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\Big).$$

This implies the following ex-ante expected input into i's utility:

$$
\begin{aligned}
& -\sum_{\omega_R \in \{0,1\}^{k_j}} \sum_{(\omega_i,\omega_{-R}) \in \{0,1\}^{D-k_j}} (a_j^{\mathsf{G}}(\omega_R) - \Omega)^2 P_i(\omega_R, \omega_i, \omega_{-R}) \\
= \; & -\sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[\mathbb{E}_j \left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right]^2 P_i(\omega_R, \omega_i, \omega_{-R}) \\
= \; & -\sum_{\omega_R} \left[\mathbb{E}_j \left(\sum_{(\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \mathbb{E}_i \left(\sum_{(\omega_i,\omega_{-R})} \omega_d | \omega_R \right) \right]^2 P_i(\omega_R) \\
& -2 \sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \{ \left[\mathbb{E}_j \left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \mathbb{E}_i \left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) \right] \cdots \\
& \cdots \left[\mathbb{E}_i \left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right] \} P_i(\omega_R, \omega_i, \omega_{-R}) \\
& -\sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[\mathbb{E}_i \left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right]^2 P_i(\omega_R, \omega_i, \omega_{-R}) \\
= \; & -A_{ij}(k_j) - B_i(k_j).
\end{aligned}
$$

The second term of the sum is the expected residual variance, expression for which was derived in the proof of Proposition 4:

$$
B_i(k_j) = \mathbb{E}_i \left[\mathrm{Var}_i \left(\sum_{\omega_d \in s_{-R}} \omega_d | \omega_R \right) \right] = \frac{\alpha_i \beta_i (\gamma + D)(D - k_j)}{\gamma(\gamma+1)(\gamma + k_j)}.
$$

The first term from the sum can be rewritten as

$$
\begin{aligned}
A_{ij}(k_j) &= (D - k_j)^2 \sum_{\omega_R} \left[\mathbb{E}_j (\theta | \omega_R) - \mathbb{E}_i (\theta | \omega_R) \right]^2 P_i(\omega_R) \\
&= (D - k_j)^2 \sum_{\omega_R} \left(\frac{\alpha_j + l}{\gamma + k_j} - \frac{\alpha_i + l}{\gamma + k_j} \right)^2 P_i(\omega_R) \\
&= (D - k_j)^2 \frac{(\alpha_i - \alpha_j)^2}{(\gamma + k_j)^2}.
\end{aligned}
$$

Because $A_{ij}(k_j)$ and $B_i(k_j)$ are positive, decreasing and convex functions of k_j, the ex-ante expected benefit $A_{ij}(k_j) + B_i(k_j) - A_{ij}(k_j+1) - B_i(k_j+1)$ is positive and decreasing in k_j.

□

C Proof of Proposition 3

Proof. Consider the truthful network g and some individual j who gets at least one truthful message. A necessary condition for individual j to have the in-degree $k_j = k_j(\mathsf{G}) < D-1$ is that for every person $i \in N_j^{-1}(\mathsf{G})$ it is incentive compatible to report truthfully, given that j believes the message. Fix some $i \in N_j^{-1}(\mathsf{G})$ and, as before, let ω_R be the set of k_j signals that individual j gets to know and ω_{-R} be the set of $D-k_j-1$ unknown signals (excluding ω_i). Assuming that individual j believes i's message, let $a_j^{\mathsf{G}}(\omega_R, \omega_i)$ and $a_j^{\mathsf{G}}(\omega_R, 1-\omega_i)$ be j's actions when i reports truthfully or lies, respectively. individual i reports his signal s_i to j truthfully if and only if it results in greater ex-interim expected payoff compared to lying:

$$\sum_{\omega_R \in \{0,1\}^{k_j}, \omega_{-R} \in \{0,1\}^{D-k_j-1}} -\left[(a_j^{\mathsf{G}}(\omega_R,\omega_i)-\Omega)^2-(a_j^{\mathsf{G}}(\omega_R,1-\omega_i)-\Omega)^2\right] P_i(\omega_R,\omega_{-R}|\omega_i) \geq 0,$$

hence,

$$-\sum_{\omega_R,\omega_{-R}} \left[(a_j^{\mathsf{G}}(\omega_R,\omega_i)-a_j^{\mathsf{G}}(\omega_R,1-\omega_i))(a_j^{\mathsf{G}}(\omega_R,\omega_i)+a_j^{\mathsf{G}}(\omega_R,1-\omega_i)-2\Omega)\right] P_i(\omega_R,\omega_{-R}|\omega_i) \\ \geq 0.$$

Let that the number of signals 1s in ω_R be l, and use the expression (4) for optimal actions $a_j^{\mathsf{G}}(\omega_R, \omega_i)$ and $a_j^{\mathsf{G}}(\omega_R, 1-\omega_i)$:

$$-\sum_{\omega_R,\omega_{-R}} P_i(\omega_R,\omega_{-R}|\omega_i)\left[2\omega_i-1+(D-k_j-1)\left(\frac{\alpha_j+l+\omega_i}{\alpha_j+\beta_j+k_j+1}-\frac{\alpha_j+l+1-\omega_i}{\alpha_j+\beta_j+k_j+1}\right)\right]\cdots \\ \cdots\left[1-2\omega_i+(D-k_j-1)\left(\frac{\alpha_j+l+\omega_i}{\alpha_j+\beta_j+k_j+1}+\frac{\alpha_j+l+1-\omega_i}{\alpha_j+\beta_j+k_j+1}\right)-2\sum_{\omega_d\in\omega_{-R}}\omega_d\right] \geq 0.$$

Expressing $P_i(\omega_R, \omega_{-R}|\omega_i) = P_i(\omega_{-R}|\omega_i, \omega_R)P_i(\omega_R|\omega_i)$, this can be simplified as

$$-(2\omega_i - 1)\frac{\alpha_j + \beta_j + D}{\alpha_j + \beta_j + k_j + 1} \cdots$$
$$\cdots \sum_{\omega_R} \left[1 - 2\omega_i + (D - k_j - 1)\frac{2\alpha_j + 2l + 1}{\alpha_j + \beta_j + k_j + 1} - 2A(\omega_i, \omega_R)\right] P_i(\omega_R|\omega_i) \geq 0,$$

where

$$\begin{aligned} A(\omega_i, \omega_R) &= \sum_{\omega_{-R}} \left(\sum_{\omega_d \in \omega_{-R}} \omega_d\right) P_i(\omega_{-R}|\omega_i, \omega_R) = \mathbb{E}_i \left(\sum_{\omega_d \in \omega_{-R}} \omega_d | \omega_i, \omega_R\right) \\ &= (D - k_j - 1)\mathbb{E}_i \left(\theta|\omega_i, \omega_R\right) = (D - k_j - 1)\frac{\alpha_i + l + \omega_i}{\alpha_i + \beta_i + k_j + 1}. \end{aligned}$$

Denote $\alpha_i + \beta_i = \gamma_i$ for all i and cancel positive term $\frac{\gamma_j + D}{\gamma_j + k_j + 1}$:

$$-(2\omega_i - 1)\sum_{\omega_R} \left[1 - 2\omega_i + (D - k_j - 1) \left(\frac{2\alpha_j + 2l + 1}{\gamma_j + k_j + 1} - 2\frac{\alpha_i + l + \omega_i}{\gamma_i + k_j + 1}\right)\right] P_i(\omega_R|\omega_i) \geq 0,$$

hence,

$$-(2\omega_i - 1) \left[1 - 2\omega_i + (D - k_j - 1) \left(\frac{2\alpha_j + 2B_i(k_j, \omega_i) + 1}{\gamma_j + k_j + 1} - 2\frac{\alpha_i + B_i(k_j, \omega_i) + \omega_i}{\gamma_i + k_j + 1}\right)\right] \geq 0,$$

where $B_i(k_j, \omega_i)$ denotes ex-ante expected number of 1s in a set of k_j signals:

$$B_i(k_j, \omega_i) = \sum_{\omega_R} l P_i(\omega_R|\omega_i) = k_j \mathbb{E}_i(\theta|\omega_i) = k_j \frac{\alpha_i + \omega_i}{\gamma_i + 1}.$$

Substituting this into incentive condition,

$$-(2\omega_i - 1) \left[1 - 2\omega_i + (D - k_j - 1) \left(\frac{2\alpha_j + 2k_j \frac{\alpha_i + \omega_i}{\gamma_i + 1} + 1}{\gamma_j + k_j + 1} - 2\frac{\alpha_i + k_j \frac{\alpha_i + \omega_i}{\gamma_i + 1} + \omega_i}{\gamma_i + k_j + 1}\right)\right] \geq 0,$$

therefore,

$$-(2\omega_i - 1) \left[1 - 2\omega_i + (D - k_j - 1) \left(\frac{2\alpha_j + 2k_j \frac{\alpha_i + \omega_i}{\gamma_i + 1} + 1}{\gamma_j + k_j + 1} - 2\frac{\alpha_i + \omega_i}{\gamma_i + 1}\right)\right] \geq 0,$$

which after some simplification becomes:

$$-(2\omega_i - 1)\left[1 - 2\omega_i + (D - k_j - 1)\frac{2\alpha_j(\gamma_i + 1) + \gamma_i + 1 - 2(\alpha_i + \omega_i)(\gamma_j + 1)}{(\gamma_i + 1)(\gamma_j + k_j + 1)}\right] \geq 0.$$

In case of $\omega_i = 1$ individual i reveals the signal iff

$$-1 + (D - k_j - 1)\frac{2\alpha_j(\gamma_i + 1) + \gamma_i + 1 - 2(\alpha_i + 1)(\gamma_j + 1)}{(\gamma_i + 1)(\gamma_j + k_j + 1)} \leq 0,$$

therefore,

$$-(\gamma_i + 1)(\gamma_j + k_j + 1) + (D - k_j - 1)(2\alpha_j(\gamma_i + 1) + \gamma_i + 1 - 2(\alpha_i + 1)(\gamma_j + 1)) \leq 0,$$

which after some algebraic transformations boils down to

$$\gamma_i - \gamma_j + \alpha_j(\gamma_i + 1) - \alpha_i(\gamma_j + 1) \leq \frac{(\gamma_i + 1)(\gamma_j + D)}{2(D - k_j - 1)}.$$

Subtracting $\frac{\gamma_i - \gamma_j}{2}$ from both sides,

$$\frac{\gamma_i - \gamma_j}{2} + \alpha_j(\gamma_i + 1) - \alpha_i(\gamma_j + 1) \leq \frac{(\gamma_i + 1)(\gamma_j + D)}{2(D - k_j - 1)} - \frac{\gamma_i - \gamma_j}{2}.$$

Consider now $\omega = 0$. The truth-telling condition in this case becomes

$$1 + (D - k_j - 1)\frac{2\alpha_j(\gamma_i + 1) + \gamma_i + 1 - 2\alpha_i(\gamma_j + 1)}{(\gamma_i + 1)(\gamma_j + k_j + 1)} \geq 0,$$

which can be simplified to

$$\alpha_j(\gamma_i + 1) - \alpha_i(\gamma_j + 1) \geq -\frac{(\gamma_i + 1)(\gamma_j + D)}{2(D - k_j - 1)}.$$

Adding $\frac{\gamma_i - \gamma_j}{2}$ to both sides,

$$\frac{\gamma_i - \gamma_j}{2} + \alpha_j(\gamma_i + 1) - \alpha_i(\gamma_j + 1) \geq -\frac{(\gamma_i + 1)(\gamma_j + D)}{2(D - k_j - 1)} + \frac{\gamma_i - \gamma_j}{2}.$$

As a result, the truth-telling condition for general signal s_i is

$$\left|\frac{\gamma_i - \gamma_j}{2} + \alpha_j(\gamma_i + 1) - \alpha_i(\gamma_j + 1)\right| \leq \frac{(\gamma_i + 1)(\gamma_j + D)}{2(D - k_j - 1)} - \frac{\gamma_i - \gamma_j}{2}. \tag{16}$$

Note that the right-hand side in (16) is greater than 0, because

$$\begin{aligned}
\frac{(\gamma_i + 1)(\gamma_j + D)}{2(D - k_j - 1)} - \frac{\gamma_i - \gamma_j}{2} &= \frac{(\gamma_i + 1)(\gamma_j + D) - (\gamma_i - \gamma_j)(D - k_j - 1)}{2(D - k_j - 1)} \\
&= \frac{\gamma_i(\gamma_j + D - D + k_j + 1) + \gamma_j(1 + D - k_j - 1) + D}{2(D - k_j - 1)} \\
&= \frac{\gamma_i(\gamma_j + k_j + 1) + \gamma_j(D - k_j) + D}{2(D - k_j - 1)} \\
&> 0.
\end{aligned}$$

Moreover, notice that as k_j gets larger, the right-hand side in (16) gets larger as well, relaxing the constraint. This means, that greater k_j allows for greater set of different priors' people to communicate. That is, we have positive effect from having many links. In this case we might expect equilibria with very little or very intensive communication.

In a particular case where the sum of prior's parameters is the same across all individuals, $\alpha_i + \beta_i = \gamma$ for any i. In this case truth-telling condition simplifies to:

$$|\alpha_j - \alpha_i| \leq \frac{\gamma + D}{2(D - k_j - 1)}.$$

□

D Proof of Proposition 4

Proof. Consider individual j who learns information $\omega_R \in \{0, 1\}^{k_j}$ ($k_j - 1$ signals coming from other individuals excluding individual i, 1 signal j receives himself) and optimally chooses action

$$a_j^{\mathsf{G}}(\omega_R) = b_j + \sum_{\omega_d \in \omega_R} \omega_d + \mathbb{E}\Big(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\Big).$$

An ex-ante expected input from individual j into i's utility is then

$$
\begin{aligned}
& - \sum_{\omega_R \in \{0,1\}^{k_j}} \sum_{(\omega_i, \omega_{-R}) \in \{0,1\}^{D-k_j}} (a_j^{\mathsf{G}}(\omega_R) - \Omega - b_i)^2 P(\omega_R, \omega_i, \omega_{-R}) \\
= & -\sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[b_j - b_i + \mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right]^2 P(\omega_R, \omega_i, \omega_{-R}) \\
= & -(b_j - b_i)^2 \\
& -2(b_j - b_i) \sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[\mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right] P(\omega_R, \omega_i, \omega_{-R}) \\
& -\sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[\mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right]^2 P(\omega_R, \omega_i, \omega_{-R}).
\end{aligned}
$$

It is easy to see that the second term in this sum is zero , because:

$$
\begin{aligned}
& \sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[\mathbb{E}\left(\sum_{\omega_d \in \omega_R} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right] P(\omega_R, \omega_i, \omega_{-R}) \\
= & \sum_{\omega_R} \sum_{(\omega_i,\omega_{-R})} \left[\mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d \right] P(\omega_i, \omega_{-R} | \omega_R) P(\omega_R) \\
= & \sum_{\omega_R} \left[\mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) - \mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R \right) \right] P(\omega_R) \\
= & \ 0.
\end{aligned}
$$

The third term is an expected residual variance $h(k_j) = \mathbb{E}\left[\mathrm{Var}\left(\sum_{\omega_d \in (\omega_i, \omega_{-R})} \omega_d | \omega_R\right)\right]$ and can be rewritten as

$$\begin{aligned}
&\sum_{\omega_R}\left[\mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R\right)\right]^2 P(\omega_R) + \sum_{\omega_R}\sum_{(\omega_i,\omega_{-R})}\left[\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d\right]^2 P(\omega_R, \omega_i, \omega_{-R}) \\
&-2\sum_{\omega_R}\mathbb{E}\left(\sum_{\omega_d \in \omega_R} \omega_d | \omega_R\right)\sum_{(\omega_i,\omega_{-R})}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d\right) P(\omega_i, \omega_{-R}|\omega_R) P(\omega_R) \\
= \ &\underbrace{-\sum_{\omega_R}\left[\mathbb{E}\left(\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d | \omega_R\right)\right]^2 P(\omega_R)}_{A} + \underbrace{\sum_{(\omega_i,\omega_{-R})}\left[\sum_{\omega_d \in (\omega_i,\omega_{-R})} \omega_d\right]^2 P(\omega_i, \omega_{-R})}_{B}.
\end{aligned}$$

Calculate the first term A, denoting l to be the number of 1s in ω_R:

$$\begin{aligned}
A &= -\sum_{\omega_R}\left[(D-k_j)\mathbb{E}(\theta|\omega_R)\right]^2 P(\omega_R) = -\frac{(D-k_j)^2}{(\gamma+k_j)^2}\sum_{\omega_R}(\alpha+l)^2 P(\omega_R) \\
&= -\frac{(D-k_j)^2}{(\gamma+k_j)^2}\left[\alpha^2 + 2\alpha\sum_{\omega_R} l P(\omega_R) + \sum_{\omega_R} l^2 P(\omega_R)\right].
\end{aligned}$$

In this expression

$$\sum_{\omega_R} l P(\omega_R) = k_j \mathbb{E}(\omega_1) = \frac{k_j \alpha}{\gamma},$$

$$\sum_{\omega_R} l^2 P(\omega_R) = \int_0^1 \left(\sum_{\omega_R} l^2 P(\omega_R|\theta)\right) f(\theta) d\theta.$$

Since l is equal to the sum of signals in ω_R, signals ω_j are identically distributed and independent conditionally on θ, the term inside the integral can be rewritten as

$$\begin{aligned}
\sum_{\omega_R} l^2 P(\omega_R|\theta) &= \mathbb{E}(l^2|\theta) = \mathrm{Var}(l|\theta) + (\mathbb{E}(l|\theta))^2 \\
&= k_j\theta(1-\theta) + k_j^2\theta^2.
\end{aligned}$$

Taking the integral,

$$\begin{aligned}\sum_{\omega_R} l^2 P(\omega_R) &= \frac{k_j}{\mathbf{B}(\alpha,\beta)}\mathbf{B}(\alpha+1,\beta+1) + \frac{k_j^2}{\mathbf{B}(\alpha,\beta)}\mathbf{B}(\alpha+2,\beta) \\ &= \frac{k_j}{\gamma(\gamma+1)}[\alpha\beta + k_j\alpha(\alpha+1)].\end{aligned}$$

Substituting these to A yields

$$\begin{aligned}A &= -\frac{(D-k_j)^2}{(\gamma+k_j)^2}\left[\alpha^2 + 2\alpha\frac{k_j\alpha}{\gamma} + \frac{k_j(\alpha\beta + k_j\alpha(\alpha+1))}{\gamma(\gamma+1)}\right] \\ &= -\frac{(D-k_j)^2\alpha}{(\gamma+k_j)\gamma(\gamma+1)}\left[\alpha(\gamma+1) + k_j(\alpha+1)\right].\end{aligned}$$

Now consider the second term B, assuming that the number of 1s in (ω_i, ω_{-R}) is $\tilde{l}$:

$$B = \sum_{(\omega_i,\omega_{-R})} \tilde{l}^2 P(\omega_i, \omega_{-R}) = \frac{(D-k_j)}{\gamma(\gamma+1)}[\alpha\beta + (D-k_j)\alpha(\alpha+1)].$$

Then, $h(k_j) = A + B$ becomes:

$$\begin{aligned}h(k_j) &= -\frac{(D-k_j)^2\alpha}{(\gamma+k_j)\gamma(\gamma+1)}\left[\alpha(\gamma+1) + k_j(\alpha+1)\right] + \frac{(D-k_j)(\alpha\beta + (D-k_j)\alpha(\alpha+1))}{\gamma(\gamma+1)} \\ &= \frac{(D-k_j)\alpha}{\gamma(\gamma+1)(\gamma+k_j)}\left[(D-k_j)(-\alpha(\gamma+1) + \gamma(\alpha+1)) + (\gamma-\alpha)(\gamma+k_j)\right] \\ &= \frac{(D-k_j)\alpha}{\gamma(\gamma+1)(\gamma+k_j)}\left[(D-k_j)(\gamma-\alpha) + (\gamma-\alpha)(\gamma+k_j)\right] \\ &= \frac{\alpha\beta(\gamma+D)(D-k_j)}{\gamma(\gamma+1)(\gamma+k_j)}.\end{aligned}$$

Finally, since the ex-ante expected input from individual j into i's payoff is $-h(k_j) - (b_j - b_i)^2$, then the benefit from improving j's information by one additional signal is $h(k_j) - h(k_j+1)$. Because $h(k_j)$ is a positive, decreasing and convex function of k_j, the benefit exceeds 0 and decreases with k_j.

□

E Proof of Proposition 5

Proof. Consider the truthful network G and some individual j who gets at least one truthful message, i.e., $N_j^{-1}(\mathsf{G}) = \{i \in N : g_{ij} = 1\} \neq \emptyset$. It must be incentive compatible for every person $i \in N_j^{-1}(\mathsf{G})$ to report truthfully, given that individual j believes their messages. Fix some $i \in N_j^{-1}(\mathsf{G})$ and let ω_R be the set of k_j signals that individual j gets to know himself and from other individuals apart from individual i; denote $D - k_j - 1$ unknown signals excluding ω_i as ω_{-R}. If i reports truthfully, j optimally chooses $a_j^{\mathsf{G}}(\omega_R, \omega_i)$; if individual i misreports and sends $m_{ij} = 1 - \omega_i$, j picks the action $a_j^{\mathsf{G}}(\omega_R, \omega_i)$. individual i reports truthfully his signal if and only if it generates a greater interim expected payoff to i compared to misreporting:

$$\sum_{\omega_R, \omega_{-R} \in \{0,1\}^{D-1}} - \left[(a_j^{\mathsf{G}}(\omega_R, \omega_i) - \Omega - b_i)^2 - (a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i) - \Omega - b_i)^2 \right] P(\omega_R, \omega_{-R} | \omega_i) \geq 0.$$

This condition can be rewritten as

$$- \sum_{\omega_R, \omega_{-R}} \left\{ \left[(a_j^{\mathsf{G}}(\omega_R, \omega_i) - a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i))(a_j^{\mathsf{G}}(\omega_R, \omega_i) + a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i) - 2\Omega - 2b_i) \right] \cdots \right.$$
$$\left. \cdots P(\omega_R, \omega_{-R} | \omega_i) \right\} \geq 0.$$

Assume that there are l signals 1 in ω_R and recall that the actions $a_j^{\mathsf{G}}(\omega_R, \omega_i)$ and $a_j^{\mathsf{G}}(\omega_R, 1 - \omega_i)$ are given by (4), then the condition for truth-telling becomes

$$- \sum_{\omega_R, \omega_{-R}} P(\omega_R, \omega_{-R} | \omega_i) \left[2\omega_i - 1 + (D - k_j - 1) \left(\frac{\alpha + l + \omega_i}{\alpha + \beta + k_j + 1} - \frac{\alpha + l + 1 - \omega_i}{\alpha + \beta + k_j + 1} \right) \right] \cdots$$
$$\cdots \left[2b_j - 2b_i + 1 - 2\omega_i + (D - k_j - 1) \left(\frac{\alpha + l + \omega_i}{\alpha + \beta + k_j + 1} + \frac{\alpha + l + 1 - \omega_i}{\alpha + \beta + k_j + 1} \right) - 2 \sum_{\omega_d \in \omega_{-R}} \omega_d \right] \geq 0.$$

Using $P(\omega_R, \omega_{-R} | \omega_i) = P(\omega_{-R} | \omega_i, \omega_R) P(\omega_R | \omega_i)$, this can be simplified to

$$- (2\omega_i - 1) \frac{\alpha + \beta + D}{\alpha + \beta + k_j + 1} \cdots$$
$$\cdots \sum_{\omega_R} \left[2(b_j - b_i) + 1 - 2s_i + (D - k_j - 1) \frac{2\alpha + 2l + 1}{\alpha + \beta + k_j + 1} - 2A(\omega_i, \omega_R) \right] P(\omega_R | \omega_i) \geq 0,$$

where

$$
\begin{aligned}
A(\omega_i, \omega_R) &= \sum_{\omega_{-R}} \left(\sum_{\omega_d \in \omega_{-R}} \omega_d \right) P(\omega_{-R} | \omega_i, \omega_R) = \mathbb{E} \left(\sum_{\omega_d \in \omega_{-R}} \omega_d | \omega_i, \omega_R \right) \\
&= (D - k_j - 1) \mathbb{E} \left(\theta | \omega_i, \omega_R \right) = (D - k_j - 1) \frac{\alpha + l + \omega_i}{\alpha + \beta + k_j + 1}.
\end{aligned}
$$

After accounting for that and canceling the positive term $\frac{\alpha+\beta+D}{\alpha+\beta+k_j+1}$, the truth-telling condition becomes:

$$
-(2\omega_i - 1) \sum_{\omega_R} \left[2(b_j - b_i) + 1 - 2\omega_i + (D - k_j - 1) \frac{1 - 2\omega_i}{\alpha + \beta + k_j + 1} \right] P(\omega_R | \omega_i) \geq 0,
$$

hence,

$$
-(2\omega_i - 1) \left[2(b_j - b_i) + (1 - 2\omega_i) \frac{\alpha + \beta + D}{\alpha + \beta + k_j + 1} \right] \geq 0.
$$

If $\omega_i = 1$, the truth-telling condition becomes

$$
b_j - b_i \leq \frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)}.
$$

If $\omega_i = 0$, the truth-telling condition becomes

$$
b_j - b_i \geq -\frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)}.
$$

As a result,

$$
|b_j - b_i| \leq \frac{\alpha + \beta + D}{2(\alpha + \beta + k_j + 1)}.
$$

Since this condition must hold for every $i \in N_j^{-1}(\mathsf{G})$, this completes the proof of Proposition 5.

□

F Proof of Proposition 6

Proof. We split the proof into three steps for each type of conflict:

Conflicting preferences, Step 1: Existence of a maximal equilibrium. Because the number of individuals and strategies is finite, the number of the pure strategy equilibria is also finite. Thus, there exists a well-defined set of numbers, $k_1, \cdots, k_n$, where k_i is the highest in-degree of individual i that can arise in some equilibrium: for any equilibrium network G', $k_i \geq k_i' = k_i(\mathsf{G}')$. Note that the in-degrees k_i and k_j, $i \neq j$, in principle, might be achieved in different equilibrium networks. To prove an existence of a maximal equilibrium, we need to show that the in-degrees $k_1, \cdots, k_n$ might be achieved in the same equilibrium, i.e., that there exists an equilibrium network G such that $k_i = k_i(\mathsf{G})$ for all i. In order to do this, we construct the equilibrium in the following way: for each $i \in N$ consider an equilibrium where k_i is achieved and let those (and only those) individuals who report to i truthfully in that equilibrium to report truthfully to i in the constructed equilibrium. Because the incentives to report truthfully depend only on the receiver's in-degree, the sender's and the receiver's preference biases, it is still incentive compatible for those individuals to report truthfully to i. Thus, this is, indeed, an equilibrium, and, by construction, it is maximal.

Conflicting preferences, Step 2: Maximality of a pairwise stable equilibrium. Consider some pairwise stable equilibrium and assume that it is not maximal. Then there exists individual i whose in-degree in the equilibrium network is lower than his maximal in-degree. Fix some maximal equilibrium; then it must be the case that there is some agent j who reports truthfully to i in this maximal equilibrium, but not in the pairwise stable equilibrium. But then it is profitable for i and j to deviate and induce a truthful communication from j to i, which contradicts the pairwise stability. Hence, every pairwise stable equilibrium must be maximal.

Conflicting preferences, Step 3: Existence of a pairwise stable equilibrium. We illustrate this statement by constructing one of (possibly multiple) pairwise stable equilibria. For each $i \in N$ perform the following procedure: order other individuals $j \in N/\{i\}$ in the increasing absolute values of their preference divergence from i, $|b_j - b_i|$; let this order be $i_1, \cdots, i_{n-1}$. Consider the maximal in-degree of individual i, k_i. If $k_i = 0$, then nobody can report truthfully to i in equilibrium. If $k_i > 0$, then let the closest k_i individuals report truthfully to i (clearly, it is incentive compatible, because closer biases relax the incentive condition of the truth-telling). Since k_i is the maximal possible in-degree of individual i, individuals $i_{k_i+1}, \cdots, i_{n-1}$ cannot be credible in communicating to i; hence, set $g_{ji} = 0$ for these individuals.

Conflicting beliefs, Step 1: Existence and uniqueness of a maximal equilibrium. The proof of existence is the same as in conflicting preferences case. We prove the uniqueness by contradiction. Assume that the set of maximal in-degrees is $k_1, \cdots, k_n$ and suppose that there are two different maximal equilibria with communication networks $\mathsf{G} \neq \mathsf{G}'$. Then there is a link ij in G that is not present in G'. individual i can truthfully communicate his signal to individual j when j gets $k_j - 1$ truthful messages from other individuals in G. Hence, by the positive externality effect, i can be credible to j when j gets k_j truthful messages from other individuals in G'. Thus, there must exist an equilibrium where j's in-degree is $k_j + 1$, which contradicts the maximality of k_j. Hence, $\mathsf{G} = \mathsf{G}'$.

Conflicting beliefs, Step 2: Pairwise stability of a maximal equilibrium. Consider some maximal equilibrium and assume that it is not pairwise stable. Then there exist individuals i and j such that $g_{ji} = 0$, but who can improve their communication pattern to truth-telling. By the positive externality effect, making communication through ji truthful, doesn't alter the credibility of communication through other links. Hence, there exists an equilibrium where individual i receives more truthful signals, which contradicts the maximality condition. Thus, every maximal equilibrium must be pairwise stable.

Conflicting beliefs, Step 3: Existence of a pairwise stable equilibrium. Steps 1 and 2 immediately imply the existence of a pairwise stable equilibrium.

□

G Proof of Proposition 7

Proof. Consider individual i, i.e., $i = 1, 2, \cdots, k_j$. Define $\mathcal{I}_j^{-i} = (\omega_1, \omega_2, \cdots, \omega_{i-1}, \omega_{i+1}, \cdots, \omega_{k_j})$. Thus, $a_j(\mathcal{I}_j^{-i,*}, \mu_i)$ (where $\mu_i \in \{\omega_i, 1 - \omega_i\}$) is characterized by

$$a_j(\mathcal{I}_j^{-i}, \mu_i) = \sum_{i \in N_j(\mathsf{G})} \omega_i + (D - k_j)\mathbb{E}_j[\theta | \omega_1, \omega_2, \cdots, \omega_S] \tag{17}$$

Hence, given (17) the interim IC constraint for agent i is satisfied when

$$-\sum_{\omega_j^{-i}}\sum_{\omega_{-j}}\left\{\left[(a_j(\omega_i)-\Omega-b_i)^2-(a_j(1-\omega_i)-\Omega-b_i)^2\right]P_i(\omega_j^{-i},\omega_{-j}|\omega_i)\right\}\geq 0 \qquad (18)$$

where $\omega_j^{-i}\in\{0,1\}^{k_j-1}$ denotes the set of signals that j will receive from its neighbors, excluding i, and $\omega_{-j}\in\{0,1\}^{D-k_j}$ denotes the set of signals observed by non-neighbors (j does not have access to these signals).

Plugging (17) into (18) gives

$$\begin{aligned}
&-\sum_{\omega_j^{-i}}\sum_{\omega_{-j}}P_i(\omega_j^{-i},\omega_{-j}|\omega_i)\Big\{\left[a_j(\mathcal{I}_j^{-i},\omega_i)-a_j(\mathcal{I}_j^{-i},1-\omega_i)\right]\cdots\\
&\qquad\qquad\cdots\left[a_j(\mathcal{I}_j^{-i},\omega_i)+a_j^*(\mathcal{I}_j^{-i},1-\omega_i)-2\Omega-2b_i\right]\Big\}\\
&=-\sum_{\omega_j^{-i}}\sum_{\omega_{-j}}P_i(\omega_j^{-i},\omega_{-j}|\omega_i)\Big\{\left[(2\omega_i-1)\left(\frac{\gamma+D}{\gamma+k_j}\right)\right]\cdots\\
&\qquad\qquad\cdots\left[1-2\omega_i-2\sum_{\omega_j\in\{0,1\}^{D-k_j}}\omega_j-2b_i+\frac{D-k_j}{\gamma+k_j}[2(\alpha_j+\zeta(\omega_j^{-i}))+1]\right]\Big\}.
\end{aligned}$$

The above equality follows because

$$\begin{aligned}
&a_j(\mathcal{I}_j^{-i},\omega_i)-a_j(\mathcal{I}_j^{-i},1-\omega_i)\\
&\quad=2\omega_i-1+(D-k_j)\left(\mathbb{E}_j[\theta|\omega_j^{-i},\omega_i]-\mathbb{E}_j[\theta|\omega_j^{-i},1-\omega_i]\right)\\
&\quad=2\omega_i-1+(D-k_j)\left(\frac{\alpha_j+\omega_i+\zeta(\omega_j^{-i})}{\gamma+k_j}-\frac{\alpha_j+1-\omega_i+\zeta(\omega_j^{-i})}{\gamma+k_j}\right)\\
&\quad=(2\omega_i-1)\left(\frac{\gamma+D}{\gamma+k_j}\right)\\
&\quad\equiv\mathcal{A},
\end{aligned}$$

where $\zeta(\omega_j^{-i})$ denotes the number of 1's in ω_j^{-i}, and

$$
\begin{aligned}
& a_j(\mathcal{I}_j^{-i}, \omega_i) + a_j(\mathcal{I}_j^{-i}, 1-\omega_i) - 2\Omega - 2b_i \\
& \quad = 1 - 2\omega_i - 2 \sum_{\omega_j \in \{0,1\}^{D-k_j}} \omega_j - 2b_i + (D-k_j)(\mathbb{E}_j[\theta|\omega_j^{-i}, \omega_i] + \mathbb{E}_j[\theta|\omega_j^{-i}, 1-\omega_i]) \\
& \quad = 1 - 2\omega_i - 2 \sum_{\omega_j \in \{0,1\}^{D-k_j}} \omega_j - 2b_i \\
& \qquad \quad + (D-k_j) \left(\frac{\alpha_j + \omega_i + \zeta(\omega_j^{-i})}{\gamma + k_j} + \frac{\alpha_j + 1 - \omega_i + \zeta(\omega_j^{-i})}{\gamma + k_j} \right) \\
& \quad = 1 - 2\omega_i - 2 \sum_{\omega_j \in \{0,1\}^{D-k_j}} \omega_j - 2b_i + \left(\frac{D-k_j}{\gamma + k_j} \right) [2(\alpha_j + \zeta(\omega_j^{-i})) + 1] \\
& \quad \equiv \mathcal{B}.
\end{aligned}
$$

Notice that $\mathcal{A}$ is independent of ω_j^{-i}, thus (18) can be rewritten as

$$
\begin{aligned}
& - \sum_{\omega_j^{-i}} \sum_{\omega_{-j}} P_i(\omega_j^{-i}, \omega_{-j}|\omega_i) \times \mathcal{A} \times \mathcal{B} \\
& \qquad = -(2\omega_i - 1) \frac{\gamma + D}{\gamma + k_j} \sum_{\omega_j^{-i}} \sum_{\omega_{-j}} P_i(\omega_j^{-i}, \omega_{-j}|\omega_i) \times \mathcal{B}.
\end{aligned}
$$

Moreover, we also have:

$$
\begin{aligned}
\sum_{\omega_j^{-i}} \sum_{\omega_{-j}} P_i(\omega_j^{-i}, \omega_{-j}|\omega_i) [\sum_{\omega_j \in \{0,1\}^{D-k_j}} \omega_j] &= \sum_{\omega_j^{-i}} \left[\sum_{\omega_{-j}} P_i(\omega_{-j}|\omega_i, \omega_j^{-i}) \left[\sum_{\omega_j \in \{0,1\}^{D-k_j}} \omega_j \right] \right] P_i(\omega_j^{-i}|\omega_i) \\
&= \sum_{\omega_j^{-i}} \left[(D-k_j) \mathbb{E}_i[\theta|\omega_i, \omega_j^{-i}] \right] P_i(\omega_j^{-i}|\omega_i) \\
&= \sum_{\omega_j^{-i}} \left[(D-k_j) \left(\frac{\alpha_i + \omega_i + \zeta}{\gamma + k_j} \right) \right] P_i(\omega_j^{-i}|\omega_i).
\end{aligned}
$$

Equipped with the above equality we next have

$$
\begin{aligned}
&\sum_{\omega_j^{-i}} \sum_{\omega_{-j}} P_i(\omega_j^{-i}, \omega_{-j}|\omega_i) \times \mathcal{B} \\
&= \sum_{\omega_j^{-i}} \sum_{\omega_{-j}} \{ P_i(\omega_j^{-i}, \omega_{-j}|\omega_i) \left[1 - 2\omega_i - 2 \sum_{\omega_j \in \{0,1\}^{D-k_j}} \omega_j - 2b_i + \left(\frac{D-k_j}{\gamma + k_j} \right) [2(\alpha_j + \zeta(\omega_j^{-i})) + 1] \right] \} \\
&= \sum_{\omega_j^{-i}} \{ P_i(\omega_j^{-i}|\omega_i) \left[1 - 2\omega_i - 2(D-k_j)(\frac{\alpha_i + \omega_i + \zeta(\omega_j^{-i})}{\gamma + k_j}) - 2b_i + \left(\frac{D-k_j}{\gamma + k_j} \right) [2(\alpha_j + \zeta(\omega_j^{-i})) + 1] \right] \} \\
&= 1 - 2\omega_i - 2b_i - 2(D-k_j) \left(\frac{\alpha_i + \omega_i}{\gamma + k_j} \right) + \left(\frac{D-k_j}{\gamma + k_j} \right) [2\alpha_j + 1] \\
&= -2b_i + (1 - 2\omega_i) \left(\frac{D+\gamma}{\gamma + k_j} \right) + 2 \left(\frac{D-k_j}{\gamma + k_j} \right) (\alpha_j - \alpha_i).
\end{aligned}
$$

Finally, the IC for individual $i \in N_j(\mathsf{G})$ is simplified to the following

$$
\begin{aligned}
&- \sum_{\omega_j^{-i}} \sum_{\omega_{-j}} P_i(\omega_j^{-i}, \omega_{-j}|\omega_i) \times \mathcal{A} \times \mathcal{B} \\
&= -(2\omega_i - 1) \frac{\gamma + D}{\gamma + k_j} \left[-2b_i + (1 - 2\omega_i) \left(\frac{D+\gamma}{\gamma + k_j} \right) + 2 \left(\frac{D-k_j}{\gamma + k_j} \right) (\alpha_j - \alpha_i) \right] \geq 0, \\
&\qquad \forall \omega_i \in \{0,1\}. \qquad (19)
\end{aligned}
$$

Next, we consider two separate cases.

Case 1: $\omega_i = 0$ Plugging $\omega_i = 0$ into the above equality implies

$$
\begin{aligned}
-2b_i + \left(\frac{D+\gamma}{\gamma + k_j} \right) - 2 \left(\frac{D-k_j}{\gamma + k_j} \right) (\alpha_i - \alpha_j) \geq 0 \quad &\Leftrightarrow \\
\frac{1}{2} (D + \gamma) \geq b_i(\gamma + k_j) + \left(D - k_j \right) (\alpha_i - \alpha_j)&
\end{aligned}
$$

Case 2: $\omega_i = 1$ Plugging $\omega_i = 1$ into (19) implies

$$2b_i + \left(\frac{D+\gamma}{\gamma+k_j}\right) + 2\left(\frac{D-k_j}{\gamma+k_j}\right)(\alpha_i - \alpha_j) \geq 0 \quad \Leftrightarrow$$
$$\frac{-1}{2}(D+\gamma) \leq b_i(\gamma+k_j) + (D-k_j)(\alpha_i - \alpha_j)$$

As a result, the above two cases together yields

$$\left|b_i(\gamma+k_j) + (D-k_j)(\alpha_i-\alpha_j)\right| \leq \frac{1}{2}(D+\gamma).$$

Thus, to ensure IC holds for all the neighbors its necessary and sufficient to have

$$\max_{i\in N_j(\mathsf{G})} \left|b_i(\gamma+k_j) + (D-k_j)(\alpha_i-\alpha_j)\right| \leq \frac{1}{2}(D+\gamma),$$

completing the proof.

□

H Proof of Proposition 8

Proof. As shown in Proposition 7, in equilibrium, we must have

$$\max_{i\in N_j(\mathsf{G})} \left|b_i(\gamma+k_j) + (D-k_j)(\alpha_i-\alpha_j)\right| \leq \frac{1}{2}(D+\gamma). \tag{20}$$

Recall that $b \equiv \max_{i\in N_j(\mathsf{G})} 2|b_i|$ and $\alpha \equiv \max_{i\in N_j(\mathsf{G})} 2|\alpha_i - \alpha_j|$. We consider two separate cases as follows.

Case 1: $b = 0$: Thus, (20) implies that $\left(\frac{D-k_j}{\gamma+k_j}\right)\alpha \leq \frac{D+\gamma}{\gamma+k_j}$. Therefore, information aggregation occurs if and only if $k_j \geq \frac{1}{\alpha}(D(\alpha-1)-\beta)$.

Case 2: $\alpha = 0$: Thus, (20) implies that $b \leq \frac{D+\gamma}{\gamma+k_j}$. Therefore, information aggregation occurs if and only if $k_j \leq \frac{1}{b}(D+\gamma(1-b))$.

Case 3: $\alpha \neq 0$ and $b \neq 0$: The result immediately follows from (9), more details are in the main text. $\square$

I Proofs of Proposition 9 and Proposition 10

Proof. We start with the following useful Remark.

Remark 8. *Suppose $\theta \sim$ **Beta**$(\alpha, \beta - \alpha)$ and $X = \{x_1, x_2, \cdots, x_n\}$ where $x_i \in \{0,1\}$ and $x_i = 1$ with probability θ. Let $\zeta(X)$ be the number of 1s in X. Then*

$$
\begin{aligned}
\mathbb{E}[\zeta^k] &= \sum_{X \in \{0,1\}^n} \zeta^k(X) P(X) \\
&= \int_0^1 \sum_{X \in \{0,1\}^n} \zeta^k(X) P(X|\theta) dP_{\mathbf{Beta}(\alpha,\beta-\alpha)}(\theta) \\
&= \int_0^1 \left[\sum_{j=0}^{n} \binom{n}{j} \theta^j (1-\theta)^{n-j} j^k \right] dP_{\mathbf{Beta}(\alpha,\beta-\alpha)}(\theta)
\end{aligned}
$$

where $P_{\mathbf{Beta}(\alpha,\beta-\alpha)}$ denotes the corresponding c.d.f. of θ. Two consequence of the above expansion useful in the sequel are:

$$
\begin{aligned}
\mathbb{E}_X[\zeta(X)] = \sum_{X \in \{0,1\}^n} \zeta(X) P(X) &= \int_0^1 n\theta \; dP_{\mathbf{Beta}(\alpha,\beta-\alpha)}(\theta) \\
&= n \frac{\mathbf{B}(\alpha+1, \beta-\alpha)}{\mathbf{B}(\alpha, \beta-\alpha)} \\
&= n \frac{\alpha}{\beta}
\end{aligned} \tag{21}
$$

and

$$
\begin{aligned}
\mathbb{E}_X[\zeta^2(X)] &= \sum_{X\in\{0,1\}^n} \zeta^2(X)P(X) \\
&= \int_0^1 n(n\theta^2 + \theta(1-\theta))\, dP_{\mathbf{B}(\alpha,\beta-\alpha)}(\theta) \\
&= n\,\frac{\mathbf{B}(\alpha+1,\beta-\alpha+1)}{\mathbf{B}(\alpha,\beta-\alpha)} + n^2\,\frac{\mathbf{B}(\alpha+2,\beta-\alpha)}{\mathbf{B}(\alpha,\beta-\alpha)} \\
&= n\alpha\,\frac{\beta-\alpha+n(\alpha+1)}{\beta(\beta+1)}
\end{aligned}
\tag{22}
$$

Note that the last equalities in (21) and (22) follow because

$$
\mathbf{B}(\alpha+k_1,\beta-\alpha+k_2) = \frac{\Gamma(\alpha+k_1)\Gamma(\beta-\alpha+k_2)}{\Gamma(\beta+k_1+k_2)}
\tag{23}
$$

where $\Gamma(\cdot)$ is the Gamma distribution, also note that $\Gamma(k+\alpha) = T(\alpha)k(k+1)\cdots(k+\alpha-1)$.

Next we prove the propositions. The proof follows in several steps.
Given that $|N_j(\mathsf{G})| = k_j$, thus, plugging the j's action into the individual i's (ex-ante) payoff gives:

$$
\begin{aligned}
\mathbb{E}_i[u_i|k_j] &= -\sum_{\omega_j}\sum_{\omega_{-j}} \left(a_j - \Omega - b_i\right)^2 P_i(\omega_j,\omega_{-j}) \\
&= \sum_{\omega_j,\omega_{-j}} \left(\sum_{i\in N_j(\mathsf{G})} \omega_i + \mathbb{E}_j\left[\sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i|\mathcal{I}_j \right] - \Omega - b_i \right)^2 P_i(\omega_j,\omega_{-j}) \\
&= \sum_{\omega_j,\omega_{-j}} \left(- \sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i + \mathbb{E}_j\left[\sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i|\mathcal{I}_j \right] - b_i \right)^2 P_i(\omega_j,\omega_{-j}) \\
&= -b_i^2 + 2b_i \sum_{\omega_j,\omega_{-j}} \left(- \sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i + \mathbb{E}_j\left[\sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i|\mathcal{I}_j \right] \right) P_i(\omega_j,\omega_{-j}) \\
&\quad - \sum_{\omega_j,\omega_{-j}} \left(- \sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i + \mathbb{E}_j\left[\sum_{\omega_i\in\{0,1\}^{D-k_j}} \omega_i|\mathcal{I}_j \right] \right)^2 P_i(\omega_j,\omega_{-j}) \\
&\equiv -b_i^2 + A + B.
\end{aligned}
\tag{24}
$$

To compute (24), we next simplify A and B.

Simplifying A:

$$
\begin{aligned}
A &= 2b_i \sum_{\omega_j, \omega_{-j}} \left(- \sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i + \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right) P_i(\omega_j, \omega_{-j}) \\
&= 2b_i \sum_{\omega_j} \left(-\mathbb{E}_i \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] + \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right) P_i(\omega_j) \\
&= 2b_i \left(\frac{D-k_j}{\gamma + k_j} \right) \sum_{\omega_j} \left(-(\alpha_i + \zeta(\omega_j)) + (\alpha_j + \zeta(\omega_j)) \right) P_i(\omega_j) \\
&= 2b_i \left(\frac{D-k_j}{\gamma + k_j} \right) (\alpha_j - \alpha_i),
\end{aligned}
$$

where $\zeta(\omega_j)$ denotes number of 1s in ω_j.

To simplify B, we first decompose it to C and D as follows:

Simplifying B:

$$
\begin{aligned}
B &= -\sum_{\omega_j,\omega_{-j}} \left[-\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i + \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right]^2 P_i(\omega_j, \omega_{-j}) \\
&= -\sum_{\omega_j,\omega_{-j}} \{ \left[(\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i)^2 + \mathbb{E}_j^2 \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \cdots \right. \\
&\qquad \left. \cdots - 2(\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i) \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right] P_i(\omega_j, \omega_{-j}) \} \\
&= -\sum_{\omega_j,\omega_{-j}} \{ \left[(\sum_{\omega_i \in \{0,1\}^{D-k_j}})^2 + \mathbb{E}_j^2 [\sum_{\omega_i \in \{0,1\}^{D-k_j}} | \mathcal{I}_j] \cdots \right. \\
&\qquad \left. \cdots - 2(\sum_{\omega_i \in \{0,1\}^{D-k_j}}) \mathbb{E}_j [\sum_{\omega_i \in \{0,1\}^{D-k_j}} | \mathcal{I}_j] \right] P_i(\omega_j, \omega_{-j}) \} \\
&= -\sum_{\omega_j} \{ \left[\mathbb{E}_i \left[(\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i)^2 | \mathcal{I}_j \right] + \mathbb{E}_j^2 \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \cdots \right. \\
&\qquad \left. \cdots - 2\mathbb{E}_i \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right] P_i(\omega_j) \} \\
&= -\mathbb{E}_i \left(\mathrm{Var}_i \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right) \\
&\qquad - \sum_{\omega_j} \left(\mathbb{E}_i \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] - \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right)^2 P_i(\omega_j) \\
&\equiv C + D.
\end{aligned}
$$

Next, we simplify D and C, respectively.

Simplifying D:

$$
\begin{aligned}
D &= -\sum_{\omega_j} \left(\mathbb{E}_i \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] - \mathbb{E}_j \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right)^2 P_i(\omega_j) \\
&= -\sum_{\omega_j} \left(\frac{D-k_j}{\gamma + k_j} (\alpha_i + \zeta(\omega_j)) - \frac{D-k_j}{\gamma + k_j} (\alpha_j + \zeta(\omega_j)) \right) P_i(\omega_j) \\
&= -\left(\frac{D-k_j}{\gamma + k_j} (\alpha_i - \alpha_j) \right)^2.
\end{aligned}
$$

To simplify C, we first decompose it to E and F.

Simplifying C:

$$
\begin{aligned}
C &= -\mathbb{E}_i \left(\mathrm{Var}_i \left[\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j \right] \right) \\
&= -\sum_{\omega_j} \left(\mathbb{E}_i \left[\Big(\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i \Big)^2 | \mathcal{I}_j \right] - \mathbb{E}_i^2 [\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j] \right) P_i(\omega_j) \\
&= -\sum_{\omega_j} \mathbb{E}_i [(\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i)^2 | \mathcal{I}_j] P_i(\omega_j) + \sum_{\omega_j} \mathbb{E}_i^2 [\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j] P_i(\omega_j) \\
&= E + F.
\end{aligned}
$$

Next, we simplify F and E, respectively.

Simplifying F:

$$
\begin{aligned}
F &= \sum_{\omega_j} \mathbb{E}_i^2 [\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i | \mathcal{I}_j] P_i(\omega_j) \\
&= \sum_{\omega_j} \left[(D-k_j) \frac{\alpha_i + \zeta(\omega_j)}{\gamma + k_j} \right]^2 P_i(\omega_j) \\
&= \left(\frac{D-k_j}{\gamma+k_j} \right)^2 \sum_{\omega_j} \left(\alpha_i + \zeta(\omega_j) \right)^2 P_i(\omega_j) \\
&= \left(\frac{D-k_j}{\gamma+k_j} \right)^2 \left(\alpha_i^2 + 2\alpha_i \sum_{\omega_j} \zeta(\omega_j) P_i(\omega_j) + \sum_{\omega_j} \zeta^2(\omega_j) P_i(\omega_j) \right) \\
&= \left(\frac{D-k_j}{\gamma+k_j} \right)^2 \left(\alpha_i^2 + 2\alpha_i k_j \frac{\alpha_i}{\gamma} + \frac{\alpha_i k_j (\gamma - \alpha_i + k_j(\alpha_i+1))}{\gamma(\gamma+1)} \right),
\end{aligned}
$$

the last equality follows by (21) and (22).

Simplifying E:

$$
\begin{aligned}
E &= -\sum_{\omega_j} \mathbb{E}_i [(\sum_{\omega_i \in \{0,1\}^{D-k_j}} \omega_i)^2 | \mathcal{I}_j] P_i(\omega_j) \\
&= -\sum_{\omega_j} \mathbb{E}_i [\zeta^2(\omega_j) | \mathcal{I}_j] P_i(\omega_j) \\
&= -\alpha_i \frac{D-k_j}{\gamma(\gamma+1)} \left[\gamma - \alpha_i + (D-k_j)(\alpha_i+1) \right]
\end{aligned}
$$

the last equality follows by (22).

Finally, putting all together and given the above simplifications, we have

$$
\begin{aligned}
\mathbb{E}_i[u_i|k_j] &= -b_i^2 + A + \underbrace{D + \underbrace{E + F}_{=C}}_{=B} \\
&= -b_i^2 + 2b_i \left(\frac{D-k_j}{\gamma+k_j} (\alpha_j - \alpha_i) \right) - \left(\frac{D-k_j}{\gamma+k_j} (\alpha_j - \alpha_i) \right)^2 \\
&\quad + \left(\frac{D-k_j}{\gamma+k_j} \right)^2 \left(\alpha_i^2 + 2\alpha_i k_j \frac{\alpha_i}{\beta} + \frac{\alpha_i k_j (\beta - \alpha_i + k_j(\alpha_i+1))}{\gamma(\gamma+1)} \right) \\
&\quad - \alpha_i \frac{D-k_j}{\gamma(\gamma+1)} \left[\gamma - \alpha_i + (D-k_j)(\alpha_i+1) \right] \\
&= -b_i^2 + 2b_i \left(\frac{D-k_j}{\gamma+k_j} \right) (\alpha_j - \alpha_i) - \left(\frac{D-k_j}{\gamma+k_j} \right)^2 (\alpha_j - \alpha_i)^2 \\
&\quad - \left(\frac{D-k_j}{\gamma+k_j} \right) \left(\frac{\alpha_i(\gamma-\alpha_i)(D+\gamma)}{\gamma(\gamma+1)} \right). \qquad (25)
\end{aligned}
$$

Define $\Lambda(k_j) = \frac{D-k_j}{\gamma+k_j}$. Then, (25) yields

$$
\begin{aligned}
\mathbb{E}_i[u_i|k_j] = -b_i^2 - 2b_i \Lambda(k_j)(\alpha_i - \alpha_j) - \big(\Lambda(k_j)(\alpha_i - \alpha_j)\big)^2 \\
- \Lambda(k_j) \frac{\alpha_i(\gamma-\alpha_i)(D+\gamma)}{\gamma(\gamma+1)} \qquad (26)
\end{aligned}
$$

Next, to prove the monotonicity and concavity of $\mathbb{E}_i[u_i|k_j]$ in k_j, first notice that the following inequalities are true:

$$
\Lambda(k_j) > 0, \quad \frac{\partial \Lambda(k_j)}{\partial k_j} < 0, \quad \frac{\partial^2 \Lambda(k_j)}{\partial k_j^2} > 0, \quad \frac{\partial \Lambda^2(k_j)}{\partial k_j} < 0, \quad \frac{\partial^2 \Lambda^2(k_j)}{\partial k_j^2} > 0. \qquad (27)
$$

Also by the assumption, $b_i(\alpha_i - \alpha_j) \geq 0$. Hence, by (27), it is immediate that (26) is a sum of monotonically increasing and concave functions. Therefore, $\mathbb{E}_i[u_i|k_j]$ is increasing and concave in S, completing the proof. □

J Pairwise stable equilibria

An equilibrium $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ is *pairwise stable*, if no pair of individuals can improve the communication pattern between them and increase their ex-ante expected utilities, while satisfying the truth-telling constraints and keeping other strategies fixed. To better understand how the communication pattern can be improved, consider the possible cases of information transmission from individual i to some individual j with in-degree k_j in G. If individual i reports truthfully to individual j, $g_{ij} = 1$, then there is no way to improve. If individual i does not report informatively to individual j, $g_{ij} = 0$, then it is possible to increase the ex-ante expected payoffs by inducing the truthful reporting through the link ij, if it is interim incentive compatible. Otherwise, there is no way of improvement. Given these alternatives, pairwise stability is formally defined as

Definition 4. *An equilibrium $\{\mathsf{G}, (\mu^{\mathsf{G}}, a^{\mathsf{G}})\}$ is pairwise stable if for any $i, j \in N$, $g_{ij} = 0$ only if, holding other strategies fixed, i cannot be credible in reporting to j, assuming that j believes i's message.*

Maximality and pairwise stability. Note, that the sets of maximal and pairwise stable equilibria might not coincide. In particular, under conflicting preferences, there might be maximal equilibria that are not pairwise stable. For example, let the prior distribution of θ be uniform on $[0, 1]$. Consider 3 individuals with the preference biases $b_1 = b_2 = 0$, $b_3 \in (\frac{2+D}{10}; \frac{2+D}{8}]$. Then there are several maximal equilibria that generate the in-degrees $k_1 = k_2 = k_3 = 1$. The examples of such truthful networks are: (1) $g_{12} = g_{21} = g_{13} = 1$, $g_{23} = g_{31} = g_{32} = 0$, and (2) $g_{12} = g_{23} = g_{31} = 1$, $g_{13} = g_{32} = g_{21} = 0$. It is easy to see that the first communication network corresponds to a pairwise stable equilibrium. In contrast, the second communication network doesn't correspond to a pairwise stable equilibrium, because individuals 1 and 2, who agree in their preferences, would deviate and induce truthful communication through the soft link 21.

Similarly, under conflicting beliefs, there exist pairwise stable equilibria that are not maximal. For such examples, see the equilibria discussed in the two communities setting.

Symmetry. Under conflicting preference, pairwise stability immediately implies that any two individuals i and j with the same preference biases must be treated in a symmetric way, i.e., they communicate truthfully with each other and receive the same number of truthful messages from other individuals. Under conflicting beliefs, individuals with the same priors might not be treated symmetrically in a pairwise stable equilibrium; but they are necessarily symmetric in any maximal equilibrium. In particular, they communicate truthfully with each other, get truthful reports from the same set of individuals and reveal their signals to the same set of individuals (by the positive externality effect).

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