Information Disclosure in Dynamic OTC Markets*

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Abstract

This paper studies how mandatory transparency (through TRACE), along with long term incentive of informed dealers, affect market price informativeness, liquidity and welfare in dynamic over-the-counter (OTC) markets. We show public disclosure of additional information about past trades, paradoxically, makes the markets more opaque, by reducing market price informativeness. Thus, surprisingly, the transparency requirements of the U.S. Dodd-Frank Act may make the markets more opaque. However, this market opacity creates liquidity and increases welfare. To enhance financial transparency via improving price informativeness as well as market liquidity and welfare, an effective way is to randomly audit dealers.

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1 Introduction

A large portion of trade occurs in dealer-based over-the-counter (OTC) markets for almost all financial assets.[1] A dealer-based OTC market does not use a centralized trading mechanism, such as an auction or limit-order book, to aggregate bids and offers and to allocate trades. An OTC trade negotiation is initiated when a trader (customer) contacts the dealer and asks for terms of trade. Next, the dealer making two-sided markets typically provides a take-it-or-leave-it pair of prices, a bid and an ask (offer), to a trader. He can then choose to accept it by immediately hitting the bid or lifting the ask, or reject it. For this reason, OTC markets are relatively opaque and traders are somewhat in the dark about the most attractive available terms and about whom to contact for those terms.[2]

A dealer-based OTC market is also complex because dealers have private information about asset values that dynamically change over time with the arrival of good and bad news (economic states).[3] This happens because the dealer frequently observes the order flow, negotiates trade terms and gathers information, while the trader (her customer) usually has more limited opportunities to trade and thus relatively less information about recent economic states. In this case, such disparity in market access is relatively common knowledge and tends to convey a bargaining power to the dealer.[4]

A common concern about OTC markets is their opaqueness. Given the important role that OTC markets played in the global financial crisis,[5] many regulators have at-

[1]For example fixed income securities are mainly traded over-the-counter, such as swaps, bonds and repos. For instance, Nagel (2016) finds that 95% of electronic swaps trades are over-the-counter. Moreover, Tuttle (2014) shows 16.99% of total dollar volume (18.75% of share volume) of National Market System (NMS) stocks is executed OTC without the involvement of an alternative trading systems. Common justifications for OTC trading include: regulatory barriers, nonstandardization and asset complexity.

[2]For empirical evidences supporting these see for example Green et al. (2007a), Ashcraft and Duffie (2007) and Massa and Simonov (2003). There are also several empirical works that show dealers quote prices have smaller spreads to customers who are likely to be uninformed (Linnainmaa and Saar (2012)), and OTC trades are less informative compared to trades on exchanges (Bessembinder and Venkataraman (2004)).

[3]Good news tends to have a positive effect on markets and one can see asset value's drift rising while its volatility falling soon after the news come out in the open. This story is quite opposite after a bad news. In fact, bad news tends to have a negative effect on markets so that asset value's drift falling while its volatility rising soon after the news come out in the open. See for example Kothari and Warner (1997), Fama (1998), Daniel et al. (1998) and Hong et al. (2000) that all have excellent synopses of the literature on stock price reactions to various events.

[4]There are several empirical evidences supporting imperfect competition in a dealer-based OTC market where a monopolistic dealer offers quote prices to (unsophisticated) customers. See, for example, Green et al. (2007a) that document dramatic variation across investors in the prices paid for the same municipal bond. See also Massa and Simonov (2003) that report dispersion in the prices at which different dealers trade the same Italian regulator bonds.

[5]For a reference, see Duffie (2012, 2017).

tempted to shed some light on those so-called dark markets. Perhaps the most notable reform aiming to increase transparency was the U.S. Dodd-Frank Act, implemented after the 2008 financial crisis.

The transparency requirements of the U.S. Dodd-Frank Act (through Trade Reporting and Compliance Engine (TRACE))[6] aims to promote market transparency through two types of regulations: (i) the public disclosure of some aggregate information on trading volumes; (ii) the public disclosure of previous transaction prices of standardized derivatives.[7] Also in order to reduce excessive risk taking behavior by managers, Dodd-Frank section 956 requires: (iii) long-term incentive based compensation schemes for managers (e.g., dealers). Can these mandates paradoxically lead to more market opacity? If Dodd-Frank transparency Act does have unintended effects then what types of policy regulations can lead to more market transparency in such environments? In this paper we provide answers to these questions.

In Section 2, we offer our parsimonious infinite-horizon discrete time model. In our model asset values change over times in a regime-switching framework. There are two sources of uncertainty that directly contribute to the asset value diffusions: (1) volatility shocks, and (2) drift shocks. To incorporate these uncertainties, we model the diffusion of the asset values by an $\mathsf{AR}(1)$ process and embed an ergodic Markov chain into it to model the economic states diffusion.[8] The current economic state is only known by the dealer. The dealer has the market power to quote prices and liquidity traders (customers) are the potential sellers or buyers of the asset in each period.[9] At each time, given the dealer's quote price a trader decides to accept or reject the proposed price offer. The dealer is strategic, forward-looking and risk-neutral, and traders are strategic and risk-averse. Therefore, any customer's trade with the dealer is motivated by the customer's risk sharing motive. Since traders are also fully rational, we focus on analyzing perfect Bayesian equilibrium in this dynamic trading game.

[6]See Title VII, Wall Street Reform and Consumer Protection Act.

[7]For some OTC markets, such as those for U.S. corporate and municipal bonds, regulators have mandated post-trade transparency (price and volume) through the publication of an almost complete record of transactions shortly after they occur. Empirical analyses of the implications of post-trade transparency (price and volume) in bond markets, through Trade Reporting and Compliance Engine (TRACE), include those of Edwards et al. (2007), Green et al. (2007a,b), Bessembinder and Maxwell (2008), Goldstein et al. (2007), Green et al. (2010) and Asquith et al. (2013). See Duffie (2012) for an excellent review of transparency requirements of the U.S. Dodd-Frank Act.

[8]Therefore, our model includes business cycles and macro economic (boom and recession) shocks as well.

[9]In the subsequent sections we let traders' trading positions stochastically change over time from a seller to a buyer and vice-versa.

Section 3 briefly considers the benchmark of the model where in each period the dealer is **privately informed** about the economic state, but she is **myopic** (i.e., dealer only cares about maximizing her within-period payoffs). We show depending on how desperate a trader is, there are two types of equilibria. When a trader is sufficiently conservative and dislikes volatility in asset values, meaning that trader's risk aversion is sufficiently high so that he is so desperate to hedge against the uncertainty shocks, there exists an opaque (pooling) equilibrium in which dealer fully **conceals** its private information. Of course, hiding information, whenever possible, is the best thing the dealer can do, as by doing so she gains the maximal information rent from her private information. Moreover, market liquidity (trade activity) and risk sharing in this opaque equilibrium increase, because trades occurs in both good times and bad times.

What if a trader's motive to hedge against uncertainty shocks is moderate? In this case the dealer is not able to conceal her private information. In fact, in order to persuade the trader to accept the price offer, the dealer should **reveal** her private information about the economic state, leading to a revealing (separating) equilibrium. Importantly, in contrast to the previous opaque equilibrium, now trades only occur in trader's favorable times (that is, in good times if traders are sellers and in bad times if traders are buyers). As a result, the amount of market liquidity, risk sharing and the dealer's ex-ante profit will all fall.

Section 4 analyzes the impacts of dealer's long term incentives. There the dealer is not only privately better **informed** about the economic state, but also **forward-looking** (i.e., the dealer also cares about future cash flows by discounting her continuation payoffs). Does the long-term trading with a forward-looking, risk-neutral, informed dealer improve price informativeness? We show that the answer is the opposite. This is because when the informed dealer is forward-looking the above opaque equilibrium becomes easier to sustain. To see this note that deviation to decline a transaction in trader's unfavorable times has a future cost for the dealer. Even though she can avoid the loss in the current period, in all future periods, the following traders observe her deviation and will expect to play the revealing equilibrium. The potential traders will adjust their expectations and she can only collect at most her revealing equilibrium payoff, achievable when she reveals her private information about the economic states in all future periods. As a result, in all future periods, she is unable to hide her private information and hence her expected continuation value is lowered after deviation. Such additional loss of continuation payoffs discourages dealer's deviation and makes the opaque pricing scheme easier to sustain under the dynamic trading setting. Therefore, price opacity increases with a long-term incentive of a forward-looking informed dealer.[10] Moreover, market liquidity and dealer's ex-ante profit will both rise.[11]

Section 5 considers how TRACE (i.e., public information disclosure of past trade history) affects dynamic OTC markets. There we add an important component of the model that **forward-looking risk-neutral informed dealer** trades with strategic traders when the past history of the **volumes and prices** may not be fully available (i.e., **private history**). Particularly, in line with the U.S. Dodd-Frank transparency Act of 2010 for corporate and municipal bonds and swaps (that follows TRACE's definition of transparency by requiring public dissemination of post-trade transaction information regarding price and volume),[12] we consider three cases:

1. Past prices are not observable (lack of post-trade price transparency).[13]
2. Both past prices and past transaction orders (trades) are not observable.
3. Past prices are not observable, but traders observe signals about past transaction orders (some aggregate information on past trading volumes).

We show that the transparency consequences of public information dissemination of **past transaction prices** can be different from **past trading volumes**. In the first case we show that the lack of knowledge about past prices has no impact on the structure of the above dynamic opaque equilibrium. Therefore, post-trade price transparency does not necessarily improve price efficiency. This result holds because the knowledge of past transaction orders are **sufficient statistics** for a previous deviation, implementing the

[10]Hence, this result suggests long-term incentive harms financial stability by reducing market price informativeness, which is in contrast with the Dodd-Frank section 956 that strongly encourages long-term incentive based compensation schemes for managers, inducing mangers (e.g., dealers) to become more forward-looking.

[11]We also show that there exist an equilibrium in which informed forward-looking dealer fully reveals its private information in all the trades. For this equilibria to exist, traders' risk aversion should be high enough to induce them to trade with the informed dealer in trader's unfavorable time as well. As a result, in contrast to the previous informative equilibrium, trade occurs in those times as well. However, whenever this equilibrium exists, the dynamic opaque equilibrium also exists. Hence, as the dynamic opaque equilibrium results in the maximum dealer's ex-ante payoff, this dynamic fully revealing equilibrium is not played.

[12]Similar reforms have been proposed for public transactions reporting known in U.S. regulation as swap execution facilities (SEFs). Japan and Europe (in the context of MiFID II and MiFIR that are more ambitious) have followed a course similar to that of the United States (Duffie (2017)).

[13]Post-trade price transparency for (almost) all U.S. corporate bonds and some of other fixed-income instruments was actually mandated by the SEC beginning in 2002, via Transaction Reporting and Compliance Engine (TRACE).

dynamic opaque equilibrium with no change. As a consequence, with post trade price transparency, trade activity may not rise.[14]

More surprisingly, in the second and the third cases, we show that the public information disclosure about previous transaction orders (volumes) paradoxically makes the market more opaque via the reduction of price informativeness. This is because when information about past transaction orders is available, the dealer can use it as a **commitment device**. As a result, the opaque trading equilibrium is easier to sustain, impairing price informativeness. Moreover, since in opaque equilibrium trades occur in both good and bad times, post-trade public disclosure regarding *volumes* via TRACE can improve liquidity (trading volume).[15]

In Section 6 we consider the social welfare and dealer's profit, and propose effective policies such as *randomly auditing dealers* to improve financial transparency through increasing the market price informativeness and at the same time achieving the maximal market liquidity with the highest feasible social welfare in the markets. We show increasing auditing intensity can force the dealer to reveal her private information about economic states more frequently and as a result, lead to more price transparency in OTC markets. Section 6.3 considers the expected surplus and welfare. We show that the expected surplus of informed dealer can increase with the disclosure of additional information about trade history. This result may appear surprising, as it goes against the general lesson of contract theory that less disclosure gives more information rent to the party with private information. However, it holds because with more information disclosure about past trades, opaque trading equilibrium is easier to sustain, impairing price efficiency and improving dealer's ex-ante profit.

Moreover, in line with Dang et al. (2012), we show that market ignorance increases market liquidity and expected welfare. In Dang et al. (2012) market ignorance is due to less public information and complexity of financial instruments. In contrast, however, our market ignorance is due to more public information disclosure about past trades that leads to less market price informativeness. Hence, our welfare result also suggests that there can be a tradeoff between price informativeness and social welfare in OTC markets.

In Section 7 we extend and demonstrate the robustness of our model in four directions. First, we show that our main conclusions do not depend on deliberate functional forms and can go beyond the mean-variance utility functions. Second, we extend the

[14] For empirical evidence, see Asquith et al. (2013).
[15] For empirical evidence, see Bessembinder and Maxwell (2008).

analysis to the case where traded orders are divisible. We show divisible trades have no impact on the opaque equilibrium structure. However, in contrast to the discrete order case, in the revealing equilibrium trade also occurs in trader's unfavorable times (however, its amount is still strictly less than that in the other times). But other than this nothing changes structurally. Third, we allow trader's demand shock stochastically change over times, meaning that the trader stochastically changes from a seller to a buyer and vice-versa.[16] We show that our results are robust to such a change. Finally, we present the sufficient and necessary condition for the existence of a semi-opaque trading equilibrium in static game. In the semi-opaque trading equilibrium, the dealer always trades with traders in trader's favorable times and mixes between trading and not trading in the other times. The expected ex-post social welfare of this class of equilibrium lies between that of opaque equilibrium and that of informative equilibrium, and increases in the probability of trading in trader's unfavorable times. This result reinforces our previous one that there is a trade-off between price informativeness and social welfare.

Finally, Section 8 concludes.

1.1 Related Literature

This paper is part of the growing literature on dealer-based OTC markets. Prior literature on OTC markets focuses on the dealers' ability to contract with customers (Grossman and Miller (1988)), discriminate based on order size (Seppi (1990)), welfare effects of decentralized trading (Malamud and Rostek (2017), Babus and Parlatore (2017)), price movements in OTC markets when block orders are large (Grossman (1992)), searching for good price in OTC markets with multiple dealers (Zhu (2012)), random search and matching in large markets among a continuum of traders (Duffie et al. (2005); Lagos and Rocheteau (2009)), adverse selection with search frictions and discrete trading opportunities (Guerrieri and Shimer (2014)) and less competition in equity trades that are sent to dark pools (Zhu (2014)).[17] In contrast to these works our focus here is on the role of public information disclosure in price efficiency (informativeness), market liquidity and social welfare.

Our work also contributes to the literature of information disclosure in financial markets.[18] A few recent papers present models in which disclosure can harm price in-

[16]That is, traders' demand shock is modeled as a binary stochastic process changing over time between seller and buyer.

[17]See Duffie (2012) for an excellent review of the literature on dealer based OTC markets.

[18]See Verrecchia (2001), Goldstein and Sapra (2013) and Goldstein and Yang (2017) for excellent reviews

formativness, although through different channels. Important works by Goldstein and Bond (2015) and Goldstein and Yang (2018) construct noisy REE models and show that the type of information being disclosed is important in determining whether disclosure is desirable. In particular, when disclosure is about a variable that the real decision maker cares to learn, disclosure can harms price informativeness. In the same realm of REE models, Banerjee et al. (2018) show that lowering the cost of information concerning the fundamentals may induce traders to learn more about noise. As a result, information efficiency may decrease. Amador and Weill (2010) construct a monetary model and show that releasing public information about productivity shocks can reduce welfare through reducing price informativeness. In contrast to these models, our framework is on dynamic OTC markets with informed forward-looking dealers (market makers) distinguishing from the literature in both the content and the techniques. Our results highlight the importance of dealer's forward-looking incentives on price informativeness. Moreover, the paper demonstrates different implications of disclosing past prices v.s. trade volumes (via TRACE). We show that public disclosure of additional information about past trades, paradoxically, makes the markets more opaque, by reducing market price informativeness. However, this market opacity creates trade activity and increases welfare.[19]

2 Model

We consider an infinite horizon dynamic trading game between an informed, risk-neutral and forward-looking **dealer** (insider, market maker) and a series of uninformed, risk-averse and myopic **traders** (sellers or buyers, customers). Time is discrete $t \in \{1, 2, 3, \cdots\}$. At each period, a trader comes to the dealer, either possessing an asset

on this topic.

[19]Moreover, our market liquidity results contribute to the literature of market liquidity with financial frictions. In particular, other studies have also argued that recent liquidity in bond markets has been improved by crisis-induced regulations (see e.g. Section 619 part of the Dodd-Frank Act) (Adrian et al. (2013); Duffie (2017)) and capital and leverage ratio requirements for banks (Caballero (2010)). In this regard other theories of market liquidity have been proposed. See Brunnermeier and Pedersen (2005) for predatory trading, Gorton and Pennacchi (1990) for adverse selection in secondary debt markets, Shleifer and Vishny (1992, 1997); Kiyotaki and Moore (1997) when the natural buyer of an asset experiences financial stress, Caballero and Krishnamurthy (2008) for Knightian uncertainty, Guerrieri and Lorenzoni (2009) for precautionary behavior on business cycle fluctuations, (Bebchuk and Goldstein, 2011; Goldstein, 2012) for panics, self-fulfilling crisis and credit freezes, Ahnert and Kakhbod (2017) for costly private-information acquisition. In contrast to these theories, our results on improving market liquidity is due to a new channel based on the extent of available information about past transactions, dealer's long-term incentive, and the customer's hedging motives.

or desiring an asset to hedge his other investments. The market asset value a_t changes with the arrival of good and bad news over time. Suppose θ_t represents the underlaying economic **state**, which is known to the dealer but not traders. The dealer has the bargaining power and in each period offers a take-it-or-leave-it price p_{t,θ_t} to the trader. The trader then makes a decision o_t of whether to accept the dealer's offer or not. The size of the asset traded in each period is normalized to 1, and we extend the analysis to divisible orders in Section 7.2.

The main components of the model are discussed in details below.

2.1 Good Times and Bad Times: Asset Value Dynamics

The asset value at time t, denoted by a_t, is given by the following dynamic:

$$a_t = \varphi a_{t-1} + \text{innov}_t$$

where $\varphi \in (0, 1]$ is the persistence coefficient, and innov_t denotes the stochastic **innovation** in the asset value at time t:

$$\text{innov}_t = J_{\theta_t} + \sigma_{\theta_t} z_t. \tag{1}$$

z_t (idiosyncratic shock) is an independent and normally distributed process with mean zero and variance one, i.e. $z_t \sim \mathcal{N}(0, 1)$. θ_t is an ergodic Markov chain that takes values from $\Theta \equiv \{\mathrm{g}, \mathrm{b}\}$, where g stands for good times and b is for bad times. In good times the asset value has a higher mean and a lower variance, while in bad times the asset value has a lower mean and a higher variance. J_{θ_t} denotes the state-dependent **drift** and σ_{θ_t} denotes the state-dependent **volatility** of the innovation shock at time t when the economic state is θ_t. In other words, we assume

$$J_{\mathrm{g}} > J_{\mathrm{b}} \quad \text{and} \quad \sigma_{\mathrm{g}} \leq \sigma_{\mathrm{b}}. \tag{2}$$

We assume that the state transition follows a Markov process (illustrated in Figure 1):

$$\begin{aligned} &\text{Prob}\{\theta_t = g | \theta_{t-1} = g\} = \alpha_g, && \text{Prob}\{\theta_t = b | \theta_{t-1} = g\} = 1 - \alpha_g, \\ &\text{Prob}\{\theta_t = g | \theta_{t-1} = b\} = \alpha_b, && \text{Prob}\{\theta_t = b | \theta_{t-1} = b\} = 1 - \alpha_b, \end{aligned} \tag{3}$$

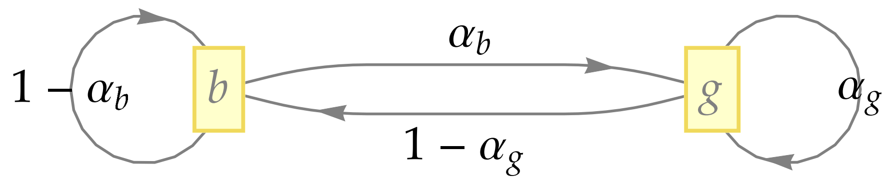

Figure 1: Markov chain dynamics (transition probabilities) between good times g and bad times b.

where θ_t is the economic state in period t and $\alpha_g, \alpha_b \in (0,1)$. In addition, z_t and θ_t are independent, i.e. $z_t \perp \theta_t$, for all t. We assume that $J_g, J_b, \sigma_g, \sigma_b, \alpha_g, \alpha_b$ and φ are common knowledge to both parties. Moreover, at the end of period t, asset value a_t and the economic state θ_t become publicly observable after the trade.

2.2 Demand Shocks

At the beginning of period t, the trader's demand shock χ_t is independently drawn from $\{-1, 1\}$ such that

$$\text{Prob}\{\chi_t = 1\} = \beta \in [0,1], \quad \text{Prob}\{\chi_t = -1\} = 1 - \beta,$$

where $\chi_t = 1$ represents that the trader coming at period t possesses an extra unit of risky asset to sell, while $\chi_t = -1$ represents that he is in need of a unit of risky asset, and without purchasing one, he has to pay the realized value of the asset to someone else. The trading position χ_t is also observable to the dealer.

Hence, at period t, a trader has three order decisions $o_t \in \{-1, 0, 1\}$: $o_t = -1$ means that the trader accepts the bid offer and sells the asset to the dealer[20] ; $o_t = 1$ means that the trader accepts the ask offer and buys the asset from the dealer; $o_t = 0$ represents that the trader declines the dealer's offer. Given the trader's trading position, a trader who is in need of an asset will never accept a bid offer from the dealer, and a trader who possesses an extra asset will not like to accept an ask offer. This puts restriction on the feasibility of $\{o_t\}_{t=1}^{\infty}$:

Case (1): When $\chi_t = 1$, the trader is a potential seller. Hence, $o_t \in \{0, -1\}$, i.e., $o_t = 0$ means keeping the asset and $o_t = -1$ means selling the asset.

[20] Here we use negative value of o_t to represent that by selling the asset, the trader actually loses one unit of the asset.

Case (2): When $\chi_t = -1$, the trader is a potential buyer. Hence, $o_t \in \{0, 1\}$, where $o_t = 0$ means rejecting dealer's price offer and $o_t = 1$ means buying the asset from the dealer.

2.3 Information Disclosure: Public vs. Private history

In order to analyze the effect of information disclosure, we compare two variants about the observability of the past history of trades.

In our **public history** model, the past history of trades (including prices and transaction orders) is publicly observable. In other words, we assume that past transactions are public and future traders can view prices offered to previous traders and whether they are accepted or not.

The assumption that previous trade history is publicly observable is relaxed in several directions in Section 5, where we present the **private history** variant of our model. In Section 5.1 we discuss the case where although past prices are unobservable, future traders can still perfectly learn whether a previous trade happens, that is, only the bilateral transaction price is kept private from the informed dealer and the trader who received it. In Section 5.2 we further relax that assumption and examine a case where future traders can only observe imperfect signals about whether trade happens before. As we will show in Section 5, learning previous prices has no effect on the equilibrium behaviors. It is the information of orders (volumes) that matters.

2.4 Beliefs

Let h^t denote the past history of the trades available at the beginnings of period t. Each trader is rational. Strictly speaking, at the beginning of each period t, he Bayesian updates his prior belief about the economic state $\theta_t = (J_{\theta_t}, \sigma_{\theta_t})$ from the past history, h^{t-1}, and the new price offer, p_{t,θ_t}, from the dealer,

$$\zeta(p_{t,\theta_t}; h^{t-1}) \equiv \text{Prob}\{\theta_t = g | h^{t-1}, p_{t,\theta_t}\} = \frac{\text{Prob}(\theta_t = g | h^{t-1})\text{Prob}(p_{t,\theta_t} | \theta_t = g, h^{t-1})}{\sum\limits_{\theta_t} \text{Prob}(\theta_t | h^{t-1})\text{Prob}(p_{t,\theta_t} | \theta_t, h^{t-1})}.$$

2.5 Payoffs

In our main model we assume that dealer is risk-neutral and trader has mean-variance preferences. In Section 7.1 we will show that our main conclusion does not depend on specific functional forms.

Traders' payoffs. Each trader's end-of-period wealth is given by

$$w_t^{\chi_t} = \chi_t a_t + o_t \cdot (a_t - p_{t,\theta_t}), \tag{4}$$

The myopic, rational trader is **risk-averse**. His ex-ante utility at the beginning of period t, given the available information about past trades, and the price offer, p_{t,θ_t}, from the dealer, is given by

$$\mathbb{E}\left[w_t^{\chi_t} | h^{t-1}, p_{t,\theta_t}\right] - \frac{\rho}{2}\mathrm{Var}\left[w_t^{\chi_t} | h^{t-1}, p_{t,\theta_t}\right], \tag{5}$$

where ρ denotes his risk aversion coefficient. Therefore, as trader dislikes the volatility of his end of period wealth, he has an incentive to trade with the dealer due to this risk sharing motive.

Dealer's payoff. The end of period wealth of the **risk-neutral** dealer is also given by

$$u_t \equiv (p_{t,\theta_t} - a_t) \cdot o_t.$$

The dealer, instead, is **forward-looking** and her utility, given the public history, h^{t-1}, and her private information about the economic state θ_t, is given by:

$$\begin{aligned} U_t &= (1-\delta)\mathbb{E}\left[\sum_{s \geq t} \delta^{s-t}(p_{s,\theta_s} - a_s) \cdot o_s \middle| h^{t-1}, \theta_t\right] \\ &= (1-\delta)\mathbb{E}[u_t | h^{t-1}, \theta_t] + \delta\mathbb{E}[U_{t+1} | h^{t-1}, \theta_t], \end{aligned} \tag{6}$$

where $\delta \in [0, 1)$ is dealer's discount factor. $\delta = 0$ represents the case where the dealer becomes fully myopic like traders.

2.6 Equilibrium Concept and Refinement Criterion

Throughout this paper, the solution concept considered is perfect Bayesian equilibrium, defined below.

Definition 1. *A perfect Bayesian equilibrium (PBE) $\{p^*(\cdot), o^*(\cdot), \zeta^*(\cdot)\}$ consists of the dealer's price offer, $p^*(\cdot)$, the trader's order decision, $o^*(\cdot)$, and the trader's posterior belief about the* *economic state θ_t, $\zeta^*(\cdot)$, such that the following properties hold:*

- *The dealer chooses her optimal price offer $p_{t,\theta_t} \equiv p^*(\theta_t; h^{t-1})$ to maximize her expected utility (see Eq. (6)) given the public transaction history h^{t-1}, her private information about θ_t, and trader's order decision $o^*(\cdot)$.*

- *Given any public history h^{t-1} and the price offer p_t proposed by the dealer, the trader updates his posterior belief about the economic state θ_t according to the Bayes's rule, whenever it applies.*

- *The trader chooses his optimal order decision $o^*(p^*_{t,\theta_t}, \zeta^*; h^{t-1}) \in \{1, 0, -1\}$ to maximize his expected utility (see Eq. (5)), given the price offer p^*_t, the public transaction history h^{t-1}, and his posterior belief $\zeta^*(\cdot)$ about the underlying economic state θ_t.*

In the case of multiple equilibria, since the informed dealer moves first, intuitively, we apply a refinement specifying the maximal ex-ante utility for the dealer. We call such equilibria *maximal PBE*. This refinement follows the same spirit as the undefeated criterion proposed by Mailath et al. (1993).[21] Moreover, the maximal PBE not only are optimal for the informed party who moves first, they also generate the highest ex-ante social welfare. A formal version of the selection criterion is presented below.

Definition 2 (Undefeated Criterion and Maximal PBE)**.** *We say that a pure strategy PBE $(p(\cdot), o(\cdot), \zeta(\cdot))$* ***defeats*** *another pure PBE $(p'(\cdot), o'(\cdot), \zeta'(\cdot))$ if and only if*

- *in a one-time signaling game, for any trader's initial prior belief ζ_0, we have*

$$U(p(\cdot), o(\cdot), \zeta(\cdot)) \geq U(p'(\cdot), o'(\cdot), \zeta'(\cdot));$$

- *in a repeated signaling game with perfect monitoring, along the equilibrium path, for any t, we have the following relationship between the continuation payoffs,*

$$U_t(p(\cdot), o(\cdot), \zeta(\cdot)) \geq U_t(p'(\cdot), o'(\cdot), \zeta'(\cdot)),$$

  and the inequality is strict for some t.

We call a pure PBE a ***maximal PBE*** *if it is defeated by no other pure PBE. A PBE outcome is called a maximal PBE outcome if it is obtained through a maximal PBE.*

[21] In fact, as shown in the proof, in the static case, the undefeated criterion in Mailath et al. (1993) picks the same set of maximal PBE. Several signaling models use this concept (or stronger versions of it), e.g., Hartman-Glaser (2017) and Carapella and Williamson (2015).

With the above setups, we are ready to show how the public disclosure of past trades (via TRACE), dealer's long-term incentive and the trader's hedging motives can drastically change the nature of market price informativeness, market liquidity and welfare in dynamic OTC markets.

Remark 1. *For the ease of explanation, we first study a case where $\beta = 1$, that is, traders are always potential sellers who possess risky assets (see Section 2.2). Then in Section 7.3, we show that this case is exactly mirrored by the one where sellers are buyers and need risky assets to hedge, i.e., $\beta = 0$. We also analyze the situation where traders' trading positions stochastically change from period to period, i.e., $\beta \in (0, 1)$.*

3 Benchmark: Static Trading

To better understand how trade history and long term incentive of dealers affect OTC markets, in this section we briefly consider our benchmark case where trade history is public, and in each period t the dealer is informed but myopic, with the discount factor $\delta = 0$. Before the trade in each period, the dealer learns the underlying economic state θ_t, which affects the asset value a_t.[22]

3.1 Opaque Static Trading Equilibrium

As formally shown later in Section 6.3, any pricing strategy that increases traders' uncertainty and hides her private information allows the dealer to extract more information rent from them. We call such a *pooling* equilibrium where the dealer can hide her private information about current underlying economic state an **opaque static trading equilibrium** (OSTE).

Definition 3. *If the dealer and traders are both myopic, then an equilibrium is an opaque static trading equilibrium (*OSTE*) if and only if on the equilibrium path, the dealer's pricing strategy $\{p_t(h^{t-1})\}_{t=0}^{\infty}$ is independent of her private knowledge about the drift and the volatility in asset values, that is, independent of the current underlying economic state θ_t.*

In other words, dealer offers opaque prices that are same in good times and in bad times. The following proposition specifies the necessary and sufficient condition for the existence of an OSTE.

[22] The case where in each period t the dealer is (like traders) fully uninformed about the realization of the economic state θ_t is considered in Appendix A.

Proposition 1 (Opaque Static Trading Equilibrium). *The opaque static trading equilibrium* (OSTE) *exists if and only if*

$$\rho \geq \rho^{\mathsf{OSTE}} = 2\frac{\alpha_{\max}(J_g - J_b)}{\alpha_{\max}(1-\alpha_{\max})(J_g - J_b)^2 + \alpha_{\max}\sigma_g^2 + (1-\alpha_{\max})\sigma_b^2} \tag{7}$$

where $\alpha_{\max} = \max\{\alpha_g, \alpha_b\}$.[23]

Proof. See Appendix C. □

Proposition 1 shows that opaque pricing strategy is feasible if and only if traders are conservative enough to hedge their risky assets via trading with the dealer. In other words, opaque pricing equilibrium can be sustained if traders' risk aversion coefficient is sufficiently large (i.e., $\rho \geq \rho^{\mathsf{OSTE}}$) and traders, who sufficiently dislike volatilities in their asset values, will accept relatively low offers from the dealer. In bad times, the dealer may have an incentive to deter the trade if it is not profitable. Therefore, when traders are risk-averse enough, the possibility to buy the asset at a low enough price then makes the transaction in bad times more likely to be profitable and prevent the rejection from the dealer.

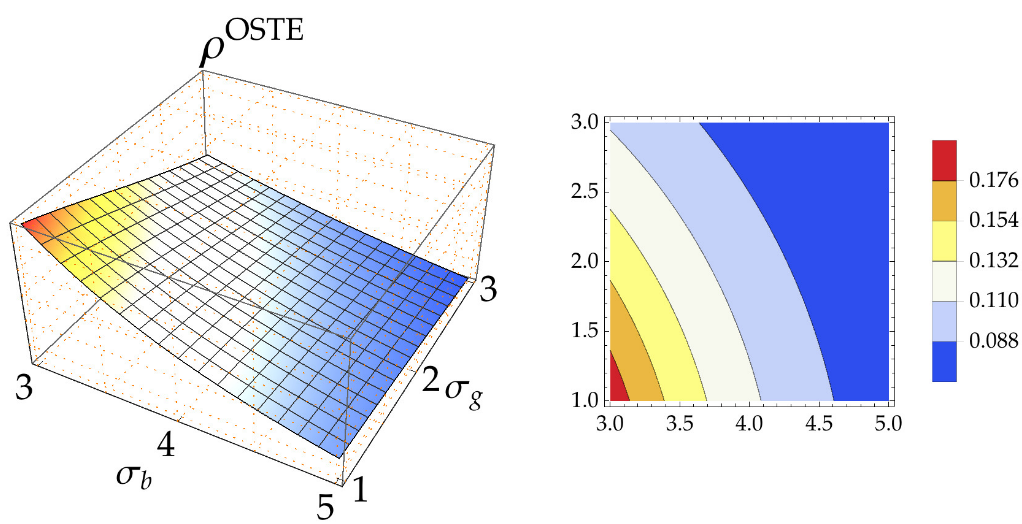

Figure 2: This chart plots ρ^{OSTE} for $\alpha_g = \alpha_b = \frac{1}{2}$ and $J_g - J_b = 1$. The area above the surface represents the region where an OSTE exists. From the graph, ρ^{OSTE} decreases in the asset innovation volatilities in both good and bad times (i.e., σ_g and σ_b).

[23] The reason that $\alpha_{\max}$ is appeared in the ρ^{OSTE} is because the trader's individual rationality constraint needs to be satisfied when he believes the current economic is good both with probability α_g and with probability α_b. Hence, the larger one is binding. See the proof for more details.

Proposition 1 also implies some interesting comparative statics, summarized in Corollary 1 (see also Figure 2). For example, as the asset innovation becomes more volatile, either in good or in bad times (i.e., σ_g or σ_b increases) traders are more likely to accept a certainty price offer and an OSTE exists even if traders are less risk-averse. In other words, the equilibrium becomes easier to sustain and the cutoff risk aversion coefficient ρ^{OSTE} decreases. In addition, the equilibrium is harder to sustain with a larger $\alpha_{\max}$. To see the intuition, consider an extreme case where $\alpha_g = \alpha_b$. If the trader believes the current period is more likely to be in good times, then his uninformative expectation about the asset is closer to the higher one in good times, hence the trader is more likely to refuse the trade in bad times. Therefore, the dealer's individual rationality constraint in bad times is harder to fulfill and the equilibrium is easier to break down.

Corollary 1. *The threshold ρ^{OSTE} always decreases in the asset innovation volatilities in both good and bad times (i.e., σ_g and σ_b). It also monotonically increases in $\alpha_{\max}$.*

Finally, it is worth noting that among all the OSTE, there exists one providing the dealer with the highest ex-ante expected payoff. By this opaque pricing strategy, dealer not only obtains the benefit of insurance due to the residual risk $\sigma_{\theta_t} z_t$ (which is equal to $\frac{\rho}{2}\left[\mathrm{Var}[\sigma_{\theta_t} z_t | h^{t-1}]\right]$) but also receives her information rent for shocks in the drift of asset innovations J_{θ_t} (which is equal to $\frac{\rho}{2}\left[\mathrm{Var}[J_{\theta_t} | h^{t-1}]\right]$). Hence, overall her ex-ante surplus in this Pareto dominant OSTE becomes

$$\begin{aligned} U_t^{\mathsf{OSTE}} &= \frac{\rho}{2}\left[\mathrm{Var}[\sigma_{\theta_t} z_t | h^{t-1}] + \mathrm{Var}[J_{\theta_t} | h^{t-1}]\right] \\ &= \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2]. \end{aligned}$$

From now on we denote this payoff as dealer's OSTE payoff.

3.2 Informative Static Trading Equilibrium

What happens if traders are not so risk-averse to hedge against their uncertainty shocks? One scenario is that they may reject the opaque offers that are sufficiently low. However, since dealer's outside option is normalized to zero, she will always be better off to trade as long as it provides non-negative interim gains. This drives our attention to another kind of equilibrium where dealer sacrifices her information rent to induce trade in good times. We call such a *separating* or *revealing* equilibrium an **informative static trading equilibrium** (ISTE).

Two important observations are worth noting.

- Our first observation is that $p_t(g; h^{t-1}) \equiv p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. On the one hand, this is trader's highest evaluation of the asset, given any belief. That is, trader values his asset most when he is most optimistic and believes now is in good times for sure. Therefore, he will accept any offer weakly above this, and it's suboptimal for the dealer to offer a price strictly above it. Therefore, in any ISTE, $p_t^g \leq \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. On the other hand, in any ISTE, dealer offers different prices at different times, and a Bayesian trader, after observing p_t^g, will believe that the current period is in good times. Therefore, his individual rationality constraint implies that $p_t^g \geq \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. Together we have $p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ in any informative equilibrium where dealer chooses to price discriminatingly.

- Our second observation is that in an ISTE, trade only occurs in good times. Because otherwise in good times the dealer can offer a lower price p_t^b and persuade the trader the current economic state is bad and he should sell his asset at a lower price. By doing so she can purchase his asset at a lower price. To discourage such deviation, trade can not happen in bad times.

Therefore, we just show all ISTE are payoff equivalent and it is without loss of generality to restrict on-path prices as follows.

Definition 4. *If the dealer and the trader are both myopic, then an equilibrium is called informative static trading equilibrium (*ISTE*) if and only if the dealer chooses a pricing strategy* $\{p_t(\theta_t, h^{t-1})\}$ *such that* $p_t(g, h^{t-1}) = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ *and* $p_t(b, h^{t-1}) < \varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2$.

Given the above observations, the next proposition characterizes the necessary and sufficient conditions for the existence of ISTE.

Proposition 2 (Informative Static Trading Equilibrium). *Suppose the dealer is myopic (i.e.* $\delta = 0$*) and is informed about the current economic state (i.e.* θ_t*). If and only if*

$$\rho < \rho^{\mathsf{ISTE}} = 2\frac{J_{\mathrm{g}} - J_{\mathrm{b}}}{\sigma_{\mathrm{g}}^2}, \tag{8}$$

there exists an ISTE.

Proof. See Appendix C. □

The intuition behind (8) is as follows. Similar to the argument that dealer in good times should not be encouraged to mimic dealer in bad times, by symmetry dealer in bad times also should not have the incentive to offer p_t^g to induce trade. This provides a lower bound on the price offered in good times, p_t^g, which characterizes the upper bound of trader's risk aversion coefficient ρ for the existence of an ISTE.

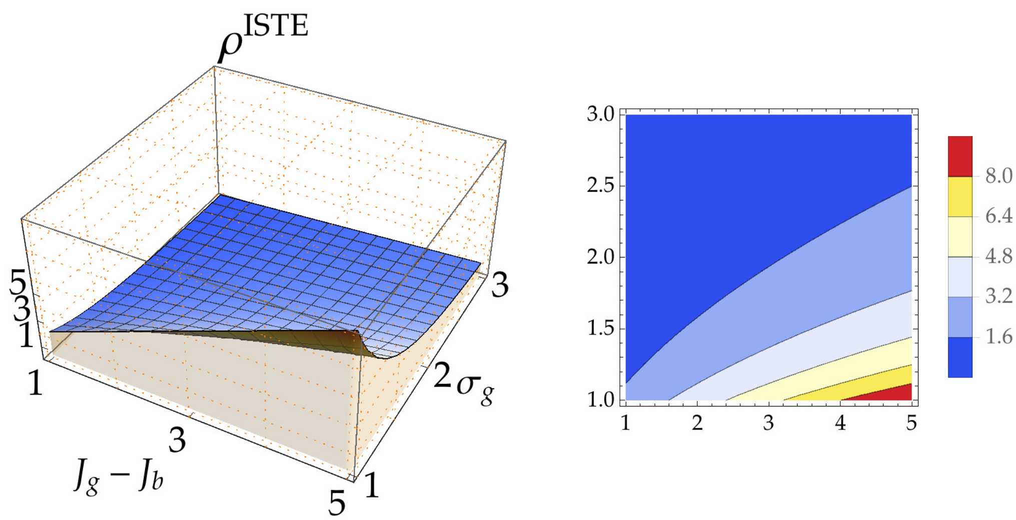

Figure 3: This chart plots ρ^{ISTE} for $\alpha_g = \alpha_b = \frac{1}{2}$. The gray area bellow the surface represents the region where ISTE exists. It also shows ρ^{ISTE} increases in the spread $J_g - J_b$ and decreases in the asset volatility in good times σ_g. Unlike ρ^{OSTE}, however, ρ^{ISTE} is independent of σ_b.

Proposition 2 also implies some interesting comparative statics, summarized in Corollary 2 (See also Figure 3). For example, it shows that the threshold ρ^{ISTE} always decreases in the asset innovation volatility in good times, i.e., σ_g, and increases in the jump spread $J_g - J_b$. Hence, decreasing the volatility σ_g and increasing the spread $J_g - J_b$ both expand the domain of trader's risk aversion coefficient (i.e., its hedging motive) for which an ISTE exists. Most importantly, since trades only occurs in good times, this threshold is independent of asset volatility in bad times, i.e., σ_b.

Corollary 2. *The threshold ρ^{ISTE} decreases in the asset innovation volatility in good times, i.e. σ_g. It does* not *depend on the asset innovation volatility in bad times, i.e., σ_b. Finally, it increases in the spread $J_g - J_b$.*

It is also worth noting that, since in any ISTE trades only occur in good times, the dealer's ex-ante surplus purely comes from her insurance due to the residual risk in good times, that is, $\alpha_{\theta_{t-1}} \frac{\rho}{2} \sigma_g^2$. Since in ISTE dealer reveals her private information about economic states, her ex-ante surplus is lower than that in the opaque equilibrium. Moreover, the price offer in good times increases in the drift of good times shocks J_g and decreases in the risk aversion coefficient ρ, as well as the volatility in good times σ_g.

Finally, the next corollary compares the thresholds ρ^{OSTE} and ρ^{ISTE}.

Corollary 3. *When the informed dealer is myopic, the threshold of* ISTE *is strictly larger than that of* OSTE*. In other words*

$$\rho^{\mathsf{ISTE}} > \rho^{\mathsf{OSTE}}. \tag{9}$$

In addition, for any $\rho \geq \rho^{\mathsf{OSTE}}$, there is a unique maximal PBE outcome, achieved only via an OSTE*. For any $\rho \in [\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$, such an* OSTE *defeats the* ISTE*.*

Proof. See Appendix C. □

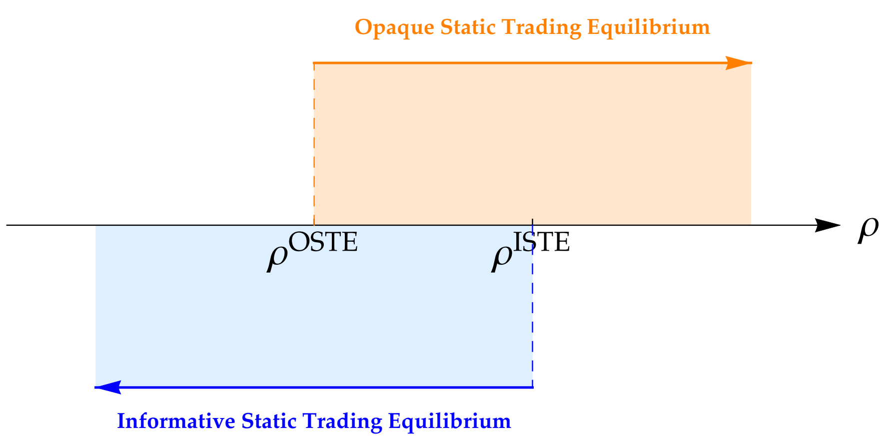

Figure 4: This chart plots the equilibria when the informed dealer is myopic. Depending on the extent of trader's hedging motive there are two equilibria. When his risk aversion is low (i.e., low hedging demand or $\rho < \rho^{\mathsf{ISTE}}$), the dealer reveals her information but trade only occurs in good times, reducing liquidity, risk sharing, and the dealer's ex-ante surplus. When trader's risk aversion coefficient is high (i.e., high hedging demand or $\rho > \rho^{\mathsf{OSTE}}$) the dealer will conceal her private information and trades happen in both times, leading to the dealer's highest ex-ante surplus. These thresholds (i.e., ρ^{ISTE} and ρ^{OSTE}) are pinned down (explicitly) by drifts and volatility of both times, as well as transition probabilities of the regime switch.

Corollary 3 implies that $\rho^{\mathsf{ISTE}} > \rho^{\mathsf{OSTE}}$. Hence, for all ρ between ρ^{OSTE} and ρ^{ISTE}, both types of equilibria (i.e., OSTE and ISTE) exist in the static trading game. Nevertheless, according to the undefeated criterion, there always exists an OSTE that defeats the

ISTE, making the OSTE the unique maximal PBE. In addition, as shown later, the former also generates higher social surplus and for any risk aversion coefficient ρ, there exists an OSTE that Pareto dominates ISTE.

Remark 2. *The results in this section hold no matter the history of past trades are public or private. In other words, if the dealer is myopic, only static trading equilibria discussed in this Section, as well as the semi-opaque ones in Section 7.5, will be played.*

4 Dynamic Trading: Public History

So far we have shown that when traders are sufficiently conservative to hedge against uncertainty shocks, an informed but myopic dealer can conceal her private information about economic states (OSTE). Next, motivated by Dodd-Frank (section 956), we consider how long-term incentives of dealers affect the sustainability of opaque pricing strategies and the transparency in OTC markets. In particular, we ask: Does long-term incentive of informed dealers improve market price informativeness? To answer this question, we extend the benchmark case and allow the informed dealer to be *forward-looking*, i.e., she also cares about future cash flows and has a positive discount factor $\delta > 0$.

4.1 Opaque Dynamic Trading Equilibrium

This section shows that dealer's forward-looking incentive enables the threat of the loss of future profits and provides another device to deter her deviation, making opaque pricing equilibria easier to sustain and reducing the price efficiency. We also characterize the sufficient and necessary condition for the existence of such equilibria.

Section 3 shows that OSTE holds in the one-shot game when traders are risk-averse enough ($\rho \geq \rho^{\mathsf{OSTE}}$). Thus, it also consists of a subgame perfect equilibrium in the dynamic trading game. Corollary 3 implies that $\rho^{\mathsf{ISTE}} > \rho^{\mathsf{OSTE}}$, so opaque pricing is a static Nash equilibrium for $\rho > \rho^{\mathsf{ISTE}}$. We now focus our attention on a non-trivial case $\rho \leq \rho^{\mathsf{ISTE}}$ and examine whether the opaque equilibrium can be sustained under the dynamic setting. That is, throughout this subsection, we make the following assumption.

Assumption 1. *Assume $\rho \leq \rho^{\mathsf{ISTE}}$ and an informative static trading equilibrium (ISTE) always exists.*

Next, we consider a class of equilibrium with forward-looking dealer and along the equilibrium path, the dealer can still conceal her private information about current economic state and provide an opaque price at any history. We call such an equilibrium an **opaque dynamic trading equilibrium** (ODTE).

Definition 5. *If the informed dealer is forward-looking and the traders are uninformed and myopic, then a PBE is called an opaque dynamic trading equilibrium (*ODTE*) if and only if on the equilibrium path, in each period, the dealer offers a price $p_t(h^{t-1})$ that does not depend on her private information about the economic state θ_t.*

Since in an ODTE price offers reveal *no* information about the economic states, dealer's offers can only depend on the public history. To sustain such an equilibrium to the greatest extent, that is, to deter the decline of the trade by the dealer in bad times, one needs to implement the harshest punishment for such deviations. First, the harshest punishment should involve grim trigger strategies. That is, once the dealer deviates, she will be punished in all future periods. Second, the deviator's expected one-shot payoff in the punishment stage should be as low as possible.

In order to have robust results against any off path construction, we want to use the minmax profile. But we also need to make sure that there is no incentive to deviate on the punishment stage. Fortunately, in this dynamic trading game, as shown in the following lemma, this minmax profile turns out to be the ISTE, which is a Nash equilibrium of the one-shot game and has the subgame perfect property in the dynamic setting.

Lemma 2.2. *For any conditional belief $\zeta(p_t;\cdot)$, the dealer's ex-post* ISTE *payoffs consists of the min-max static payoffs given that the trader best responses to her strategy. Specifically, we have*

$$
\begin{aligned}
\min_{\zeta(p_t;h^{t-1})} \max_{p_g,p_b} \min_{o_t \in \mathsf{BR}_{\zeta(p_t;h^{t-1})}} (\varphi a_{t-1} + J_g - p_g)\cdot o_t(p_g) &= \frac{\rho}{2}\sigma_g^2 \\
\min_{\zeta(p_t;h^{t-1})} \max_{p_g,p_b} \min_{o_t \in \mathsf{BR}_{\zeta(p_t;h^{t-1})}} (\varphi a_{t-1} + J_b - p_b)\cdot o_t(p_b) &= 0.
\end{aligned}
$$

Proof. See Appendix C. □

Then when dealer is forward-looking, deviation to decline a transaction in bad times has future costs for her and makes it more costly to deviate. In fact, after providing a low price and declining the trade in bad times, although the dealer can avoid the loss in current period, in all future periods, the following traders observe her deviation and will expect to play ISTE equilibrium. The potential traders will adjust their expectations and thus dealer can collect at most ISTE payoff, only achievable when she reveals her private information about the economic states in all future periods. As a result, in all future periods, she is unable to hide her private information and hence her expected continuation value is lowered after deviation. Such concern discourages dealer's deviation and makes the opaque equilibrium easier to sustain under this dynamic trading setting, reducing market price informativeness.

The following proposition captures this idea and characterizes the necessary and sufficient condition for the existence of an opaque dynamic trading equilibrium (ODTE). We leave the specific characterization of the threshold, the construction of such an equilibrium and the necessity proof in the Appendix.

Proposition 3 (Opaque Dynamic Trading Equilibrium). *There exists a $\rho^{\mathsf{ODTE}} \leq \rho^{\mathsf{OSTE}}$ such that if and only if $\rho \geq \rho^{\mathsf{ODTE}}$, there exists an opaque dynamic trading equilibrium (ODTE) in which the dealer always conceals her private information about current economic state and trades occur in both* good *times and* bad *times. Moreover, for any $\rho \geq \rho^{\mathsf{ODTE}}$, there is a unique maximal PBE outcome, achieved only through an* ODTE*. Specifically, for any $\rho \in [\rho^{\mathsf{ODTE}}, \rho^{\mathsf{ISTE}}]$, there exists an* ODTE *that defeats and Pareto dominates the PBE where* ISTE *is played in every period.*

Proof. See Appendix C. □

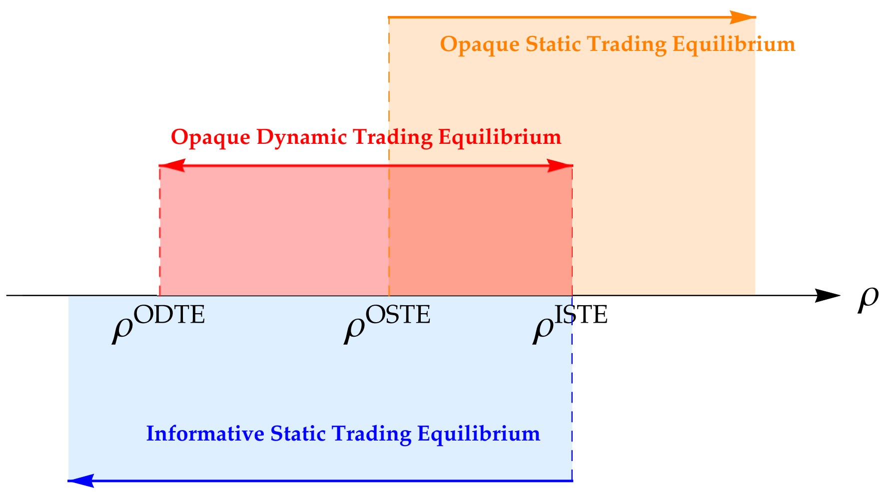

Figure 5: This chart plots ρ^{ISTE}, ρ^{OSTE} and ρ^{OSTE} and shows that $\rho^{\mathsf{ODTE}} \leq \rho^{\mathsf{OSTE}}$. Hence, when dealer is forward-looking, opaque pricing equilibrium is easier to sustain, reducing the price efficiency.

Finally, the comparison between ρ^{ODTE} and ρ^{OSTE} implies that ODTE is easier to sustain than OSTE. Therefore, long-term incentives of dealers may reduce financial transparency via the decrease of market price efficiency (see also Figure 5).

Remark 3. *For completeness, we also characterize another kind of equilibrium where traders always distinctly price the assets in good times and in bad times. We call it an informative dynamic trading equilibrium (*IDTE*) and leave the details of the discussion and construction in the Appendix B. However, whenever both types of equilibria exist, there is always an* ODTE *that defeats and Pareto dominates any* IDTE*.*

5 Information Disclosure via TRACE: Private vs. Public History

To analyze the effect of information disclosure on OTC markets, we now consider an important extension of our model where the informed dealer is still forward-looking but previous trade history is **not** publicly available. Particularly, in line with the Dodd-Frank transparency Act of 2010 for corporate bonds and swaps (implemented by TRACE by requiring public dissemination of post-trade transaction information regarding price and volumes), we assume that previous economic states are still publicly observable and discuss two kinds of imperfect monitoring about previous transactions:

- The first one is that although future traders cannot observe exact price offers provided by the dealer, they can perfectly observe whether there is a transaction between the dealer and the trader in a previous period.
- The second kind of imperfect monitoring involves the situation where future traders can observe neither previous price offers nor whether or not there is a transaction in a previous period. Nevertheless, they can obtain a noisy signal y_t about o_t, whether a trade occurs in period t or not. Suppose y_t is independently drawn from a distribution $F_{o_t}(\cdot)$.

In particular, in this section, we ask:

> Does TRACE improve financial transparency by increasing the price efficiency (informativeness)? Moreover, how does TRACE affect market liquidity?

5.1 Private Prices and Public Volumes History

The next proposition shows that in the first case, the ODTE constructed for Proposition 3 can still hold. In fact, from the analysis of ODTE, one can learn that offering a price weakly higher than $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ will not discourage traders from accepting the offers, but the dealer derives lower profits and trigger future punishments. Therefore, such strategies are always sub-optimal. Then the only kind of possible profitable deviation is to offer a price strictly lower than $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ and other than on-path price p_t^*. But in such a deviation, traders will always decline the offer and there is no transaction. Therefore, if the information about o_t is publicly observable, then players can perfectly monitor whether there are previous deviations and use the same punishment device as that in the ODTE. In other words, o_t is a sufficient statistic for previous deviations.

Proposition 4. *If future traders can not observe previous price offers, but can perfectly observe whether or not there is a trade in any previous period, then there exists an opaque dynamic trading equilibrium if and only if* $\rho \geq \rho^{\mathsf{ODTE}}$.

Proof. See Appendix C. □

Therefore, past orders are sufficient statistics for future traders to determine whether a dealer deviates or not. Proposition 4 implies that post trade price transparency does not affect the existence of opaque trading equilibrium. Releasing these details to the public does not make it easier or harder to elicit dealers' inside knowledge about the economic states. Moreover, since in any opaque trading equilibrium, trades occur in both good times and bad times, the post trade price transparency via TRACE need not decrease the market liquidity.

5.2 Private Prices and Private Volumes History

We now turn our focus on a more interesting case, where future traders can not observe previous price offers but can have noisy signals about whether transaction happens or not in any previous period. We show that the opaque equilibrium has similar structure as before and compare its threshold with the ones in previous sections.

We model this case as in the imperfect monitoring literature,[24] where all pure strategy Nash equilibria are payoff equivalent to the set of perfect public equilibria. Thus, it is without loss of generality to focus on the equilibria where players use public strategies only. Formally speaking, we only consider cases where both the dealer and the trader choose their strategies and beliefs based on past public signals $y^t \equiv \{y_1, ..., y_t\}$ and past economic states $\{\theta_1, ..., \theta_{t-1}\}$.

[24] Here we do not solve the whole game for all possible public equilibrium, like in Fudenberg and Levine (1994), as we are only interested in when opaque ones can be sustained.

We are interested in how private history will affect the existence of opaque dynamic trading equilibrium. In opaque dynamic trading game, on the equilibrium path the dealer will conceal her private information about the current economic state and trade always occurs. To deter dealer's deviations to the greatest extent, we need to implement the harshest punishment for her off the equilibrium path, which according to Lemma 2.2 informative static trading equilibrium (ISTE) will be played forever.

Denote Y_{1t} as the set of signals such that upon observing such signals, the dealer will still offer an opaque price and the trader will form the corresponding belief. Denote Y_{0t} as the set of the rest signals, upon observing which the players will play an ISTE equilibrium. Denote $f_i^t \equiv \text{Prob}(y_t \in Y_{1t} | o_t = i), i = 0, 1$. Ideally if we can observe o_t directly, then simply setting $y_t = o_t$ can give us $f_1^t = 1$ and $f_0^t = 0$. Another extreme is that the signal is completely informativeless, that is, $F_{o_t=0} = F_{o_t=1}$, then we will have $f_0^t = f_1^t$ no matter how we partition the signal space Y.

To simplify the analysis and make the model tractable, we restrict our attention to the equilibria where the division of signals is fixed from period to period. So we can drop the subscript and instead write Y_1, Y_0, f_1 and f_0. We, specifically, define the opaque pricing equilibrium associated with this partition as a **private history equilibrium**.

Definition 6. *Consider an environment where future traders can view neither the details of previous offers nor whether there is a transaction at period t, but just a noisy signal $y_t \in Y$ about o_t. A private history equilibrium consists of a set of strategies $p(y^t)$ and beliefs $\xi(\cdot|y^t)$ such that for some non-empty subset $Y_1 \subset Y$, if $y^s \in Y_1, \forall s \le t-1$, then in period t, the dealer chooses to offer a price $p_t^*(h^{t-1})$ that is independent of her private information about θ_t.*

Then, we ask whether opaque pricing strategies can still form an equilibrium under this private history setup. The answer is that the private history equilibrium defined above exists when trader's risk aversion coefficient is above a certain level.

Proposition 5 (private transaction history). *Suppose future traders can only observe whether a trade occurs or not at period t via a noisy signal y_t, which is independently drawn from the distribution $F_{o_t}(\cdot)$. There exists a $\rho^{\mathsf{private}} \in [\rho^{\mathsf{ODTE}}, \rho^{\mathsf{OSTE}}]$ such that the following statement is true: if and only if $\rho \ge \rho^{\mathsf{private}}$, then there exists a private history equilibrium defined above. Moreover, for any $\rho \in [\rho^{\mathsf{private}}, \rho^{\mathsf{ISTE}}]$, there exists a private history equilibrium that defeats and Pareto dominates the equilibrium where* ISTE *is played in every period.*

Proof. See Appendix C. $\square$

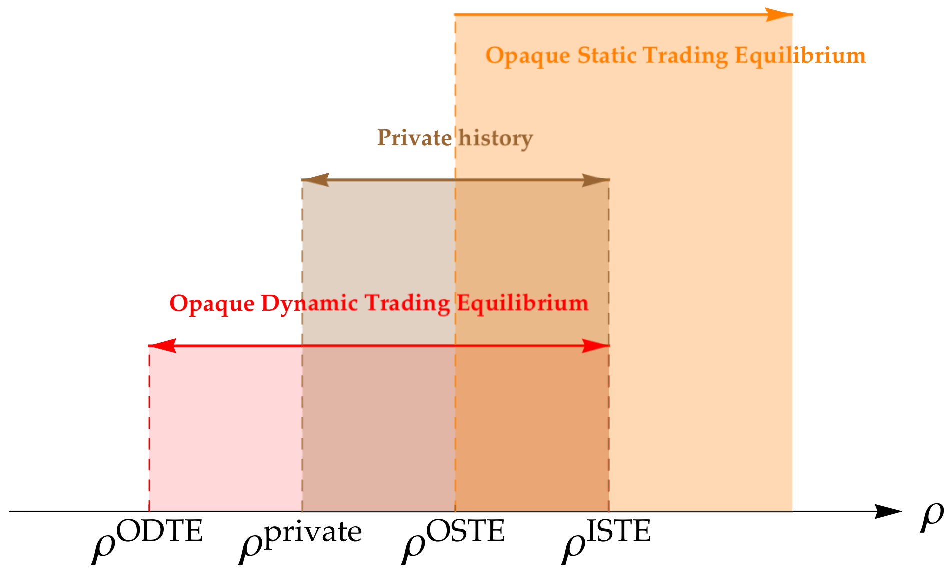

Figure 6: This chart plots three kind of opaque pricing equilibria, OSTE (when dealer is myopic), ODTE (when dealer is forward-looking and previous orders are publicly observable) and private history equilibrium (when dealer is forward-looking but previous orders are coarsely observed). We always have ρ^{private} between ρ^{ODTE} and ρ^{OSTE}.

Proposition 5 has important policy implications for whether the regulation institution should require dealers and traders to report information about private transactions to the public. On the one hand, in private history equilibrium, occasionally unlucky signals are produced and the game moves into the punishment phase, where ISTE is played. Therefore, even when traders are sufficiently risk-averse ($\rho > \rho^{\mathsf{private}} > \rho^{\mathsf{ODTE}}$) such that both ODTE and private history equilibrium can be sustained, the private history equilibrium still leads to higher price transparency due to its occasional derailment to the punishment stage. In conclusion, when traders are intermediately risk-averse, requiring dealers and traders to accurately report past transaction orders will alter the equilibrium outcome drastically from ISTE to ODTE. When traders are sufficiently risk-averse and opaque pricing strategies can be sustained before and after this regulation, it will reduce the occurrence of informative pricing outcomes. From both points of view, asking the dealer to fully reveal her previous transaction orders (volumes) paradoxically impairs the transparency and the informativeness of the market prices.

On the other hand, from private history to public history, with more information about past transaction orders, the domain shrinks of traders' risk aversion coefficient where opaque trading equilibria can be supported. In fact, the more informative signals of transaction orders become, the more credible the following claim from the dealer:

> "I will not deviate in the current period because if I did so, in following periods the future traders would figure this out and punish me for this deviation."

In other words, the dealer can use the information of past orders as a commitment device to convince traders that she will not price discriminatingly. Consequently, opaque pricing strategies along the equilibrium path can be supported. This is why for $\rho \in (\rho^{\mathsf{ODTE}}, \rho^{\mathsf{private}})$, public history of past orders enables opaque pricing and impairs the post price transparency in the market.

In conclusion, our analysis above of post-trade disclosure regarding prices and volumes shows that post-trade transparency via TRACE, paradoxically, makes markets more opaque, by reducing the market price efficiency.

Finally, as in Section 5.1, we obtain a similar conclusion about the market liquidity (trade activity and volume). This result is immediate because in any opaque equilibrium, trades occur in both good and bad times, thus post-trade disclosure regarding volumes via TRACE need not reduce the market liquidity, and may even increase it.

5.3 Signal Structure and Price Informativeness

Sometimes it is unrealistic to completely ban or completely release the information about past transactions. The next proposition discusses the effect of one type of change in signal structure on the existence of opaque pricing equilibrium, as well as the price transparency.

Proposition 6. *For a fixed f_0, the cutoff of the existence of a private history equilibrium ρ^{private} characterized in Proposition 5 decreases in f_1. If $f_1 - f_0 = 1$, then ρ^{private} coincides with ρ^{OSTE}. If $f_1 - f_0 = 0$, then ρ^{private} coincides with ρ^{ODTE}.*

Proof. See Appendix C. □

Hence, making the signal more distinctive by purely increasing f_1 will relax the requirement on traders' risk aversion coefficient, and the opaque pricing equilibrium can be sustained with less risk-averse traders. An increase of f_1 also leads to more lenient strategies, lower frequency of triggering the punishment stage and a more transparent pricing scheme. In other words, the effect of this signal structure change is twofold. First it makes opaque pricing equilibrium easier to hold. Second, even with traders sufficiently risk-averse enough, an increase in f_1 makes it less likely to have a signal in Y_0, so less likely to have price transparency.

Finally, it is worth noting that Kandori (1992) shows that as signals of past actions are improved in Blackwell's sense, the set of PBE payoffs expands. With certain conditions, our result above reinforces such message. If f_0 is fixed and f_1 decreases, past history signals become less informative, and ρ^{private} will increase. Since there always exists a private history equilibrium Pareto dominating the ISTE, for any ρ the equilibrium payoff set weakly decreases (for intermediate ρ this decrease is strict). In other words, less informative signals shrink the possible equilibrium payoff set.

6 Policy

To explore potential policy interventions, we consider a policymaker who is concerned about the transparency in OTC markets. We discuss policy implications of our results and provide suggestions such as *stochastic auditing* dealers to improve the price informativeness in these markets. We also compare the welfare of all equilibria discussed above and show that there is a tradeoff between the price informativeness and the social welfare in OTC markets.

6.1 Information Releasing

Proposition 4 and Proposition 5 together imply that if the policymaker (regulator) wants to increase the price transparency in OTC markets, one way is to control what kind of information future traders can get access to. Simply restricting the disclosure of details of past transactions is not enough. To delink the dynamic incentive of dealers, it is necessary to hide whether there are transactions, or information of orders, as well.

One way to implement the above strategy is to ban the discussion of past transactions with future traders. The regulator can also forbid the use of past deals to advertise and attract new traders. Moreover, the regulator can encrypt the identity or past order history of dealers when they are approached by traders.

6.2 Stochastic Auditing

If restricting the disclosure of information about past orders is hard to implement in some circumstances, the regulator can also improve the price informativeness of the OTC market via supervision and punishment. As shown in this subsection, increasing the auditing intensity will make the opaque equilibrium harder to sustain.

In this subsection we model the case where the regulator steps in to combat the information hiding of dealers and at the beginning of each period, the regulator randomly audits a fraction of dealers and checks whether they provide opaque, uninformative orders in last period. If caught, then a dealer is prohibited from participating in financial activities and all her future period payoffs are gone.

Specifically, at the beginning of period $t+1$, the regulator randomly picks up q fraction of dealers to check. As past information about economic states, asset values and price offers are all available to the regulator, it can then check whether in period t, the dealer offers a price of $p_t^g \equiv \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ or something strictly below $p_t^b \equiv \varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2$. If not, then the regulator can ban the participation of this dealer into future trades. Otherwise the dealer remains in the market. We call this q the auditing intensity.

We show in the following proposition that as auditing intensity increases, the opaque equilibrium becomes harder to sustain.

Proposition 7. *Let q represents the regulator's auditing intensity, that is, with probability q, at the end of a period t, a dealer is audited. If she provides a price weakly above $\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2$ but not $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$, then all her future payoffs are forfeited. Denote P_q as the set of trader's risk aversion coefficients when* ODTE *can be sustained. If $q_1 < q_2$, then $P_{q_2} \subseteq P_{q_1}$. Specifically, $P_0 = [\rho^{\mathsf{ODTE}}, \infty]$ and $P_1 \subsetneq [\rho^{\mathsf{OSTE}}, \infty]$.*

Proof. See Appendix C. □

In other words, stochastic auditing can also force the dealer to reveal her private information about economic states more frequently and as a result, lead to more price transparency in the market. This coincides with the general intuition of censorship literature that the higher the detection effort, the less likely the individuals will misbehave. The regulator can then choose the optimal auditing intensity given her desire of price informativeness and the limited auditing resources.

Remark 4 (Maximizing Price Efficiency and Liquidity). *In fact, if the informative dynamic trading equilibrium exists (see Remark 3 and Appendix B), then there is a policy that can achieve* maximal *price efficiency as well as market liquidity at the same time. The regulator can punish the dealer who does not follow the* IDTE*. Specifically, if the regulator wants to implement an* IDTE *where in bad times the dealer collect a payoff of π^b (π^b satisfies the condition in Appendix-Proposition 1), then the regulator can ban all future participation of the dealer if she is observed not to provide on-path prices $p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ in good times or $p_t^b = \varphi a_{t-1} + J_b - \pi^b$*

in bad times. In such an equilibrium, the dealer is pricing discriminatingly and the offered price reflects her private information. Moreover, in IDTE *trade also occurs in bad times, the market liquidity is the same as when there is no auditing. Such policy also gives a higher level of social surplus than the one we constructed above. In fact, even this social surplus is still below the highest possible level, it is the most the regulator can get if he wants full price efficiency. The intuition is that the post-trade price transparency discourages traders to hedge and lowers the benefit of trade, impairing the social surplus and implying that there is a trade-off between the price efficiency and the social welfare.*

6.3 Welfare

In this section we compare the social welfare and the dealer's profits of different equilibria specified above. We first point out that there is a tradeoff between the social welfare and the price informativeness in OTC markets. We then show that with more transparency and public disclosure of trade volumes, the privately informed dealer can gather a higher expected profit. The latter result appears surprising, as it goes against the general lesson of contract theory that less disclosure gives more information rent to the party with private information.

Our results hold for the following reason. In our model more public disclosure about past trades enables the dealer to hide her private information in the current period. Consequently, opaque trading equilibrium is easier to sustain, impairing price efficiency and improving total social welfare. The dealer's expected profits depend on the division of this total surplus between her and the trader. But for each equilibrium under certain level of transparency of past orders, with more public information about past trade volumes, there always exists an equilibrium that Pareto dominates the old one, and the dealer's expected profit increases.

We now analyze the social welfare in two kinds of equilibrium. In ISTE, the dealer fully reveals her private information and trade only occurs in good times. As a result, the ex-ante welfare reduces to its minimum level. In opaque equilibria (either static or dynamic), however, the ex-ante welfare achieves its maximum. The difference between these extreme scenarios, in regions where both types are available, is what we call the **welfare gap**. The next proposition shows the exact amount of this welfare gap.

To make the social surplus comparable, we here define the social surplus as the sum of the dealer's and trader's per period expected payoffs given their equilibrium behaviors. Formally speaking, W^{ODTE} represents the social surplus in a certain period given that both parties follow their on-path ODTE strategies. We are going to compare this with the social surplus in another equilibrium in the same period.

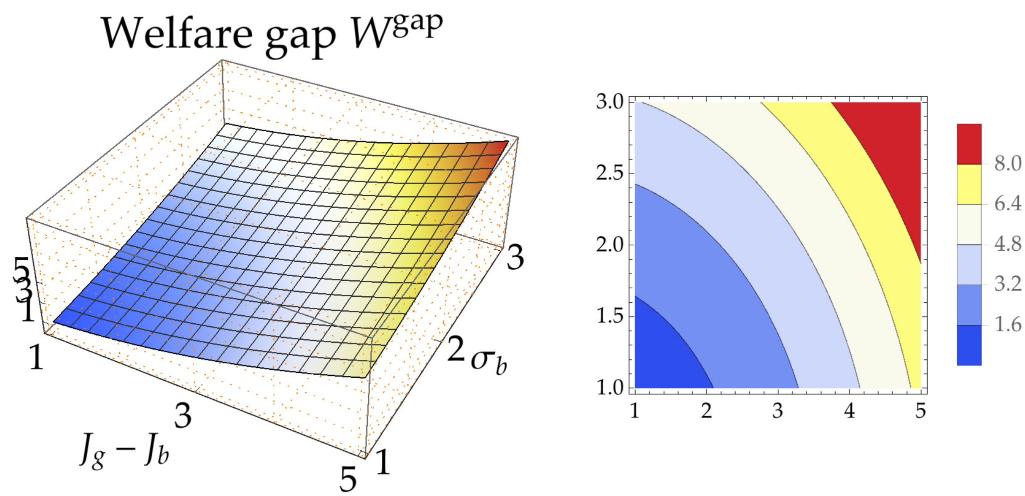

Figure 7: This chart plots welfare gap W^{gap}. It shows welfare gap is increasing in the spread $J_g - J_b$ and the asset volatility in bad times σ_b. Note that W^{gap} is independent of the asset volatility in good times σ_g.

Proposition 8. *In period t, the expected social welfare for each equilibrium is explicitly given by:*

$$W^{\mathsf{OTE}} \equiv W^{\mathsf{OSTE}} = W^{\mathsf{ODTE}} = \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right],$$
$$W^{\mathsf{ISTE}} = \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2\right].$$

In the region where ISTE *and either* OSTE *or* ODTE *exist (i.e.,* $\rho \in [\rho^{\mathsf{ODTE}}, \rho^{\mathsf{ISTE}}]$*), the welfare gap is*

$$W^{gap} \equiv W^{\mathsf{OTE}} - W^{\mathsf{ISTE}} = \frac{\rho}{2}\left[(1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right],$$

which is independent of σ_g *and is convex and increasing in* σ_b *as well as the spread* $J_g - J_b$*.*

Proof. See Appendix C. □

The ex-ante social welfare of ISTE is smaller than that of OSTE or ODTE for two reasons. First, there is no trade in bad times in ISTE. Although the dealer still has a higher evaluation than the trader, who possesses the asset, the incentive compatibility constraint prevents both parties from exchanging the asset and leads to a loss of the social welfare, captured by $\frac{\rho}{2}(1-\alpha_{\theta_{t-1}})\sigma_b^2$. The residual term in the welfare gap, $\frac{\rho}{2}\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2$ comes from the information rent. In fact, in informative equilibrium traders learn about economic state θ_t before the transaction (after observing dealer's bid price offer). Therefore, the only uncertainty in his evaluation comes from the residual risk, and this reduce of risk decreases benefits of trade as well as his desire to hedge. As a result, social surplus coming from the trade decreases and we lose the social insurance derived from the uncertainty of drift of innovation.

It then follows immediately that the welfare gap is independent of the asset volatility in good times (i.e. σ_g) but monotonically increases in the asset volatility in bad times σ_b and the spread $J_g - J_b$ (see also Figure 7). This is because the dealer gains the insurance from residual risks (i.e., due to $\sigma_{\theta_t} z_t$) in good times in both informative equilibrium and opaque equilibrium. Hence, it cancels out in the welfare gap.

We next discuss the dealer's expected profits. As shown in previous sections, there exist multiple opaque equilibria, and each differs from the others on specific shares of the total surplus the dealer and the trader can get. Therefore, the dealer's profits with different levels of public disclosure of past trades are not comparable. However, we can fix this technical difficulty by comparing the equilibrium with the same division ratio between the dealer and the trader. By doing so, for example, in opaque equilibrium the total surplus is higher than that in informative equilibrium, and as a result, dealer can gain higher profits in an opaque equilibrium than in an informative one. Next, recall that Proposition 5 shows that if information about past orders is available, the dealer can use such signals as a commitment device and opaque equilibrium is easier to hold. Therefore, disclosing the dealer's information about past order volumes can actually increase her expected payoffs. Surprisingly, such result is apart from the general lesson of contract theory, where private information always provides information rent for the insider.

7 Robustness and Extensions

In this section we extend our model in several other directions and analyze the robustness of our main findings. In Section 7.1, we show that our main conclusions do not depend on deliberate functional forms and can go beyond the mean-variance preferences. Section 7.2 extends the analysis to the case where traded orders are divisible. In Section 7.3 we allow trader's demand shock to stochastically change over times, meaning that trader changes from a seller to a buyer or vice-versa from period to period. Finally, in Section 7.5 we characterize a semi-pooling equilibrium in the one-shot trading game.

7.1 More General Forms of Utilities

In this subsection we show that our main conclusions are not the result of particulate functional forms and can be applied to more general utility functions. But with more general forms of utilities, we need a standard to compare the risk aversion of different agents. In this paper, we introduce the following definition and argue that all the results in this paper still hold if we relax the mean-variance utility form and use the following standard to compare the risk preference of traders.

Definition 7. *Suppose $\tilde{x}_g$ and $\tilde{x}_b$ are lotteries with normally distributed outcomes, $\tilde{x}$ is a lottery of lotteries, such that with probability α, the outcome is drawn according to the lottery $\tilde{x}_g$ and with probability $1-\alpha$, the outcome is drawn according to the lottery $\tilde{x}_b$. An utility function V* ***exhibits more risk aversion*** *than an utility function U if and only if*

$$0 \leq CE_U(\tilde{x}_g) - CE_V(\tilde{x}_g) \leq CE_U(\tilde{x}) - CE_V(\tilde{x}),$$

where $CE_f(\cdot)$ is the certainty equivalence function for $f(\cdot)$, that is, $CE_f(\tilde{y}) = f^{-1}(\mathbb{E}(f(\tilde{y})))$ for any random variable $\tilde{y}$ and increasing function f.

The intuition behind the above definition is as follows. $\tilde{x}$ is a more volatile lottery than $\tilde{x}_g$, and if the agent is more risk-averse, then he would further dislike risks and value $\tilde{x}_g$ over $\tilde{x}$ more than a less risk-averse agent. In other words, more risk-averse trader requires a higher risk premium for the risky asset than less risk-averse traders. The difference is more salient for asset with a higher risk level. For example, in our baseline model, the utility function is mean-variance utility and

$$CE_U(\tilde{x}) = \mathbb{E}(\tilde{x}) - \frac{\rho^U}{2}\mathrm{Var}(\tilde{x}).$$

Then a mean-variance utility function V exhibits more risk aversion than a mean-variance utility function U if and only if $\rho^V \geq \rho^U$, which coincides with the traditional comparison of risk aversion for mean-variance utilities.

With the above definition, we can derive the following propositions and argue that the general message of our analysis is not altered by this generalization.

Proposition 9.

(a) If OSTE *exists with trader's utility function U, then there still exists an* OSTE *when trader's utility function is V and V exhibits more risk aversion than U.*

(b) If ISTE *exists with trader's utility function V, then there still exists an* ISTE *when trader's utility function is U and V exhibits more risk aversion than U.*

(c) If ODTE *exists with trader's utility function U, then there still exists an* ODTE *when trader's utility function is V and V exhibits more risk aversion than U.*

(d) If a private history equilibrium exists with trader's utility function U, then there still exists a private history equilibrium when trader's utility function is V and V exhibits more risk aversion than U.

Proof. See Appendix C. □

Therefore, the structure of the equilibria described in previous sections (OSTE, ODTE, ISTE and private history equilibrium) all remain the same. If traders are extremely risk-averse, then opaque equilibrium can be sustained, no matter whether it is under static setting, dynamic setting with public history, or dynamic setting where the order history is imperfectly observed. The informative equilibrium in static trading game exists when traders are sufficiently risk-loving.

Proposition 10. *Fix a trader's utility function U,*

(a) if an OSTE *and an* ISTE *exist, then there exists a private history equilibrium;*

(b) if a private history equilibrium and an ISTE *exist, then there exists an* ODTE*.*

Proof. See Appendix C. □

Proposition 10 implies that the relative relation between OSTE, ODTE, ISTE and private history equilibrium also remain the same. Specifically speaking, dynamic trading environment provides a commitment device for the dealer and makes the opaque pricing strategies easier to sustain, while the imperfect observation of past volumes impairs the supervision by future traders on the dealer's action, reduces the cost of her deviation and increases the potential of the breakdown of opaque equilibrium.

7.2 Divisible Orders

In this subsection we extend our analysis to the case where traded orders can be divisible, i.e., $o_t \in [0,1]$. At the beginning of period t, the informed dealer offers a take-it-or-leave-it bundle (o_t, p_t) to the trader, where o_t and p_t are, respectively, the order (volume) and and the per unit (bid) price. As we next show, nothing fundamental changes in terms of the structure of opaque equilibria. For the informative static trading equilibrium (ISTE), however, things work differently and the trade now can also occur in bad times, in contrast to the discrete order case presented in Proposition 2, where the trade only occurs in good times.

If selling o_t unit of his risky asset to the dealer, a trader with the prior belief ζ_t can collect an ex-ante payoff of

$$\begin{aligned}
&p_t o_t + (1-o_t)\left[\varphi a_{t-1} + \zeta_t J_g + (1-\zeta_t) J_b\right] \\
&-\frac{\rho}{2}(1-o_t)^2\left[\zeta_t(1-\zeta_t)(J_g-J_b)^2 + \zeta_t\sigma_g^2 + (1-\zeta_t)\sigma_b^2\right].
\end{aligned}$$

OSTE. We start with the analysis of the opaque static trading equilibrium. We claim that the one constructed in Section 3 remains an equilibrium. Since in an OSTE the dealer's offer contains no information about the current economic state, after observing it the trader still holds his prior beliefs, $\zeta_t = \alpha_{\theta_{t-1}}$.

In this case the dealer solves the following mechanism design problem:

$$\begin{aligned}
&\max_{o_t\in[0,1],p_t} \; o_t(\varphi a_{t-1} + \alpha_{\theta_{t-1}} J_g + (1-\alpha_{\theta_{t-1}}) J_b - p_t) \\
s.t.\quad & p_t o_t + (1-o_t)\left[\varphi a_{t-1} + \alpha_{\theta_{t-1}} J_g + (1-\alpha_{\theta_{t-1}}) J_b\right] \\
&-\frac{\rho}{2}(1-o_t)^2\left[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g-J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2\right] \geq 0
\end{aligned}$$

The trader's individual rationality (IR) constraint provides a lower bound for p_t. Plugging this into the objective function, the problem is equivalent to

$$\begin{aligned}
&\max_{o_t\in[0,1]} \quad -\frac{\rho}{2}(1-o_t)^2\left[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g-J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2\right] + \\
&\qquad\qquad (o_t + 1 - o_t)(\varphi a_{t-1} + \alpha_{\theta_{t-1}} J_g + (1-\alpha_{\theta_{t-1}}) J_b) \\
\Leftrightarrow &\max_{o_t\in[0,1]} \quad -\frac{\rho}{2}(1-o_t)^2\left[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g-J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2\right]
\end{aligned}$$

Therefore, the optimal order to provide is still $o_t = 1$, which coincides with the equilibrium offers in the discrete case. The cutoff for the existence of such an equilibrium remains the same.

ISTE. Next we look at the ISTE. We first restrict that trader's risk aversion coefficient in the range where ISTE exists for the discrete order case, i.e., $\rho \leq \rho^{\mathsf{ISTE}} = 2\frac{J_g - J_b}{\sigma_g^2}$. Then denote $(o_{t,\theta_t}, p_{t,\theta_t}), \theta_t \in \{g, b\}$ as an ISTE. The dealer maximizes

$$o_{t,g}(\varphi a_{t-1} + J_g - p_{t,g}) + o_{t,b}(\varphi a_{t-1} + J_b - p_{t,b})$$

subject to the following constraints

$$\begin{aligned}
&\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 \leq o_{t,b}p_{t,b} + (1 - o_{t,b})[\varphi a_{t-1} + J_b] - \frac{\rho}{2}(1 - o_{t,b})^2\sigma_b^2 && \textbf{trader's IR at } \theta_t = b \\
&\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2 \leq o_{t,g}p_{t,g} + (1 - o_{t,g})[\varphi a_{t-1} + J_g] - \frac{\rho}{2}(1 - o_{t,g})^2\sigma_g^2 && \textbf{trader's IR at } \theta_t = g \\
&o_{t,b}(\varphi a_{t-1} + J_b - p_{t,b}) \geq 0 && \textbf{dealer's IR at } \theta_t = b \\
&o_{t,g}(\varphi a_{t-1} + J_g - p_{t,g}) \geq 0 && \textbf{dealer's IR at } \theta_t = g \\
&o_{t,b}(\varphi a_{t-1} + J_b - p_{t,b}) \geq o_{t,g}(\varphi a_{t-1} + J_b - p_{t,g}) && \textbf{dealer's IC at } \theta_t = b \\
&o_{t,g}(\varphi a_{t-1} + J_g - p_{t,g}) \geq o_{t,b}(\varphi a_{t-1} + J_g - p_{t,b}) && \textbf{dealer's IC at } \theta_t = g
\end{aligned}$$

We claim that with divisible orders, the ISTE constructed in previous sections fail to exist. Moreover, there exists another informative equilibrium that Pareto dominates it. We prove this by contradiction. Suppose $o_{t,g} = 1$ and $p_{t,g} = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. The dealer's problem then becomes

$$\begin{aligned}
&\max_{o_{t,b}, p_{t,b}} \; o_{t,b}(\varphi a_{t-1} + J_b - p_{t,b}) \\
\text{s.t.} \quad & o_{t,b}[p_{t,b} - \varphi a_{t-1} - J_b + \frac{\rho}{2}\sigma_b^2(2 - o_{t,b})] \geq 0 && (10) \\
& o_{t,b}[\varphi a_{t-1} + J_b - p_{t,b}] \geq 0 && (11) \\
& o_{t,b}(\varphi a_{t-1} + J_g - p_{t,b}) \leq \frac{\rho}{2}\sigma_g^2 && (12) \\
& o_{t,b}(\varphi a_{t-1} + J_b - p_{t,b}) \geq \frac{\rho}{2}\sigma_g^2 + J_b - J_g && (13)
\end{aligned}$$

In the above program, inequality (10) is from trader's IR at $\theta_t = b$, inequality (11) is dealer's IR at $\theta_t = b$, inequality (12) is from dealer's IC at $\theta_t = b$, and inequality (13) is from dealer's IC at $\theta_t = g$.

We show that $o_{t,b} = 0$ fails to be a solution to dealer's optimization problem. To see this, consider an $o_{t,b} \in (0,1)$, and a price $p_{t,tb} < \varphi a_{t-1} + J_b$. If we can check that this pair satisfies conditions (10)-(13), then it gives the dealer positive payoff in bad times and dominates the no-trade bundle where $o_{t,b} = 0$.

For this bundle to work we need

$$
\begin{aligned}
p_{t,b} &\geq \varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2(2 - o_{t,b}) && (14)\\
p_{t,b} &< \varphi a_{t-1} + J_b && (15)\\
p_{t,b} &\geq \varphi a_{t-1} + J_g - \frac{\rho}{2o_{t,b}}\sigma_g^2 && (16)\\
p_{t,b} &\leq \varphi a_{t-1} + J_b - \frac{\frac{\rho}{2}\sigma_g^2 + J_b - J_g}{o_{t,b}} && (17)
\end{aligned}
$$

Since $\rho \leq \rho^{\mathsf{ISTE}}$, $-\frac{\rho}{2}\sigma_g^2 - J_b + J_g \geq 0$. Condition (17) is not binding. Also notice that for $o_{t,b}$ small enough, $\varphi a_{t-1} + J_g - \frac{\rho\sigma_g^2}{2o_{t,b}} < \varphi a_{t-1} + J_b$. Therefore, one can pick a $p_{t,b}$ in the interval $[\max\{\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2(2 - o_{t,b}), \varphi a_{t-1} + J_g - \frac{\rho}{2o_{t,b}}\sigma_g^2\}, \varphi a_{t-1} + J_b]$, then conditions (10)-(13) are all satisfied, and the dealer can still collect positive payoffs in bad times. This equilibrium gives the dealer a higher payoff than that in the discrete case.

ODTE and private history equilibrium. We claim the ODTE and the private history equilibrium discussed in this paper still exist. From the analysis of OSTE, we learn that there are no immediate benefits to offer divisible orders $o_t \in (0,1)$. Furthermore, deviating to divisible orders can trigger future punishment, although this punishment is less severe when continuous quantity is allowed. Hence, the dealer does not want to deviate and offer something with $o_t \in (0,1)$ in the repeated setting.

The discussion in the previous sections then directly follows. ODTE and private history equilibrium still exist for some $\rho \geq \rho^{\mathsf{ODTE}'}$ and $\rho \geq \rho^{\mathsf{private}'}$, respectively. Moreover, we still have the following relationship

$$\rho^{\mathsf{ODTE}'} \leq \rho^{\mathsf{private}'} \leq \rho^{\mathsf{OSTE}}.$$

7.3 Different Trading Positions

In this section we allow traders' positions to change stochastically between sellers and buyers. Specifically, we assume demand shocks follow an i.i.d. process such that

$$\text{Prob}\{\chi_t = 1\} = \beta \in (0,1), \quad \text{Prob}\{\chi_t = -1\} = 1 - \beta.$$

The trading position χ_t is public information for both the dealer as well as traders since period t.

OSTE. In a static model, if $\chi_t = 1$, i.e., the trader is a potential seller, this is exactly what happens in our baseline model and results stay the same. Proposition 11 shows what will happen when $\chi_t = -1$, that is, when the trader is a potential buyer. The opaque equilibrium has similar structure as in previous sections.

Proposition 11. *When the trader's trading position is $\chi_t = -1$, that is, he is in demand of a unit of asset and needs to buy it from the dealer, there exists an opaque static trading equilibrium* (OSTE) *if and only if*

$$\rho \geq \rho^{\mathsf{OSTE}}_{\chi_t=-1} \equiv 2\frac{(1-\alpha_{\min})(J_g - J_b)}{\alpha_{\min}\sigma_g^2 + (1-\alpha_{\min})\sigma_b^2 + \alpha_{\min}(1-\alpha_{\min})(J_g - J_b)^2}, \quad \alpha_{\min} = \min\{\alpha_g, \alpha_b\}$$

Proof. See Appendix C. □

ISTE. As in OSTE, the analysis of $\chi_t = 1$ case for the ISTE is exactly same as in our baseline model. Proposition 12 shows that the informative equilibrium also shares the same structure as the case when traders are potential sellers.

Proposition 12. *When the trader's trading position is $\chi_t = -1$, that is, he is in demand of a unit of asset and needs to buy it from the dealer, there exists an informative static trading equilibrium* (ISTE) *if and only if*

$$\rho \leq \rho^{\mathsf{ISTE}}_{\chi_t=-1} \equiv 2\frac{J_g - J_b}{\sigma_b^2}$$

Proof. See Appendix C. □

As shown by the following Corollary, the comparison relationship between $\rho^{\mathsf{ISTE}}_{\chi_t=-1}$ and $\rho^{\mathsf{OSTE}}_{\chi_t=-1}$ remains unchanged.

Corollary 4. *When the dealer is myopic and is a potential buyer, we have*

$$\rho^{\mathsf{ISTE}}_{\chi_t=-1} > \rho^{\mathsf{OSTE}}_{\chi_t=-1}$$

Proof. See Appendix C. □

In this informative equilibrium, trade only occurs in bad times and the dealer can collect an expected payoff of $\frac{\rho}{2}\sigma_b^2$ during bad times and 0 otherwise.

ODTE. We now consider the dynamic game, i.e., $\delta > 0$. Similarly as in our baseline model, we construct an equilibrium as below.

- If the dealer has not deviated in previous periods, then
  1. the dealer offers a price $p_t(\theta_t, \chi_t, h^{t-1}) = p_t^*(h^{t-1}, \chi_t)$; if $\chi_t = 1$, then this is a bid offer; if $\chi_t = -1$, then this is an ask offer;
  2. the trader holds his prior belief after observing $p_t^*(h^{t-1}; \chi_t)$ and believes for sure that the economic state is good for all other prices, i.e.,
  $$\zeta(p_t; h^{t-1}) = \begin{cases} \alpha_{\theta_{t-1}} & \text{if } p_t = p_t^*(h^{t-1}; \chi_t) \\ 1 & \text{if } p_t \neq p_t^*(h^{t-1}; \chi_t) \end{cases}, \forall h^{t-1};$$
  3. traders will accept any prices weakly above p_t^g or exactly at $p_t^*(h^{t-1})$, and decline all other prices.
  4. if $\chi_t = 1$, then the trader will accept any bid price that is weakly above $p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ or exactly at $p_t^*(h^{t-1}, \chi_t = 1)$;
  5. if $\chi_t = -1$, then the trader will accept any ask price that is weakly below $p_t^b = \varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$ or exactly at $p_t^*(h^{t-1}, \chi_t = -1)$.
- If the dealer deviates previously, then both parties observe this and play an informative static trading equilibrium (ISTE).
  1. If $\chi_t = 1$, then in this ISTE, the trade only occurs in good times and the dealer can collect an expected payoff of $\frac{\rho}{2}\sigma_g^2$ during good times and 0 otherwise.
  2. If $\chi_t = -1$, then in this ISTE, the trade only occurs in bad times and the dealer can collect an expected payoff of $\frac{\rho}{2}\sigma_b^2$ during bad times and 0 otherwise.

Following the same technical steps as in the analysis of ODTE, we can show the following result.

Proposition 13. *Given that $\sigma_g^2 - \sigma_b^2 + (1 - \alpha_{\max})(J_g - J_b)^2 \geq 0$. Suppose the trader has a trading position χ_t in period t, then there exists a $\rho_{\chi_t}^{\mathsf{ODTE}} \leq \rho_{\chi_t}^{\mathsf{OSTE}}$ such that if and only if $\rho \geq \rho_{\chi_t}^{\mathsf{ODTE}}$, there exists an* ODTE.

Proof. See Appendix C. □

Private history equilibrium. The construction of the private history equilibrium now becomes

- If $y^s \in Y_1$ for all $s \leq t-1$, then

  1. the dealer offers a price $p_t(\theta_t, \chi_t, h^{t-1}) = p_t^*(h^{t-1}, \chi_t)$; if $\chi_t = 1$, then this is a bid offer; if $\chi_t = -1$, then this is an ask offer;
  2. the trader holds his prior belief after observing $p_t^*(h^{t-1}; \chi_t)$ and believes for sure that the economic state is good for all other prices, i.e.,

  $$\zeta(p_t; h^{t-1}) = \begin{cases} \alpha_{\theta_{t-1}} & \text{if } p_t = p_t^*(h^{t-1}; \chi_t) \\ 1 & \text{if } p_t \neq p_t^*(h^{t-1}; \chi_t) \end{cases}, \forall h^{t-1};$$

  3. traders will accept any prices weakly above p_t^g or exactly at $p_t^*(h^{t-1})$, and decline all other prices.
  4. if $\chi_t = 1$, then the trader will accept any bid price that is weakly above $p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ or exactly at $p_t^*(h^{t-1}, \chi_t = 1)$;
  5. if $\chi_t = -1$, then the trader will accept any ask price that is weakly below $p_t^b = \varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$ or exactly at $p_t^*(h^{t-1}, \chi_t = -1)$.

- If $y^s \in Y_0$ for some $s \leq t-1$, then both parties observe this and play an informative static trading equilibrium (ISTE).

  1. If $\chi_t = 1$, then in this ISTE, the trade only occurs in good times and the dealer can collect an expected payoff of $\frac{\rho}{2}\sigma_g^2$ during good times and 0 otherwise.
  2. If $\chi_t = -1$, then in this ISTE, the trade only occurs in bad times and the dealer can collect an expected payoff of $\frac{\rho}{2}\sigma_b^2$ during bad times and 0 otherwise.

The analysis of the private history equilibrium when traders have independent trading positions across periods follows the same spirit of that of ODTE. In fact, we can show the following result.

Proposition 14. *Given that $\sigma_g^2 - \sigma_b^2 + (1-\alpha_{\max})(J_g - J_b)^2 \geq 0$. Suppose the trader has a trading position χ_t in period t, then there exists a $\rho_{\chi_t}^{\text{private}} \in [\rho_{\chi_t}^{\text{ODTE}}, \rho_{\chi_t}^{\text{OSTE}}]$ such that if and only if $\rho \geq \rho_{\chi_t}^{\text{private}}$, there exists a private history equilibrium.*

Proof. See Appendix C. □

The structure of ODTE and the private history equilibrium, as well as the comparative relations between ρ^{ODTE}, ρ^{private} and ρ^{OSTE} remains unchanged. Therefore, different trading positions do not change the message our main conclusions send.

7.4 Bid-Ask Spread

In this subsection we consider the bid-ask spread in good times and in bad times for both equilibria. First, in opaque equilibrium, the dealer offers the same price $p_t^{*\mathsf{UI}}(\theta_{t-1}, \chi_t)$ in both times. Then the bid price p^{bid} is the equilibrium price when $\chi_t = -1$, or when the trader is a potential seller:

$$\begin{aligned} p^{bid} &= p_t^{*\mathsf{UI}}(\theta_{t-1}, \chi_t = -1) \\ &= \varphi a_{t-1} + \alpha_{\theta_{t-1}} J_g + (1 - \alpha_{\theta_{t-1}}) J_b - \frac{\rho}{2}(\alpha_{\theta_{t-1}} \sigma_g^2 + (1 - \alpha_{\theta_{t-1}}) \sigma_b^2 + \alpha_{\theta_{t-1}}(1 - \alpha_{\theta_{t-1}})(J_g - J_b)^2). \end{aligned}$$

Similarly the ask price p^{ask} is the one when $\chi_t = 1$ or when the trader is a potential buyer:

$$\begin{aligned} p^{ask} &= p_t^{*\mathsf{UI}}(\theta_{t-1}, \chi_t = 1) \\ &= \varphi a_{t-1} + \alpha_{\theta_{t-1}} J_g + (1 - \alpha_{\theta_{t-1}}) J_b + \frac{\rho}{2}(\alpha_{\theta_{t-1}} \sigma_g^2 + (1 - \alpha_{\theta_{t-1}}) \sigma_b^2 + \alpha_{\theta_{t-1}}(1 - \alpha_{\theta_{t-1}})(J_g - J_b)^2). \end{aligned}$$

Therefore, the bid-ask spread in opaque equilibrium becomes:

$$p^{ask} - p^{bid} = \rho(\alpha_{\theta_{t-1}} \sigma_g^2 + (1 - \alpha_{\theta_{t-1}}) \sigma_b^2 + \alpha_{\theta_{t-1}}(1 - \alpha_{\theta_{t-1}})(J_g - J_b)^2).$$

In informative equilibrium, however, the trade never occurs in bad times when the trader possesses a risky asset ($\chi_t = -1$) or in good times when the trader is in demand of a risky asset ($\chi_t = 1$). If the dealer can learn about the trader's trading position before making an offer, then such knowledge actually decreases the liquidity in OTC markets.

If the dealer is uninformed about whether she is facing a seller or a buyer, then she must offer a bid price and an ask price that makes the trader indifferent between accepting and rejecting. The bid-ask spread now becomes, in good times:

$$p_t^g(\theta_{t-1}, \chi_t = 1) - p_t^g(\theta_{t-1}, \chi_t = -1) = \rho \sigma_g^2;$$

whereas in bad times:

$$p_t^b(\theta_{t-1}, \chi_t = 1) - p_t^b(\theta_{t-1}, \chi_t = -1) = \rho\sigma_b^2.$$

However, if the dealer learns about the trading position of the other party, then the bid-ask spread will be larger than above ones and the market becomes less efficient. This is because the dealer can deliberately offer non-acceptable prices to deter trades that are not profitable.

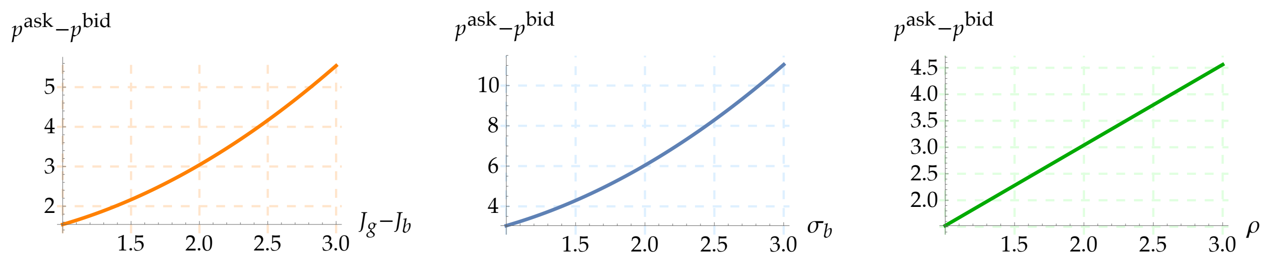

Figure 8: Bid-Ask spread in the opaque equilibrium increases in the risk aversion ρ, the asset volatility σ_b and σ_g, and the jump spread $J_g - J_b$. In the plots $\alpha_g = \alpha_b = \frac{1}{2}$.

7.5 Semi-opaque Equilibrium

In static trading game it is possible to have a semi-opaque equilibrium. For completeness, in this subsection we present the sufficient and necessary condition for the existence of such kind of semi-pooling equilibrium.

Definition 8 (semi-opaque static trading equilibrium)**.** *Assume the dealer is informed and myopic, then $(p(\cdot), o(\cdot), \zeta(\cdot))$ forms a semi-opaque static trading equilibrium if and only if*

1. *When the current economic state is good, the dealer offers a price that induces transaction,*

$$p_t(\theta_t = g; h^{t-1}) = \varphi a_{t-1} + J_b.$$

2. *When the current economic state is bad, the dealer mixes between hiding her information (offering the same price as in good times) and declining the trade (offering a low enough*

price),

$$p_t(\theta_t = b; h^{t-1}) = \begin{cases} \varphi a_{t-1} + J_b & \text{with probability } q \\ \varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 - \epsilon & \text{with probability } 1-q \end{cases}.$$

3. *After observing $\varphi a_{t-1} + J_b$ or $\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 - \epsilon$, trader Bayesian updates his belief. After observing all other prices, trader believes that the current economic state is in good times.*

$$\zeta_t(\varphi a_{t-1} + J_b; h^{t-1}) = \frac{\alpha_{\theta_{t-1}}}{\alpha_{\theta_{t-1}} + (1-\alpha_{\theta_{t-1}})q},$$
$$\zeta_t(\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 - \epsilon; h^{t-1}) = 0,$$
$$\zeta_t(p_t; h^{t-1}) = 1, \quad \textit{otherwise.}$$

4. *Trader accepts any price offer weakly above $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ or exactly at $\varphi a_{t-1} + J_b$. That is, $o_t(p_t) = -1$ if and only if $p_t \geq \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ or $p_t = \varphi a_{t-1} + J_b$.*

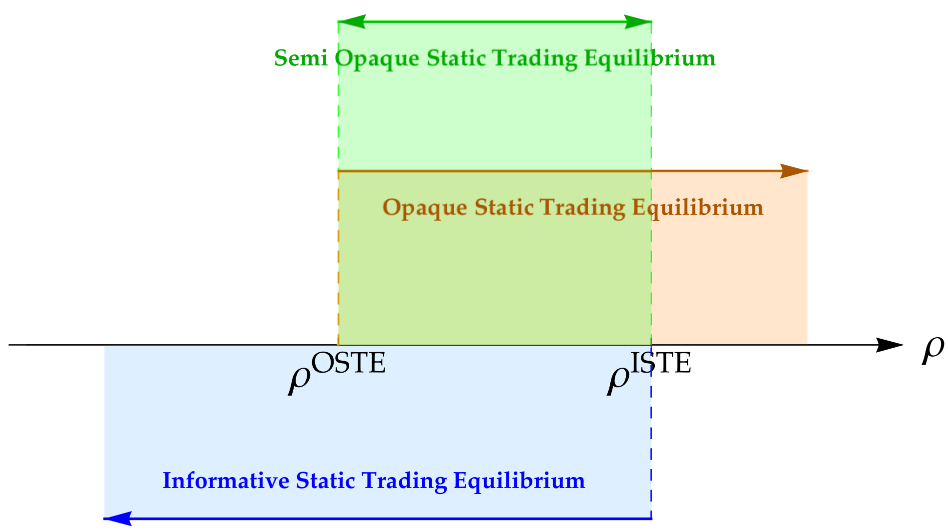

Figure 9: This Figure depicts OSTE, ISTE as well as the semi opaque static trading equilibrium. When traders are sufficiently risk-averse ($\rho > \rho^{\text{OSTE}}$), only OSTE can be sustained. When traders are sufficiently risk-neutral ($\rho < \rho^{\text{ISTE}}$), only ISTE exists. When traders have intermediate risk aversion coefficients ($\rho^{\text{ISTE}} \leq \rho \leq \rho^{\text{OSTE}}$), all three kinds of equilibria exist.

In other words, in such semi opaque equilibrium, with probability q dealer can hide her private information and offer an uninformative price, but with probability $(1-q)$

she will decline the trade in bad times. The indifferent condition implies that in bad times, both declining the trade and offering the opaque price should give her the same ex-post payoff (which is 0 in this case). It then follows that this opaque price has to be her evaluation of the asset in bad times. Finally trader's IR constraint implies that

$$\varphi a_{t-1} + \hat{\alpha} J_g + (1-\hat{\alpha}) J_b - \frac{\rho}{2}[\hat{\alpha}(1-\hat{\alpha})(J_g - J_b)^2 + \hat{\alpha}\sigma_g^2 + (1-\hat{\alpha})\sigma_b^2] \leq \varphi a_{t-1} + J_b,$$

where $\hat{\alpha} = \frac{\alpha_{\theta_{t-1}}}{\alpha_{\theta_{t-1}} + (1-\alpha_{\theta_{t-1}})q}$. This characterizes the condition for the existence of semi-opaque static trading equilibrium, as shown in the following Proposition.

Proposition 15. *If and only if trader's risk aversion coefficient ρ lies in the interval $(\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$, there exists a semi-opaque static trading equilibrium. Moreover, there exists a semi-opaque static trading equilibrium that Pareto dominates the informative static trading equilibrium* (ISTE)*, and is Pareto dominated by an opaque static trading equilibrium* (OSTE)*. In conclusion,*

$$W^{\mathsf{ODTE}} = W^{\mathsf{OSTE}} > W^{\mathsf{semi}} > W^{\mathsf{ISTE}}.$$

Finally, the expected social surplus at time t in a semi-opaque static trading equilibrium increases in the trading probability in bad times q.

Proof. See Appendix C. □

Hence, for $\rho \in [\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$, both opaque equilibrium and informative equilibrium exists, and Proposition 15 shows that there exists equilibria in between. Since the dealer now does not fully reveal or fully hide her private information about the current economic state, the social surplus of such semi-opaque equilibrium lies between that of the opaque one and that of the informative one.

8 Conclusion

After 2008 financial crisis new regulations requiring mandatory transparency (via TRACE) have been implemented in many financial markets. In this paper, we study dynamic OTC markets with time varying regime-switching fundamentals between an informed, forward-looking, strategic and risk-neutral dealer (market maker), and uninformed, strategic and risk-averse traders. We develop a tractable, yet rich, model to study the role of dealer's long term incentive and TRACE (i.e., the public information disclosure of trade history (price and trading volumes)) on the price informativeness (efficiency), the market liquidity and the social welfare.

We show that forward-looking incentive of informed dealers *hurts* the market price informativeness. Moreover, more transparency via the public disclosure of additional information about past trades, paradoxically, increases market ignorance and makes the markets more opaque, by reducing the market price informativeness. As a result, the transparency requirements such as U.S. Dodd-Frank Act may, paradoxically, make the markets more opaque, by reducing price informativeness. Importantly, however, this market ignorance can create liquidity (trade activity) and increase social welfare. Therefore, there exists a tradeoff between the price informativeness and the social welfare in OTC markets. To enhance financial transparency via improving the price informativeness and at the same time boost the market liquidity and achieve the highest feasible social welfare, an effective way is to randomly audit dealers.

We demonstrate the robustness of our findings in several directions. First, we show that our main conclusions do not depend on deliberate functional forms and can go beyond the mean-variance utility functions. Second, we extend the analysis to the case where traded orders are divisible. Third, we allow trader's demand shock stochastically change over time, meaning that the trader stochastically changes from a seller to a buyer and vice-versa. Fourth, we discuss the existence of semi-opaque equilibria that lie between opaque and informative equilibria.

Thanks to our explicit characterizations, we also derive several testable implications about how asset volatilities, extent of jump spread in good times and bad times, and risk sharing motives (trader's risk aversion) affect the price informativeness and the market liquidity in OTC markets. Finally, the qualitative predictions of our model are consistent with much of the empirical evidence observed in swap and bond markets.

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Appendix

A Uninformed Dealer

In this section, we briefly consider the set up in Section 3 in a simple case where in each period t the dealer is (like traders) fully **uninformed** about the realization of the economic state θ_t.

Clearly, when the dealer is uninformed about the economic state θ_t in period t, her price offer reveals *no* extra information about the drift and volatility (i.e. J_{θ_t} and σ_{θ_t}, respectively) of the innovation part (i.e. innov_t) in the asset value a_t. Thus, in this fully risk sharing environment, the dealer obtains all the ex-ante insurance due to residual risks $\sigma_{\theta_t} z_t$ as well as the drift innovation shocks J_{θ_t}. Therefore, in each period, the uninformed dealer, denoted by UI, offers a bid price that makes the trader indifferent between selling and keeping the asset:

$$\begin{aligned} p_t^{*\mathsf{UI}}(h^{t-1}) &= \mathbb{E}[a_t|h^{t-1}] - \frac{\rho}{2}\text{Var}[a_t|h^{t-1}] \\ &= \varphi a_{t-1} + \mathbb{E}[J_{\theta_t}|h^{t-1}] - \frac{\rho}{2}\left[\text{Var}[\sigma_{\theta_t} z_t|h^{t-1}] + \text{Var}[J_{\theta_t}|h^{t-1}]\right] \\ &= \varphi a_{t-1} + J_b + \alpha_{\theta_{t-1}}(J_g - J_b) - \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2]. \end{aligned}$$

Any bid price above $p_t^{*\mathsf{UI}}$ is ex-ante individually rational for traders and will be accepted. Given this optimal pricing offer $p_t^{*\mathsf{UI}}$, the dealer's ex-ante payoff becomes

$$\begin{aligned} U_t^{\mathsf{UI}} &= \frac{\rho}{2}\left[\text{Var}[\sigma_{\theta_t} z_t|h^{t-1}] + \text{Var}[J_{\theta_t}|h^{t-1}]\right] \\ &= \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2]. \end{aligned} \tag{18}$$

The above analysis is independent of dealer's discount factor ρ. That is, no matter whether the dealer is myopic or forward-looking, she will choose to offer a price $p_t^{*\mathsf{UI}}$ and obtains all the (ex-ante) insurance.

It then immediately follows that the dealer's ex-ante surplus monotonically increases with trader's hedging motive (captured by his risk aversion coefficient ρ), the variance of residual risks (captured by σ_g and σ_b) and the variance of drift shocks (captured by $J_g - J_b$). The following lemma summarizes these comparative statics.

Lemma 1. *If the dealer (either myopic or forward-looking) is uninformed about the current economic state θ_t, then she can obtain all the (ex-ante) insurance due to residual risks as well as* *innovation shocks. In addition, her ex-ante surplus monotonically increases with trader's hedging motive ρ, volatility of good times and bad times (i.e. $\sigma_{\mathrm{g}}, \sigma_{\mathrm{b}}$ respectively) and the additional drift of good times compared to that of bad times $J_{\mathrm{g}} - J_{\mathrm{b}}$.*

Proof. When the dealer is uninformed about the economic state θ_t in period t, she can not depend her price offer on it. In other words, dealer's price offer reveals *no* information about the drift and volatility (i.e. J_{θ_t} and σ_{θ_t}, respectively) in the asset value a_t. Therefore, we focus our attention on an equilibrium such that along the equilibrium path, the trader will not update his posterior belief of θ_t from the prior, $\zeta(\theta = g|\theta_{t-1})$, and will accept any price offers that is no lower than the evaluation of a_t based on this posterior. Thus, in each period t, the uninformed dealer, denoted by UI, in order to maximize her ex-ante payoff, offers a minimum price that makes the trader indifferent between selling and keeping the asset:

$$\begin{aligned}
p_t^{*\mathsf{UI}}(\theta_{t-1}) &= \mathbb{E}[a_t|h^{t-1}] - \frac{\rho}{2}\mathrm{Var}[a_t|h^{t-1}] \\
&= \varphi a_{t-1} + \mathbb{E}[J_{\theta_t}|h^{t-1}] - \frac{\rho}{2}\left[\mathrm{Var}[\sigma_{\theta_t} z_t|h^{t-1}] + \mathrm{Var}[J_{\theta_t}|h^{t-1}]\right] \\
&\overset{\text{(a)}}{=} \varphi a_{t-1} + J_b + \alpha_{\theta_{t-1}}(J_g - J_b) - \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right]
\end{aligned} \tag{19}$$

where (a) fallows because

$$\begin{aligned}
\mathbb{E}[J_{\theta_t}|h^{t-1}] &= J_b + \alpha_{\theta_{t-1}}(J_g - J_b), \\
\mathrm{Var}[\sigma_{\theta_t} z_t|h^{t-1}] &= \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2, \\
\mathrm{Var}[J_{\theta_t}|h^{t-1}] &= \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2.
\end{aligned} \tag{20}$$

In this equilibrium, the dealer will offer $p_t^{*\mathsf{UI}}$. Then we can calculate her ex-ante surplus as:

$$
\begin{aligned}
U_t &= (1-\delta)\mathbb{E}[u_t|h^{t-1},\theta_t] + \delta\mathbb{E}[U_{t+1}|h^{t-1}] \\
&= (1-\delta)\mathbb{E}\left[\sum_{s\geq 0}\delta^s\left((a_{t+s} - p^{*\mathsf{UI}}_{t+s,\theta_{t+s}})\,\mathbf{1}\{o_{t+s}(p^{*\mathsf{UI}}_{t+s,\theta_{t+s}}) = \text{Sell}\}\right)\Big|h^{t-1}\right] \\
&=(1-\delta)\sum_{s\geq 0}\delta^s\mathbb{E}\left[(a_{t+s} - p^{*\mathsf{UI}}_{t+s,\theta_{t+s}})|h^{t-1}\right] \\
&=\frac{\rho}{2}(1-\delta)\sum_{s\geq 0}\delta^s\left[\text{Var}[\sigma_{\theta_{t+s}}z_{t+s}|h^{t-1}] + \text{Var}[J_{\theta_{t+s}}|h^{t-1}]\right] \\
&=\frac{\rho}{2}(1-\delta)\sum_{t=0}^{\infty}\delta^t\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right]
\end{aligned}
$$

The comparative statics results then immediately follow. □

B Informative Dynamic Trading Equilibrium

In this section, we also characterize the informative dynamic trading equilibrium where the dealer always distinctly prices the assets in good times and in bad times. This equilibrium (whenever is available) can be applied as a policy to improve financial transparency through the price informativeness and at the same time can attain the maximal market liquidity and social welfare. We next specify when this equilibrium exists. Then we argue that it may not be played without external force by regulators. Finally, following the discussions of Section 6.2, we explain how regulators can implement it. But we need to point it out that whenever both exist, there is an ODTE that defeats and Pareto dominates any IDTE.

We already know that as OSTE, ISTE is also a Nash equilibrium of the one-shot game, which is subgame perfect and remains as the PBE in the dynamic trading setting. We now look at whether this enrichment of the environment can provide us some other kinds of equilibria. For example, in ISTE, transactions only occur in good times. We now ask whether in the dynamic trading setting, can trade also occur in bad times while the dealer reveals her private information of economic states and prices the asset differently in good times and bad times?

Suppose that in a PBE, at period t, on the equilibrium path the dealer offers distinct prices $p_t^g(h^t)$ and $p_t^b(h^t)$ under good times and bad times, respectively. Then after observing an offer of $p_t^g(h^t)$, the trader will believe that the underlying economic state

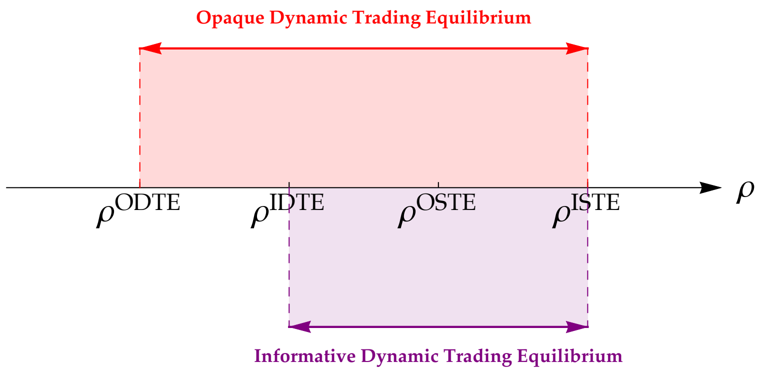

Figure 10: This chart considers dynamic equilibria when insider is forward-looking and $\alpha_g \geq \alpha_b$. Importantly, $\rho^{\mathsf{ODTE}} < \rho^{\mathsf{IDTE}} < \rho^{\mathsf{OSTE}}$, and the region in which ODTE exists covers the region for which IDTE exists.

is good with certainty. His evaluation about the asset will be $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$, and he will reject any offer below this value. But also remember that this value is the highest possible evaluation from traders. Therefore, all price offers above $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ are dominated by $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$, and to induce transaction and gather positive profits in each period, the dealer will not price below this value. As a result, we learn that $p_t^g(h^t) = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ for any price history and in good times the dealer always claims a profit of $\frac{\rho}{2}\sigma_g^2$.

If in such an equilibrium, trade only occurs in good times, then in bad times dealer claims zero profit, which is the same as the ISTE payoff. Therefore, we cannot punish dealer's deviation by using payoffs in future periods. As a result, the problem is equivalent to that under one-shot game and this kind of equilibrium is equivalent to ISTE.

Then the next question to ask is, with the dynamic trading environment, can trade also occur in bad times? The following proposition summarizes the necessary and sufficient conditions for the existence of such equilibria. Then the following corollary shows that whenever such an IDTE is played, as long as $\alpha_g \geq \alpha_b$, there exists an ODTE. In other words, if economic states are persistent, then IDTE is always Pareto suboptimal.

Appendix-Proposition 1 (Informative Dynamic Trading Equilibrium). *If there exists a $\pi^b \in [0, \frac{\rho}{2}\sigma_b^2]$ such that*

$$(1 - \frac{\delta(1-\alpha_g)}{(1-\delta)(1-\delta(\alpha_g - \alpha_b))})\pi^b \leq \frac{\rho\sigma_g^2}{2} - J_g + J_b \leq [1 + \delta\frac{1 - \delta\alpha_g - (1-\delta)\alpha_b}{(1-\delta)(1-\delta(\alpha_g-\alpha_b))})]\pi^b,$$

then there exists an informative dynamic trading equilibrium (IDTE) *where the dealer fully reveals* *her private information about current economic state by providing distinct price offers at different times, and trade occurs in both* good *times and* bad *times.*

Proof. See the following Appendix for the proof. □

Appendix-Corollary 1. *If $\alpha_g \geq \alpha_b$, then whenever an* IDTE *exists, there also exists an* ODTE*.*

Proof. See the following Appendix for the proof. □

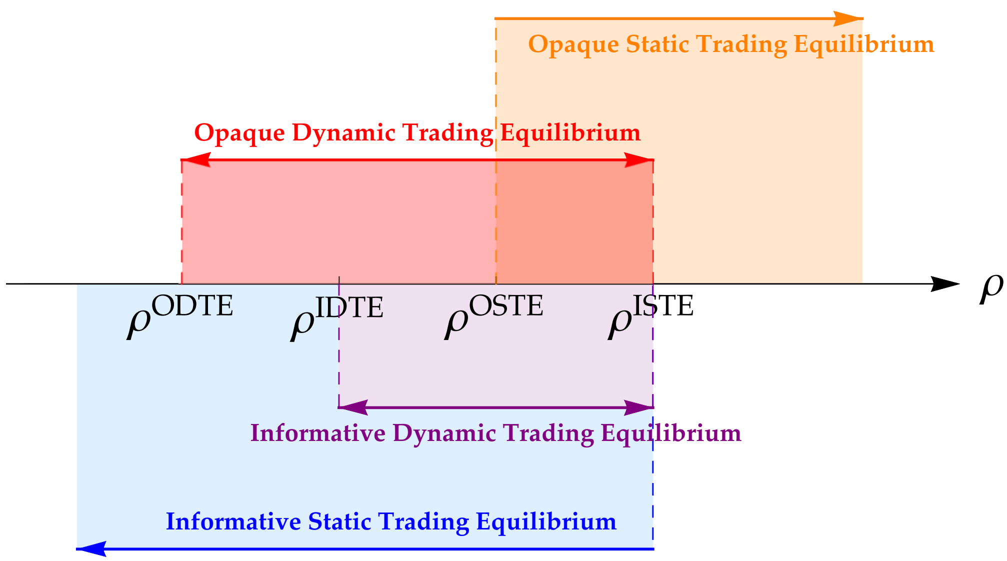

Figure 11: This chart considers all the equilibria (when previous orders are publicly observable). In particular, this plot shows how the dealer's forward-looking incentive can lead to market inefficiency by offering more opaque trade prices. When the economic state is persistent (i.e., $\alpha_g \geq \alpha_b$), we always have $\rho^{\mathsf{ODTE}} < \rho^{\mathsf{IDTE}} < \rho^{\mathsf{ISTE}}$.[26] When traders' risk aversion is sufficiently low (i.e., $\rho < \rho^{\mathsf{ODTE}}$), the only equilibrium available is ISTE, where the dealer reveals her private information in each period.

We now consider the social welfare in IDTE. As trade also occurs in bad times, it gives a higher level of social welfare than that of ISTE. However, as the dealer's offer still reveals her private information, the trader has lower incentive to hedge his asset and the benefit of the trade is impaired. It then follows that the social welfare under an IDTE is still below the maximum level as in OSTE and ODTE. The next Proposition formally displays this result.

[26]Figure 11 depicts the circumstance where $\rho^{\mathsf{IDTE}} < \rho^{\mathsf{OSTE}}$, but it is also possible that $\rho^{\mathsf{IDTE}} \geq \rho^{\mathsf{OSTE}}$

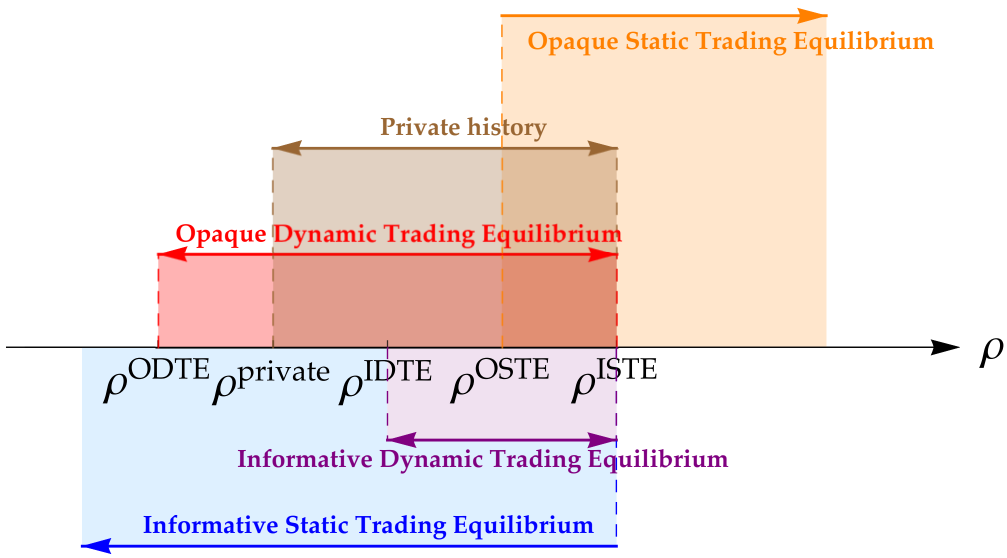

Figure 12: A complete chart that depicts all equilibria discussed in this paper.

Appendix-Proposition 2. *If an* IDTE *exists, then the expected social welfare in period t is*

$$W^{\mathsf{IDTE}} = \frac{\rho}{2}[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2].$$

Moreover, if the dealer is pricing discriminatingly, then this is the most social welfare one can get.

Proof. See the following Appendix for the proof. □

Finally, as mentioned in Section 6.2, whenever it exists, the regulator can implement an IDTE, which provides the maximum price efficiency and market liquidity, and the highest social welfare a market with post-price transparency can get. Specifically, in each period t, the regulator can randomly check a dealer with probability q. The regulator can get access to all past economics states and price offers. If the dealer has been caught not pricing p_t^g in good times or not pricing p_t^b in bad times, then she is banned to participate in all future trade. Similarly, we show that the market transparency increases with the auditing intensity q.

Appendix-Proposition 3. *If for certain parameters and the auditing intensity q, there exists an* IDTE*, then for any $q' \in [q, 1]$, there also exists an* IDTE*.*

Proof. See the following Appendix for the proof. □

C Internet Appendix [Proofs]

Proposition 1 (Opaque Static Trading Equilibrium). *The opaque static trading equilibrium* (OSTE) *exists if and only if*

$$\rho \geq \rho^{\mathsf{OSTE}} = 2\frac{\alpha_{\max}(J_g - J_b)}{\alpha_{\max}(1-\alpha_{\max})(J_g - J_b)^2 + \alpha_{\max}\sigma_g^2 + (1-\alpha_{\max})\sigma_b^2} \tag{7}$$

where $\alpha_{\max} = \max\{\alpha_g, \alpha_b\}$.[27]

Proof. We present our proof in two steps. First, we construct an OSTE to show the "if" part of the statement. As we will discuss later, this equilibrium also happens to be the OSTE with the highest dealer's ex-ante expected payoff. Second, we characterize dealer's incentive compatibility (IC) constraints and trader's individual rationality (IR) constraints and prove the "only if" part.

(if part:)

We prove by construction. We show that the following strategies and beliefs form a PBE:

1. In each period, the dealer offers a price $p_t(\theta_t; h^{t-1}) = p_t^{*\mathsf{UI}}(\theta_{t-1})$.

2. The trader believes that the current underlying economic state is good with probability $\alpha_{\theta_{t-1}}$. That is, $\zeta_t(p_t; h^{t-1}) = \alpha_{\theta_{t-1}}, \forall p_t, h^{t-1}$.

3. The trader will accept a price offer as long as it is weakly higher than $p_t^{*\mathsf{UI}}(\theta_{t-1})$.

To show this is a PBE, we first show it's sequential rational. That is, $p_t^{*\mathsf{UI}}(\theta_{t-1})$ is a best response to the trader's belief $\zeta_t(p_t; h^{t-1})$ and his order decision. In fact, if the dealer wants to trade the asset with the trader, the lowest price she can ask should make him indifferent between keeping or selling the asset. This cutoff price is $p_t = p_t^{*\mathsf{UI}}(\theta_{t-1})$. Condition (7) implies that $p_t^{*\mathsf{UI}}(\theta_{t-1})$ does not exceed $\varphi a_{t-1} + J_b$, and hence is weakly smaller than $\varphi a_{t-1} + J_g$. Therefore, trading is weakly preferred by the dealer than not trading, in both good times and bad times. Whence, the dealer has no other pricing strategies that are more optimal.

[27] The reason that $\alpha_{\max}$ is appeared in the ρ^{OSTE} is because the trader's individual rationality constraint needs to be satisfied when he believes the current economic is good both with probability α_g and with probability α_b. Hence, the larger one is binding. See the proof for more details.

To check for the consistency, actually we are going to show a stronger version of consistency. We will show that constructed beliefs will satisfy the consistency requirement under PBE. We show this by constructing the following sequence of totally-mixed strategies: $\{p^n(\theta_{t-1})\}$ such that $p^n(\theta_{t-1})$ puts $(1-\frac{1}{n})$ probability on $p_t^{*\mathsf{UI}}(\theta_{t-1})$ and uniformly takes values from all other possible strategies. The sequence of beliefs are $\zeta^n = \zeta_t$. Again, since $\{p^n(\theta_{t-1})\}$ does not depend on θ_t, observing the on-path price reveals zero information about θ_t. Consequently, the Bayesian updating posterior after observing any price remains at the prior level ζ_t. In other words, the constructed $(p_t^{*\mathsf{UI}}(\theta_{t-1}), \zeta_t(p_t; h^{t-1}))$ is consistent.

Therefore, we just show the constructed strategies and beliefs form a PBE.

(only if part:)

Suppose $\{p_t(h^{t-1})\}$ and $\zeta_t(p_t; h^{t-1})$ forms an equilibrium and trade always occurs on the equilibrium path. After observing the equilibrium price $p_t(h^{t-1})$, the trader will update his belief according to the Bayes' rule. That is,

$$\zeta_t(p_t(h^{t-1}); h^{t-1}) = \frac{\text{Prob}(\theta_t = g)\text{Prob}(p_t = p_t(h^{t-1})|\theta_t = g)}{\sum\limits_{\theta_t = g,b} \text{Prob}(\theta_t)\text{Prob}(p_t = p_t(h^{t-1})|\theta_t)} = \frac{\alpha_{\theta_{t-1}}}{\alpha_{\theta_{t-1}} + 1 - \alpha_{\theta_{t-1}}} = \alpha_{\theta_{t-1}}.$$

In fact, dealer's offer does not depend on the underlying economic state θ_t, hence contains no information about it. After observing this offer, trader will not update his prior.

The dealer should offer the trader a high enough price such that he is willing to sell his asset. In other words, the price offered by the dealer should be weakly larger than the trader's evaluation of the asset, given his on-path belief $\zeta_t(p_t(h^{t-1}); h^{t-1}) = \alpha_{\theta_{t-1}}$. Thus the trader's individual rationality constraint implies that

$$p_t(h^{t-1}) \geq p_t^{*\mathsf{UI}}(\theta_{t-1}). \tag{21}$$

On the other hand, the dealer should not have an incentive to offer a low price and prevent the trade, both in good times and in bad times. Hence the individual rationality constraints are:

$$p_t(h^{t-1}) \leq \mathbb{E}(a_t|\theta_{t-1} = g) = \varphi a_{t-1} + J_g \tag{22}$$

$$p_t(h^{t-1}) \leq \mathbb{E}(a_t|\theta_{t-1} = b) = \varphi a_{t-1} + J_b \tag{23}$$

Combine equation 21 and 23 together we get

$$
\begin{aligned}
& p_t^{*\mathsf{UI}}(\theta_{t-1}) \le p_t(h^{t-1}) \le \varphi a_{t-1} + J_b \\
\Leftrightarrow\ & p_t^{*\mathsf{UI}} - \varphi a_{t-1} - J_b \le 0 \\
\Leftrightarrow\ & \alpha_{\theta_{t-1}}(J_g - J_b) - \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right] \le 0, \quad \forall \theta_{t-1} \\
\Leftrightarrow\ & \rho \ge \rho^{\mathsf{OSTE}} \equiv 2\frac{\alpha_{max}(J_g - J_b)}{\alpha_{\max}(1-\alpha_{\max})(J_g - J_b)^2 + \alpha_{\max}\sigma_g^2 + (1-\alpha_{\max})\sigma_b^2}.
\end{aligned}
$$

□

Proposition 2 (Informative Static Trading Equilibrium). *Suppose the dealer is myopic (i.e. $\delta = 0$) and is informed about the current economic state (i.e. θ_t). If and only if*

$$\rho < \rho^{\mathsf{ISTE}} = 2\frac{J_{\mathrm{g}} - J_{\mathrm{b}}}{\sigma_{\mathrm{g}}^2}, \tag{8}$$

there exists an ISTE.

Proof. We prove by constructing the following strategies and beliefs:

1. The dealer offers

$$p_t(\theta_t, h^{t-1}) = \begin{cases} p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2 & \text{if } \theta_t = g; \\ p_t^b = \varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 - \varepsilon & \text{if } \theta_t = b. \end{cases}$$

2. At observing a price offer p_t, the trader will believe that the current underlying economic state is good unless $p_t = p_t^b$, in which case he will believe that $\theta_t = b$.

3. At period t, the trader accepts the offer if and only if the price is weakly above p_t^g.

First, we show that trader's strategies are sequential rational given the pricing strategy of the dealer and his own belief of trader. If he is offered p_t^b, then he will believe it is in bad times and his evaluation of the asset will be strictly greater than p_t^b. He will reject an offer p_t^b. For other price offers, his evaluation of the asset will be p_t^g. Therefore, he will accept any price above p_t^g, and reject any price below p_t^g.

Second, we show dealer's strategies constructed above are sequential rational. In fact, in good times, given the trader's belief and strategy:

- if the dealer offers a price at p_t^g, then she can collect an expected payoff of $\frac{\rho}{2}\sigma_g^2 > 0$;

- if the dealer offers a price higher than p_t^g, then the trader will still accept it. But now the dealer is worse off since she pays more for the asset;
- if the dealer offers a price lower than p_t^g. If it is p_t^b, the trader will reject it since it is smaller than the trader's evaluation given his belief that the current period is in bad times. If it is not p_t^b, then it is still lower than p_t^g, which is the trader's evaluation given his belief that it is in good times. Hence, the dealer will get 0, which is strictly lower than $\frac{\rho}{2}\sigma_g^2$, what she can get from offering p_t^g.

Similarly, in bad times:

- if the dealer offers a price at p_t^b, then the trader will reject it and the dealer gets 0;
- if the dealer offers a price other than p_t^b and the trader still rejects it, then the dealer still gets 0;
- if the dealer offers a price other than p_t^b but the trader accepts it, then since the trader believes it is in good times, this price should be weakly higher than p_t^g. But now the dealer's expected payoff has become

$$\varphi a_{t-1} + J_b - p_t^g < 0$$

  due to the inequality condition 8.

Finally, to show consistency, now construct totally mixed strategies $\{p^n(\theta_t)\}$: $p^n(g)$ puts $(1 - \frac{1}{n})$ probability on p_t^g, and takes vales from all other possible strategies uniformly; whereas $p^n(b)$ puts $(1 - \frac{1}{n^2})$ probability on p_t^g, and takes vales from all other possible strategies uniformly. We now show that the limit of the Bayesian posteriors given these strategies will be the one we constructed above. In fact, if observing a price other than p_t^g or p_t^b, then the posterior becomes

$$\begin{aligned}
\lim_{n\to\infty} \text{Prob}(\theta_t = 1|p_t, h^{t-1}) &= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}\text{Prob}(p_t|\theta_t = g)}{\alpha_{\theta_{t-1}}\text{Prob}(p_t|\theta_t = g) + (1-\alpha_{\theta_{t-1}})\text{Prob}(p_t|\theta_t = b)} \\
&= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}\frac{1}{n}}{\alpha_{\theta_{t-1}}\frac{1}{n} + (1-\alpha_{\theta_{t-1}})\frac{1}{n^2}} \\
&= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}}{\alpha_{\theta_{t-1}} + (1-\alpha_{\theta_{t-1}})\frac{1}{n}} = 1.
\end{aligned}$$

If observing a price offer at p_t^g, then the Bayesian posterior becomes

$$\begin{aligned}
\lim_{n\to\infty} \text{Prob}(\theta_t = 1 | p_t^g, h^{t-1}) &= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}} \text{Prob}(p_t^g | \theta_t = g)}{\alpha_{\theta_{t-1}} \text{Prob}(p_t^g | \theta_t = g) + (1 - \alpha_{\theta_{t-1}}) \text{Prob}(p_t^g | \theta_t = b)} \\
&\geq \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}(1 - \frac{1}{n})}{\alpha_{\theta_{t-1}}(1 - \frac{1}{n}) + (1 - \alpha_{\theta_{t-1}})\frac{1}{n^2}} \\
&= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}(n^2 - n)}{\alpha_{\theta_{t-1}}(n^2 - n) + (1 - \alpha_{\theta_{t-1}})} = 1.
\end{aligned}$$

If observing a price offer at p_t^b, then the Bayesian posterior becomes

$$\begin{aligned}
\lim_{n\to\infty} \text{Prob}(\theta_t = 1 | p_t^b, h^{t-1}) &= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}} \text{Prob}(p_t^b | \theta_t = g)}{\alpha_{\theta_{t-1}} \text{Prob}(p_t^b | \theta_t = g) + (1 - \alpha_{\theta_{t-1}}) \text{Prob}(p_t^b | \theta_t = b)} \\
&\leq \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}\frac{1}{n}}{\alpha_{\theta_{t-1}}\frac{1}{n} + (1 - \alpha_{\theta_{t-1}})(1 - \frac{1}{n^2})} \\
&= \lim_{n\to\infty} \frac{\alpha_{\theta_{t-1}}}{\alpha_{\theta_{t-1}} + (1 - \alpha_{\theta_{t-1}})(n - \frac{1}{n})} = 0.
\end{aligned}$$

Therefore, Bayesian posteriors calculated from $\{p^n(\theta_{t-1})\}$ approaches to the trader's beliefs, and we have proved that the construction at the beginning of this proof forms a PBE. □

Corollary 3. *When the informed dealer is myopic, the threshold of* ISTE *is strictly larger than that of* OSTE. *In other words*

$$\rho^{\mathsf{ISTE}} > \rho^{\mathsf{OSTE}}. \tag{9}$$

In addition, for any $\rho \geq \rho^{\mathsf{OSTE}}$, there is a unique maximal PBE outcome, achieved only via an OSTE. *For any $\rho \in [\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$, such an* OSTE *defeats the* ISTE.

Proof. As shown, respectively, in Propositions 1 and 2

$$\rho^{\mathsf{OSTE}} = 2\frac{(J_g - J_b)}{(1 - \alpha_{\max})(J_g - J_b)^2 + \sigma_g^2 + \frac{1-\alpha_{\max}}{\alpha_{\max}}\sigma_b^2},$$

and

$$\rho^{\mathsf{ISTE}} = 2\frac{J_g - J_b}{\sigma_g^2}$$

By a little algebra it is immediate that

$$\frac{(J_g - J_b)}{(1-\alpha_{\max})(J_g - J_b)^2 + \sigma_g^2 + \frac{1-\alpha_{\max}}{\alpha_{\max}}\sigma_b^2} < \frac{J_g - J_b}{\sigma_g^2},$$

completing the proof of the first part.

We now show that the constructed OSTE is a maximal PBE. The first observation is that in static game, there are only two types of equilibria, fully separating ones and fully pooling ones. For $\theta_t = g$, as shown in the analysis of ISTE the equilibrium price is $p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ Given $\rho \geq \rho^{\mathsf{OSTE}}$, it is not hard to check that $p_t^{*\mathsf{UI}} < p_t^g$. For fully pooling equilibrium, $p_t^{*\mathsf{UI}}$ is the lowest price offer the trader can accept. Therefore, the constructed equilibrium generates the maximal payoff for the dealer with private information $\theta = g$. Moreover, conditional on $\rho \geq \rho^{\mathsf{OSTE}}$, when $\theta = b$, the dealer also collects non-negative profits in the constructed equilibrium, which is higher than 0, what she can get in fully equilibrium. Hence, the constructed equilibrium also maximizes dealer's payoff when $\theta = b$.

In fact, we can actually show that when $\rho \in (\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$, the undefeated criterion in Mailath et al. (1993) selects the same set of maximal PBE. Mailath et al. (1993) defines the undefeated criterion in the following way.

Definition 9 (maximal PBE static Mailath et al. (1993))**.** *A pure PBE $(p(\cdot), o(\cdot), \zeta(\cdot))$ defeats another pure PBE $(p'(\cdot), o'(\cdot), \zeta'(\cdot))$ if and only if there exists a price $\hat{p}$ such that*

1. $\forall\theta : p'(\theta) \neq \hat{p}$ *and* $K \equiv \{\theta | p(\theta) = \hat{p}\} \neq \emptyset$;
2. $\forall\theta \in K : U(p(\cdot)|\theta) \geq U(p'(\cdot)|\theta)$, *and* $\exists\theta \in K : U(p(\cdot)|\theta) > U(p'(\cdot)|\theta)$;
3. $\exists\theta \in K : \zeta'(\theta|\hat{p}) \neq \frac{\alpha_\theta \pi(\theta)}{\sum_{\theta=g,b} \alpha_\theta \pi(\theta)}$ *for any* $\pi : \{g, b\} \to [0, 1]$ *satisfying*
   - $\theta' \in K$ *and* $U(p(\cdot)|\theta') > U(p'(\cdot)|\theta') \Rightarrow \pi(\theta') = 1$;
   - $\theta' \notin K \Rightarrow \pi(\theta') = 0$.

We say a pure PBE is undefeated if there is no other pure PBE that defeats it.

To see that an OSTE defeats an ISTE in the sense of Definition 9, we fix a price $\hat{p} = p_t^{*\mathsf{UI}}$. Then $K = \{g, b\}$. For $\forall\theta \in \{g, b\}$, $U_t^{\mathsf{ISTE}}(\theta = g) = \varphi a_{t-1} + J_g - p_t^g = \frac{\rho}{2}\sigma_g^2$,

$U_t^{\mathsf{ISTE}}(\theta = b) = 0$; and

$$
\begin{aligned}
& U_t^{\mathsf{OSTE}}(\theta) \\
= \;& \varphi a_{t-1} + J_\theta - p_t^{*\mathsf{UI}} \\
= \;& \begin{cases} (1-\alpha_{\theta_{t-1}})(J_g - J_b) + \frac{\rho}{2}[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2] & \text{if } \theta = g; \\ -\alpha_{\theta_{t-1}}(J_g - J_b) + \frac{\rho}{2}(\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2) & \text{if } \theta = b. \end{cases}
\end{aligned}
$$

If $\rho > \rho^{\mathsf{OSTE}}$, then one can easily check that $U_t^{\mathsf{OSTE}}(\theta = b) > U_t^{\mathsf{ISTE}} = 0(\theta = b)$. When $\theta = g$,

$$
\begin{aligned}
& U_t^{\mathsf{OSTE}}(\theta = g) - U_t^{\mathsf{ISTE}}(\theta = g) \\
= \;& (1-\alpha_{\theta_{t-1}})[J_g - J_b + \frac{\rho}{2}(\sigma_b^2 - \sigma_g^2 + \alpha_{\theta_{t-1}}(J_g - J_b)^2)] \\
> \;& 0.
\end{aligned}
$$

Second, as $U_t^{\mathsf{OSTE}}(\theta) > U_t^{\mathsf{ISTE}}(\theta)$ for both $\theta = g$ and $\theta = b$, the only feasible $\pi : \{g, b\} \to [0, 1]$ is that $\pi(\theta) = 1$, for all $\theta \in \{g, b\}$. Then

$$
\begin{aligned}
\frac{\alpha_{\theta_{t-1}}\pi(g)}{\alpha_{\theta_{t-1}}\pi(g) + (1-\alpha_{\theta_{t-1}})\pi(b)} &= \alpha_{\theta_{t-1}} \neq \mu^{\mathsf{ISTE}}(\theta = g | p_t^{*\mathsf{UI}}) = 1; \\
\frac{(1-\alpha_{\theta_{t-1}})\pi(b)}{\alpha_{\theta_{t-1}}\pi(g) + (1-\alpha_{\theta_{t-1}})\pi(b)} &= 1 - \alpha_{\theta_{t-1}} \neq \mu^{\mathsf{ISTE}}(\theta = b | p_t^{*\mathsf{UI}}) = 0.
\end{aligned}
$$

Finally, in static case there are only two types of pure strategy equilibria, OSTE and ISTE, and the former defeats the latter, it is obviously that OSTE are the only equilibria that survive the refinement in Mailath et al. (1993). □

Proposition 3 (Opaque Dynamic Trading Equilibrium). *There exists a $\rho^{\mathsf{ODTE}} \leq \rho^{\mathsf{OSTE}}$ such that if and only if $\rho \geq \rho^{\mathsf{ODTE}}$, there exists an opaque dynamic trading equilibrium (*ODTE*) in which the dealer always conceals her private information about current economic state and trades occur in both* good *times and* bad *times. Moreover, for any $\rho \geq \rho^{\mathsf{ODTE}}$, there is a unique maximal PBE outcome, achieved only through an* ODTE. *Specifically, for any $\rho \in [\rho^{\mathsf{ODTE}}, \rho^{\mathsf{ISTE}}]$, there exists an* ODTE *that defeats and Pareto dominates the PBE where* ISTE *is played in every period.*

Proof. We prove the proposition in three steps. First, we show the "if" part by construction. Second, we prove the "only if" part by showing the off path payoff in our construction above is the harshest feasible punishment one can put for deviation. Third, we show that he cutoff for the existence of ODTE is weakly below that of OSTE. Finally, we show the constructed ODTE is a maximal PBE when there is equilibrium multiplicity.

(I) if part

We will first construct a set of strategies and beliefs and characterize the sufficient conditions for this set to become a PBE. We then show that more risk-averse traders will make these conditions easier to sustain and this sufficient condition provides a lower bound for ρ, trader's risk aversion coefficients.

We now show the following construction forms a PBE.

- If the dealer has not deviated in previous periods, then
  1. the dealer offers a price $p_t(\theta_t, h^{t-1}) = p_t^{*\mathsf{UI}}(\theta_{t-1})$;
  2. the trader believes that the current underlying economic state is good with probability $\alpha_{\theta_{t-1}}$ i.e., $\zeta(p_t; h^{t-1}) = \alpha_{\theta_{t-1}}, \forall p_t, h^{t-1}$;
  3. the trader will accept any price offers that is weakly above $p_t^{*\mathsf{UI}}(\theta_{t-1})$.
- If the dealer deviates previously, then both parties observe this and play an informative static trading equilibrium (ISTE).

(i) consistency

It is easy to check that the belief construction follows the Bayes rule whenever it applies.

(ii) sequential rationality

First, there is no incentives for trader to deviate or dealer to deviate off the equilibrium path. It follows immediately that given his belief, the trader has no incentive to deviate on the equilibrium path. Since ISTE is a one-shot Nash equilibrium, both parties have no incentives to deviate if previous deviation has been observed.

Second, for dealer along the equilibrium path, at period t, offering a price strictly higher than $p_t^{*\mathsf{UI}}(\theta_{t-1})$ will not change trader's response, but will lower her payoff for the current period and trigger punishment in all future periods. As a result, she has no incentive to do that.

Third, the only possible deviation left is that the dealer may want to offer a price lower than $p_t^{*\mathsf{UI}}(\theta_{t-1})$ and refuse the trade. Again in good times, this lead to a loss of positive profit in the current period, and trigger punishment in all future periods. Hence it is not profitable to deviate in good times.

Finally, in bad times, dealer's incentive compatibility constraint becomes:

$$U_t^{\mathsf{ODTE}}(b,\theta_{t-1}) \geq \underbrace{(1-\delta)\times 0}_{\substack{\text{dealer's static payoff}\\ \text{after rejecting the trade}}} + \underbrace{\delta[\alpha_b U_t^{\mathsf{off}}(g)+(1-\alpha_b)U_t^{\mathsf{off}}(b)]}_{\substack{\text{dealer's continuous}\\ \text{off-path payoff}}}, \forall\theta_{t-1} \tag{24}$$

where $\forall\theta_t,\theta_{t-1}$,

$$\begin{aligned} U_t^{\mathsf{ODTE}}(\theta_t,\theta_{t-1}) &= (1-\delta)(\varphi a_{t-1}+J_{\theta_t}-p_t^{*\mathsf{UI}}(\theta_{t-1}))+\delta[\alpha_{\theta_t}U_{t+1}^{\mathsf{ODTE}}(g,\theta_t)+(1-\alpha_{\theta_t})U_{t+1}^{\mathsf{ODTE}}(b,\theta_t)] \\ U_t^{\mathsf{off}}(\theta_t) &= (1-\delta)\mathbf{1}(\theta_t=g)\cdot\frac{\rho}{2}\sigma_g^2+\delta[\alpha_{\theta_t}U_{t+1}^{\mathsf{off}}(g)+(1-\alpha_{\theta_t})U_{t+1}^{\mathsf{off}}(b)]. \end{aligned}$$

are dealer's on-path and off-path continuous payoffs respectively, given the current economic state is θ_t and the last period economic state is θ_{t-1}.

(iii) lower bound of ρ

We now show that condition (29) provides a lower bound for ρ, trader's risk aversion coefficient.

First, observe that both

$$\varphi a_{t-1}+J_g-p_t^{*\mathsf{UI}}(\theta_{t-1})=(1-\alpha_{\theta_{t-1}})(J_g-J_b)+\frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g-J_b)^2+\alpha_{\theta_{t-1}}\sigma_g^2+(1-\alpha_{\theta_{t-1}})\sigma_b^2]$$

and

$$\varphi a_{t-1}+J_b-p_t^{*\mathsf{UI}}(\theta_{t-1})=\alpha_{\theta_{t-1}}(J_b-J_g)+\frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g-J_b)^2+\alpha_{\theta_{t-1}}\sigma_g^2+(1-\alpha_{\theta_{t-1}})\sigma_b^2]$$

are independent of the calender time t. Since both dealer's static payoffs and her on-path equilibrium strategies solely depend on the current economic state θ_t and last period economic state θ_{t-1}, so do her on-path continuous payoffs in this equilibrium . Let's denote $\Lambda^{\theta_t,\theta_{t-1}} \equiv \varphi a_{t-1}+J_{\theta_t}-p_t^{*\mathsf{UI}}(\theta_{t-1})$ and $U^{\mathsf{ODTE}}(\theta_t,\theta_{t-1}) \equiv U_t^{\mathsf{ODTE}}(\theta_t,\theta_{t-1})$. One can then solve the on-path payoffs recursively from the equation system:

$$U^{\mathsf{ODTE}}(\theta_t,\theta_{t-1})=(1-\delta)\Lambda^{\theta_t,\theta_{t-1}}+\delta[\alpha_{\theta_t}U^{\mathsf{ODTE}}(g,\theta_t)+(1-\alpha_{\theta_t})U^{\mathsf{ODTE}}(b,\theta_t)], \forall\theta_t,\theta_{t-1}.$$

Or in the matrix form,

$$\begin{bmatrix} 1-\delta\alpha_g & 0 & -\delta(1-\alpha_g) & 0 \\ -\delta\alpha_g & 1 & -\delta(1-\alpha_g) & 0 \\ 0 & -\delta\alpha_b & 1 & -\delta(1-\alpha_b) \\ 0 & -\delta\alpha_b & 0 & 1-\delta(1-\alpha_b) \end{bmatrix} \cdot \begin{bmatrix} U^{\mathsf{ODTE}}(g,g) \\ U^{\mathsf{ODTE}}(g,b) \\ U^{\mathsf{ODTE}}(b,g) \\ U^{\mathsf{ODTE}}(b,b) \end{bmatrix} = (1-\delta) \cdot \begin{bmatrix} \Lambda^{g,g} \\ \Lambda^{g,b} \\ \Lambda^{b,g} \\ \Lambda^{b,b} \end{bmatrix}.$$

Similarly $U_t^{\mathsf{off}}(\theta_t)$ is stationary and can be solved from

$$\begin{bmatrix} 1-\delta\alpha_g & -\delta(1-\alpha_g) \\ -\delta\alpha_b & 1-\delta(1-\alpha_b) \end{bmatrix} \cdot \begin{bmatrix} U^{\mathsf{off}}(g) \\ U^{\mathsf{off}}(b) \end{bmatrix} = (1-\delta) \begin{bmatrix} \frac{\rho}{2}\sigma_g^2 \\ 0 \end{bmatrix}$$

The following lemma shows that an increase in ρ will relax inequality (29), the condition for the existence of an ODTE. In fact, the exact cutoff ρ^{ODTE} is obtained when equality holds.

Lemma 2.1. *If for trader's risk aversion coefficient ρ_1, the PBE constructed above forms to be an* ODTE, *then for any $\rho_2 > \rho_1$,it still remains as an* ODTE *if trader's risk aversion coefficient is ρ_2.*

Proof. Given the construction and the analysis right after, we only need to check that dealer's IC constraint in bad times still holds for ρ_2. Fix any realization of economic states $\{\theta_\tau\}_{\tau=t}^{\infty}$ starting from the current period t, then at any future period $\tau > t$, the loss due to the deviation punishment becomes $\Lambda^{\theta_\tau,\theta_{\tau-1}} - \mathbf{1}(\theta_\tau = g) \cdot \frac{\rho_2}{2}\sigma_g^2$. When $\theta_\tau = g$, this difference

$$\begin{aligned} \Lambda^{g,\theta_{\tau-1}} - \frac{\rho_2}{2}\sigma_g^2 &= (1-\alpha_{\theta_{\tau-1}})(J_g - J_b) + \frac{\rho_2(1-\alpha_{\theta_{\tau-1}})}{2}[\alpha_{\theta_{\tau-1}}(J_g-J_b)^2 + \sigma_b^2 - \sigma_g^2] \\ &\geq (1-\alpha_{\theta_{\tau-1}})(J_g - J_b) + \frac{\rho_1(1-\alpha_{\theta_{\tau-1}})}{2}[\alpha_{\theta_{\tau-1}}(J_g-J_b)^2 + \sigma_b^2 - \sigma_g^2] \end{aligned}$$

is greater than the deviation punishment with less risk-averse traders. Similarly when $\theta_\tau = b$, then the difference

$$\begin{aligned} \Lambda^{b,\theta_{\tau-1}} - 0 &= \alpha_{\theta_{\tau-1}}(J_b - J_g) + \frac{\rho_2}{2}[\alpha_{\theta_{\tau-1}}(1-\alpha_{\theta_{\tau-1}})(J_g-J_b)^2 + (1-\alpha_{\theta_{\tau-1}})\sigma_b^2 + \alpha_{\theta_{\tau-1}}\sigma_g^2] \\ &\geq \alpha_{\theta_{\tau-1}}(J_b - J_g) + \frac{\rho_1}{2}[\alpha_{\theta_{\tau-1}}(1-\alpha_{\theta_{\tau-1}})(J_g-J_b)^2 + (1-\alpha_{\theta_{\tau-1}})\sigma_b^2 + \alpha_{\theta_{\tau-1}}\sigma_g^2] \end{aligned}$$

is also greater than that under the case of ρ_1.

Furthermore, at period t, the deviation gain becomes $0 - \Lambda^{b,\theta_{t-1}}$ and decreases in

ρ, implying the deviation gain is smaller with more risk-averse trader.

Therefore, with more risk-averse trader, or as ρ increases, dealer's current gain from declining to trade is smaller, but she now faces harsher punishment in all future periods in all realization of economic state shocks. Hence, condition (24) is easier to sustain with more risk-averse trader. In other words, if condition (24) holds for ρ_1, then it also holds for ρ_2 and with ρ_2 there exists anODTE as constructed. $\square$

(II) only if part

First, we observe that ISTE is the harshest punishment for dealer's deviation. Then, we show that the ODTE constructed in the "if" part is actually the one easiest to sustain and provides a lower bound for the existence of an ODTE (even without the highest dealer's payoff restriction).

Before we move on to prove that, the first observation is that .

Lemma 2.2. *For any conditional belief $\zeta(p_t; \cdot)$, the dealer's ex-post* ISTE *payoffs consists of the min-max static payoffs given that the trader best responses to her strategy. Specifically, we have*

$$\begin{aligned} \min_{\zeta(p_t;h^{t-1})} \max_{p_g, p_b} \min_{o_t \in \mathsf{BR}_{\zeta(p_t;h^{t-1})}} (\varphi a_{t-1} + J_g - p_g) \cdot o_t(p_g) &= \frac{\rho}{2}\sigma_g^2 \\ \min_{\zeta(p_t;h^{t-1})} \max_{p_g, p_b} \min_{o_t \in \mathsf{BR}_{\zeta(p_t;h^{t-1})}} (\varphi a_{t-1} + J_b - p_b) \cdot o_t(p_b) &= 0. \end{aligned}$$

Proof of the Lemma. In bad times, first observe that the dealer can always offer a low enough price to decline the trade and collect an ex-post payoff of 0.

Next, we show that for the following belief, she can do no better than that. Suppose the trader believes that the current period is in good times unless $\varphi a_{t-1} + J_b$ is offered, at which he believes it is in bad times, then he will only accept any offer above $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ or exactly at $\varphi a_{t-1} + J_b$. Offering a price weakly higher than $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ leads to non-positive payoff, while offering $\varphi a_{t-1} + J_b$ leads to zero payoff. Hence, with this trader belief and trader best responding to it, the best case dealer's ex-post payoff is 0. In other words,

$$\max_{p_b} \min_{o_t \in \mathsf{BR}_{\zeta(p_t;\cdot)}} (\varphi a_{t-1} + J_b - p_b) \cdot o_t(p_b) \geq 0, \quad \forall \zeta(p_t; \cdot).$$

In good times, first observe that the dealer's ex-post payoff will be at least $\frac{\rho}{2}\sigma_g^2$. In fact, for any $\zeta(p_t;\cdot)$,

$$\begin{aligned}\mathbb{E}_{\zeta(\cdot|p)}(a_t|h^{t-1}) &= \varphi a_{t-1} + J_b + \zeta(p_t;\cdot)(J_g - J_b) - \frac{\rho}{2}\zeta(p_t;\cdot)(1-\zeta(p_t;\cdot))(J_g - J_b)^2 \\ &\quad -\frac{\rho}{2}[\zeta(p_t;\cdot)\sigma_g^2 + (1-\zeta(p_t;\cdot))\sigma_b^2] \\ &\leq \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2.\end{aligned}$$

Therefore, the dealer can always offer a price $p'_g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ to confirm the order from the trader and guarantee an ex-post payoff of $\frac{\rho}{2}\sigma_g^2$ for sure.

Next, this is the best she can get if the trade believes that the current period is always in good times. As only prices weakly above p'_g will be accepted by the trader. So we have,

$$\max_{p_g} \min_{o_t \in \mathsf{BR}_{\zeta(p_t;\cdot)}} (\varphi a_{t-1} + J_g - p_g)\cdot o_t(p_g) \geq \frac{\rho}{2}\sigma_g^2, \quad \forall \zeta(p_t;\cdot).$$

Finally, these ex-post payoffs can be obtained in ISTE. In conclusion, for any arbitrary beliefs trader may hold, dealer's ex-post ISTE payoffs ($\frac{\rho}{2}\sigma_g^2$ in good times and 0 in bad times) are no larger than the min-max static payoffs given that the trader best responses to her strategy. □

We now go back to prove the "only if" part. Suppose in an ODTE the dealer offers p_t^* at time t, and trade always occurs. Therefore, the trader learns no information via observing the on-path price history, and for any consistent belief system, her posteriors should stay the same as the prior, that $\zeta(p_t^*; h^{t-1}) = \alpha_{\theta_{t-1}}$. Therefore, for the trader to accept this offer, we need $p_t^* \geq p_t^{*\mathsf{UI}}(\theta_{t-1})$.

From the dealer's perspective, the continuous payoff $\hat{U}_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1})$, given the current and the last economic states are θ_t and θ_{t-1}, will be no more than $U^{\mathsf{ODTE}}(\theta_t, \theta_{t-1})$, because in each period starting from t, the dealer is offering a price greater than the on-path price in ODTE constructed in the if part above. Therefore, we have

$$\begin{aligned}& U^{\mathsf{ODTE}}(b, \theta_{t-1}) \\ \geq\ & \hat{U}_t^{\mathsf{ODTE}}(b, \theta_{t-1}) \\ \overset{(1)}{\geq}\ & (1-\delta)\times 0 + \sum_{k=1}^{\infty}\delta^k \\ & \sum_{h^{t+k}|h^t} \Pr(h^{t+k}|h^t) \max_{p_g, p_b} \min_{o_{t+k}\in \mathsf{BR}_{\zeta_{t+k}(p_t;\cdot)}} (\varphi a_{t+k-1} + J_{\theta t+k} - p_{t+k}(\theta_{t+k}, h^{t+k-1}))\cdot o_{t+k}(p_{t+k}(\theta_{t+k}, h^{t+k-1})) \\ \overset{(2)}{\geq}\ & (1-\delta)\times 0 + \delta[\alpha_b U^{\mathsf{off}}(g) + (1-\alpha_b)U^{\mathsf{off}}(b)]\end{aligned}$$

Inequality (1) follows from the incentive compatibility of the dealer in bad times. inequality (2) comes from Lemma 2.2, that the lowest min-max payoffs in each following history are weakly larger than ISTE payoffs, which are the off-path payoffs. Therefore, whenever an ODTE exists, condition (29) holds, and solving the above inequality we get $\rho \geq \rho^{\mathsf{ODTE}}$.

(III) $\rho^{\mathsf{OSTE}} \geq \rho^{\mathsf{ODTE}}$

we prove the following claim. That is, whenever an OSTE can sustain, so can an ODTE.

Lemma 3. *Fix all parameter values, whenever there is an* ODTE *equilibrium, there also exists an* OSTE *equilibrium.*

Proof. Suppose σ^* is such an OSTE, then dealer's IC constraint in bad times implies that $\varphi a_{t-1} + J_b - p_t(h^t) \geq 0$. Trader's IR constraint implies that $p_t(h^{t-1}) \geq p_t^{*\mathsf{UI}}(\theta_{t-1})$. Therefore,

$$\varphi a_{t-1} + J_b - p^{\mathsf{UI}}(\theta_{t-1}) \geq 0$$

Consider the ODTE constructed in the "if" part of this proof. From above we know this construction consists of an equilibrium in our dynamic trading game if the following inequality holds, $\forall \theta_{t-1}$,

$$\begin{aligned} & (1-\delta)(\varphi a_{t-1} + J_b - p^{\mathsf{UI}}(\theta_{t-1})) + \delta[\alpha_b U^{\mathsf{ODTE}}(g,b) + (1-\alpha_b)U^{\mathsf{ODTE}}(b,b)] \\ \geq \ & (1-\delta) \times 0 + \delta[\alpha_b U^{\mathsf{off}}(g) + (1-\alpha_b)U^{\mathsf{off}}(b)]. \end{aligned} \tag{25}$$

But from Lemma 2.2, $U^{\mathsf{off}}(\theta_t)$ is the lowest possible dealer's ex-post payoff given traders best response to dealer's offers. Therefore, in all future periods and at all future histories, dealer derives weakly lower payoff on the punishment stage than on the equilibrium path.

$$U^{\mathsf{ODTE}}(\theta_t, b) \geq U^{\mathsf{off}}(\theta_t), \forall \theta_t$$

and inequality (25) is satisfied. $\square$

(IV) check for undefeated criterion

First, in the ODTE constructed at the beginning of the proof, σ^{ODTE}, along the equilibrium path, opaque pricing is always provided by the dealer, thus the maximal ex-ante social surplus is achieved in every period from our analysis in Section 6.3. The next observation is that in σ^{ODTE}, the dealer collects all the social surplus. Thus, along the equilibrium path, she achieves the maximal payoff she can ever get in a PBE. Therefore, this PBE consists of an maximal PBE, which defeats and Pareto dominates the equilibrium where ISTE is played in every period.

More importantly, we show that this maximal PBE outcome is unique and only achieved via an ODTE. We prove by contradiction. Suppose not and there is another PBE $\sigma' = (p'(\cdot), o'(\cdot), \zeta'(\cdot))$ that is a maximal PBE. In this PBE, in some periods, along the equilibrium path, the dealer offers discriminatingly. Suppose t is the earliest one of such a period. Then again follow the same analysis in Section 6.3, as in period t the dealer reveals her private information about θ_t, she will collect strictly lower ex-ante payoff than in σ^{ODTE}. The continuation payoff of σ^{ODTE}, as it already achieves the highest feasible level, is also weakly higher than that of σ'. Thus, we have $U_t(p'(\cdot), o'(\cdot), \zeta'(\cdot)) < U_t(p^{\mathsf{ODTE}}(\cdot), o^{\mathsf{ODTE}}(\cdot), \zeta^{\mathsf{ODTE}}(\cdot))$, which contradicts with our definition of maximal PBE and completes the proof.

□

Proposition 5 (private transaction history). *Suppose future traders can only observe whether a trade occurs or not at period t via a noisy signal y_t, which is independently drawn from the distribution $F_{o_t}(\cdot)$. There exists a $\rho^{\mathsf{private}} \in [\rho^{\mathsf{ODTE}}, \rho^{\mathsf{OSTE}}]$ such that the following statement is true: if and only if $\rho \geq \rho^{\mathsf{private}}$, then there exists a private history equilibrium defined above. Moreover, for any $\rho \in [\rho^{\mathsf{private}}, \rho^{\mathsf{ISTE}}]$, there exists a private history equilibrium that defeats and Pareto dominates the equilibrium where* ISTE *is played in every period.*

Proof. As in the proof of Proposition 3, we first present the "if" part, followed by the proof of "only if" part, and then compare the threshold of trader's risk aversion coefficient in this private order history setup with those of ODTE and OSTE. Finally, we check for maximal PBE.

(I) if part

We prove the "if" part by construction in three steps. First let's fix a partition (Y_1 and Y_0) of the signal space Y, and denote the corresponding probabilities as $f_i = \text{Prob}(y_s \in Y_1 | o_t = i), i = 1, 0, \forall s$. We then construct specific type of private equilibrium displayed below. Next, we check for consistency and the sequential rationality, and characterize $\rho^{\mathsf{private}}(f_1, f_0)$. Finally, we search over all partitions of the signal space Y to find the lower bound of trader's risk aversion coefficient ρ^{private} for the existence of an opaque equilibrium in private order history setup.

The private equilibrium we construct is as follows:

1. If $y^s \in Y_1, \forall s \leq t-1$, then in period t

   - the dealer chooses to offer a price $p_t^*(h^{t-1})$;
   - the trader holds his prior belief after observing $p_t^*(h^{t-1})$ and believes for sure that the economic state is good for all other prices, i.e.,

$$\zeta(p_t; h^{t-1}) = \begin{cases} \alpha_{\theta_{t-1}} & \text{if } p_t = p_t^*(h^{t-1}) \\ 1 & \text{if } p_t \neq p_t^*(h^{t-1}) \end{cases}, \forall h^{t-1};$$

   - traders will accept any prices weakly above p_t^g or exactly at $p_t^*(h^{t-1})$, and decline all other prices.

2. If $y^s \in Y_0$ for some $s \leq t-1$, then at period t both parties play an ISTE equilibrium.[28]

Next, to see sequential rationality, we characterize dealer's incentive compatibility constraint in bad times and check other individual rationality and incentive compatibility constraints for both players at all times are fulfilled. One can check that no profitable deviations off the path. On the equilibrium path, the trader is best responding and the dealer has no incentive to offer a higher price at all times or to offer a lower price in good times. Therefore, the only condition left to check becomes the incentive compatibility constraint of the dealer in bad times: for $\forall \theta_{t-1}$,

$$\begin{aligned} U_t^{\text{private}}(b, \theta_{t-1}; f_1, f_0) \geq & \underbrace{(1-\delta) \times 0}_{\text{dealer's static payoff after rejecting the trade}} \\ & + \underbrace{\delta \cdot f_0 \cdot [\alpha_b U_{t+1}^{\text{private}}(g, b; f_1, f_0) + (1-\alpha_b) U_{t+1}^{\text{private}}(b, b; f_1, f_0)]}_{\text{dealer's continuous if } y_t \in Y_1} \\ & + \underbrace{\delta \cdot (1-f_0) \cdot [\alpha_b U_{t+1}^{\text{off}}(g) + (1-\alpha_b) U_{t+1}^{\text{off}}(b)]}_{\text{dealer's continuous payoff if } y_t \in Y_0} \end{aligned} \tag{26}$$

[28]Here we assume once the game moves into the punishment stage, it never comes back. Future works can study more complicated structure like the one in Ellison (1994), where there is low probability that the game moves back to the first stage from the punishment stage.

where $\forall \theta_t, \theta_{t-1}$,

$$\begin{aligned} U_t^{\text{private}}(\theta_t, \theta_{t-1}; f_1, f_0) &= (1-\delta)(\varphi a_{t-1} + J_{\theta_t} - p_t^{*\text{UI}}(\theta_{t-1})) \\ &\quad + \delta f_1 [\alpha_{\theta_t} U_{t+1}^{\text{private}}(g, \theta_t; f_1, f_0) + (1-\alpha_{\theta_t}) U_{t+1}^{\text{private}}(b, \theta_t; f_1, f_0)] \\ &\quad + \delta \cdot (1-f_1) \cdot [\alpha_b U_{t+1}^{\text{off}}(g) + (1-\alpha_b) U_{t+1}^{\text{off}}(b)]. \end{aligned}$$

and $U_t^{\text{off}}(\theta_t)$ is the continuous payoff on the punishment stage or in the ISTE equilibrium, which is no different from that in public history environment.

Similarly we have

$$\Lambda^{\theta_t, \theta_{t-1}} = \begin{cases} (1-\alpha_{\theta_{t-1}})(J_g - J_b) - \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2] & \text{if } \theta_t = g \\ \alpha_{\theta_{t-1}}(J_g - J_b) - \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2] & \text{if } \theta_t = b, \end{cases}$$

so by symmetry we have $U^{\text{private}}(\theta_t, \theta_{t-1}; f_1, f_0) \equiv U_t^{\text{private}}(\theta_t, \theta_{t-1}; f_1, f_0)$.

Rearranging we get condition 26 is equivalent to

$$\begin{aligned} & (1-\delta)\Lambda^{b, \theta_{t-1}} + \delta(f_1 - f_0)[\alpha_b U^{\text{private}}(g, b; f_1, f_0) + (1-\alpha_b) U^{\text{private}}(b, b; f_1, f_0)] \\ \geq\ & \delta(f_1 - f_0) \cdot [\alpha_b U_{t+1}^{\text{off}}(g) + (1-\alpha_b) U_{t+1}^{\text{off}}(b)]. \end{aligned} \tag{27}$$

$\rho^{\text{private}}(f_1, f_0)$ is constructed as the smallest ρ such that condition 27 is satisfied, i.e., the ρ that makes the equality holds in condition 27. Following the similar steps as in Lemma 2.1, we can claim that for any $\rho \geq \rho^{\text{private}}(f_1, f_0)$, the dealer's IC constraints in bad times are satisfied. The reason is that an increase in ρ reduces the deviation gain in current period as well as enlarges the punishment loss in all future periods at all realizations.

The check of consistency is the same as in the proof of Proposition 3. Therefore, the strategies and beliefs constructed at the beginning of this proof form a perfect Bayesian equilibrium.

Finally, we find the optimal partition of the signal space Y that derives the lowest possible $\rho^{\text{private}}(f_1, f_0)$:

$$\rho^{\text{private}} = \min_{\substack{Y_1, Y_0 \ s.t. \\ Y_1 \cap Y_0 = \emptyset, Y_1 \cup Y_0 = Y, Y_1 \neq \emptyset}} \rho^{\text{private}}(f_1, f_0).$$

(II) only if part

To check that $\rho \geq \rho^{\text{private}}$ is necessary for the existence of an opaque dynamic trading equilibrium, fix any ODTE σ in this model with imperfect observation of previous orders. Suppose in this equilibrium the dealer offers p_t on the equilibrium path, and the probability of not triggering a punishment (staying on-path) is f_1' and f_0', given that the dealer not deviates and deviates at period t correspondingly. We adopt the same idea as in Proposition 3. Because playing ISTE forever is the harshest punishment that can be put on the dealer, and $p_t^{*\mathsf{UI}}(\theta_{t-1})$ is the lowest price the dealer can charge to still keep the trader. We have

$$\begin{aligned}
& U_t^{\mathsf{private}}(b,\theta_{t-1};f_1',f_0') \\
= \;& (1-\delta)(\varphi a_{t-1}+J_b-p_t)+\delta f_{1t}'[\alpha_b U_{t+1}^{\mathsf{private}}(g,b;f_1',f_0')+(1-\alpha_b)U_{t+1}^{\mathsf{private}}(b,b;f_1',f_0')]+ \\
& \delta(1-f_1')\cdot[\min_{\zeta(p;\cdot)}\max_{p_g,p_b}\min_{o_t\in\mathsf{BR}_{\zeta(p;\cdot)}}\alpha_b(\varphi a_{t-1}+J_g-p_g)\cdot o_t(p_g)+(1-\alpha_b)(\varphi a_{t-1}+J_b-p_b)\cdot o_t(p_b)] \\
\overset{(1)}{\geq} \;& (1-\delta)\times 0+\delta f_0'[\alpha_b U_{t+1}^{\mathsf{private}}(g,b;f_1',f_0')+(1-\alpha_b)U_{t+1}^{\mathsf{private}}(b,b;f_1',f_0')]+ \\
& \delta(1-f_0')\cdot[\min_{\zeta(p;\cdot)}\max_{p_g,p_b}\min_{o_t\in\mathsf{BR}_{\zeta(p;\cdot)}}\alpha_b(\varphi a_{t-1}+J_g-p_g)\cdot o_t(p_g))+(1-\alpha_b)(\varphi a_{t-1}+J_b-p_b)\cdot o_t(p_b)],
\end{aligned}$$

$$\begin{aligned}
\overset{(2)}{\Leftrightarrow} \;& (1-\delta)(\varphi a_{t-1}+J_b-p_t)+\delta f_1'[\alpha_b U_{t+1}^{\mathsf{private}}(g,b;f_1',f_0')+(1-\alpha_b)U_{t+1}^{\mathsf{private}}(b,b;f_1',f_0')]+ \\
& \delta(1-f_1')\cdot\frac{\rho}{2}\alpha_b\sigma_g^2 \\
\geq \;& (1-\delta)\times 0+\delta f_0'[\alpha_b U_{t+1}^{\mathsf{private}}(g,b;f_1',f_0')+(1-\alpha_b)U_{t+1}^{\mathsf{private}}(b,b;f_1',f_0')]+\delta(1-f_0')\cdot\frac{\rho}{2}\alpha_b\sigma_g^2
\end{aligned}$$

Hence

$$\begin{aligned}
& (1-\delta)\Lambda^{b,\theta_{t-1}}+\delta(f_1-f_0)[\alpha_b U^{\mathsf{private}}(g,\theta_t;f_1,f_0)+(1-\alpha_b)U^{\mathsf{private}}(b,\theta_t;f_1,f_0)-\frac{\rho}{2}\alpha_b\sigma_g^2] \\
\overset{(3)}{\geq} \;& (1-\delta)(\varphi a_{t-1}+J_b-p_t)+\delta(f_1-f_0)[\alpha_b U_{t+1}^{\mathsf{private}}(g,b;f_1,f_0)+(1-\alpha_b)U_{t+1}^{\mathsf{private}}(b,b;f_1,f_0)-\frac{\rho}{2}\alpha_b\sigma_g^2] \\
\overset{(4)}{\geq} \;& (1-\delta)(\varphi a_{t-1}+J_b-p_t)+\delta(f_1'-f_0')[\alpha_b U_{t+1}^{\mathsf{private}}(g,b;f_1',f_0')+(1-\alpha_b)U_{t+1}^{\mathsf{private}}(b,b;f_1',f_0')-\frac{\rho}{2}\alpha_b\sigma_g^2] \\
\geq \;& 0.
\end{aligned}$$

Here inequality (1) comes from dealer's IC constraint in bad times, equivalence (2) is from Lemma 2.2, inequality (3) comes from the fact that $p_t \geq p_t^{*\mathsf{UI}}$, and inequality (4) is due to the construction of f_1 and f_0, and the fact that $\frac{\rho}{2}\alpha_b\sigma_g^2$ is the min-max payoff.

Therefore, we derive a necessary condition for the existence of opaque equilibrium in private order history setup and provide a lower bound ρ^{private} for ρ.

(III) $\rho^{\mathsf{ODTE}} \leq \rho^{\mathsf{private}} \leq \rho^{\mathsf{OSTE}}$

First, to show that $\rho^{\text{private}} \leq \rho^{\text{OSTE}}$, notice that for any ρ such that there exists an opaque static trading equilibrium $(s^{\text{OSTE}}, \zeta^{\text{OSTE}}(\cdot))$, then there exists a private history equilibrium where the dealer is forward-looking and imperfectly observing about whether the trade occurs. In fact, given an OSTE equilibrium, on the equilibrium path, both players choose the OSTE strategies and beliefs, off the equilibrium path, both players choose the ISTE strategies and beliefs. As shown previously in the proof, consistency follows with similar construction. There is no incentives for the trader to deviate, nor for both players if the signal to play off-path is revealed. It also follows immediately that the dealer has no incentives to enhance the price offer, or lower it in good times. Therefore, the only condition left to check is that dealer's IC constraint in bad times, i.e., condition (27). However, given the on-path strategies is an OSTE, we have $\Lambda^{b,\theta_{t-1}} \geq 0$. Offering a lower price to deter trade in bad times doesn't even provide a static gain, and enhances the probability of punishment in future periods. Hence, no dealer will do this in bad times and the condition is satisfied. We just show that whenever $\rho \geq \rho^{\text{OSTE}}$, a private history equilibrium exists, implying that $\rho^{\text{private}} \leq \rho^{\text{OSTE}}$.

Second, to check that $\rho^{\text{private}} \geq \rho^{\text{ODTE}}$, we show that for any $\rho \geq \rho^{\text{private}}$, there exists an ODTE. In fact, consider the equilibrium we constructed at the beginning of this proof. To see that this forms an ODTE, we only need to check dealer's IC constraint in bad times. That is,

$$(1-\delta)\Lambda^{b,\theta_{t-1}} + \delta[\alpha_{\theta_t} U^{\text{ODTE}}(g,\theta_t) + (1-\alpha_{\theta_t})U^{\text{ODTE}}(b,\theta_t)] \geq \delta\frac{\rho}{2}\alpha_b\sigma_g^2. \tag{28}$$

Now notice with the same strategies and beliefs, in ODTE, if the dealer follows the on-path pricing rule, the probability that a punishment is triggered is higher than that in private history equilibrium. Therefore, we have $U^{\text{ODTE}}(\theta_t,\theta_{t-1}) \geq U^{\text{private}}(\theta_t,\theta_{t-1})$. Dealer's IC constraint in bad times in private history equilibrium implies that

$$\begin{aligned}
& (1-\delta)\Lambda^{b,\theta_{t-1}} + \delta f_1[\alpha_b U^{\text{private}}(g,b) + (1-\alpha_b)U^{\text{private}}(b,b)] + \delta(1-f_1)\cdot\frac{\rho}{2}\alpha_b\sigma_g^2 \\
\geq \quad & (1-\delta)\times 0 \quad + \delta f_0[\alpha_b U^{\text{private}}(g,b) + (1-\alpha_b)U^{\text{private}}(b,b)] + \delta(1-f_0)\cdot\frac{\rho}{2}\alpha_b\sigma_g^2
\end{aligned}$$

Thus, we can show that

$$\begin{aligned}
& (1-\delta)\Lambda^{b,\theta_{t-1}} + \delta[\alpha_{\theta_t} U^{\mathsf{ODTE}}(g,\theta_t) + (1-\alpha_{\theta_t})U^{\mathsf{ODTE}}(b,\theta_t) - \frac{\rho}{2}\alpha_b\sigma_g^2] \\
\geq\ & (1-\delta)\Lambda^{b,\theta_{t-1}} + \delta[\alpha_{\theta_t} U^{\mathsf{private}}(g,\theta_t) + (1-\alpha_{\theta_t})U^{\mathsf{private}}(b,\theta_t) - \frac{\rho}{2}\alpha_b\sigma_g^2] \\
\geq\ & (1-\delta)\Lambda^{b,\theta_{t-1}} + \delta(f_1-f_0)[\alpha_{\theta_t} U^{\mathsf{private}}(g,\theta_t) + (1-\alpha_{\theta_t})U^{\mathsf{private}}(b,\theta_t) - \frac{\rho}{2}\alpha_b\sigma_g^2] \\
\geq\ & 0,
\end{aligned}$$

which is exactly dealer's bad times IC constraint for the existence of ODTE. Therefore, whenever $\rho \geq \rho^{\mathsf{private}}$, the equilibrium constructed at the beginning of this proof also consists of an ODTE, implying that $\rho^{\mathsf{ODTE}} \leq \rho^{\mathsf{private}}$.

(IV) comparison with the always-ISTE equilibrium

Similarly, as shown in the analysis of Section 6.3, the private history equilibrium generates strictly higher social surplus than ISTE. Moreover, the equilibrium we constructed in this proof allows the dealer to collect all the social surplus from the trade. Thus, such a PBE defeats (in the sense of higher ex-ante dealer's payoff in every period in equilibrium) and Pareto dominates the equilibrium where ISTE is always played.

□

Proposition 6. *For a fixed f_0, the cutoff of the existence of a private history equilibrium ρ^{private} characterized in Proposition 5 decreases in f_1. If $f_1 - f_0 = 1$, then ρ^{private} coincides with ρ^{OSTE}. If $f_1 - f_0 = 0$, then ρ^{private} coincides with ρ^{ODTE}.*

Proof. We first show the second half of the proposition. If $f_1 - f_0 = 0$, then condition 27 is equivalent to $\Lambda^{b,\theta_{t-1}} \geq 0$, which is the sufficient and necessary condition for the existence of OSTE, and hence ρ^{private} coincides with ρ^{OSTE}. If $f_1 - f_0 = 1$, then condition 27 is equivalent to condition 24, which is the necessary and sufficient condition for ODTE to hold. Whence ρ^{private} coincides with ρ^{ODTE}.

We now prove the first half of the proposition. Fix f_0, an increase in f_1 implies that in all future periods, the punishment stage is triggered less frequently. As dealer's static on-path payoff is strictly greater than that off the path, an increase in f_1 thus increases her continuation payoff. Condition (27) is equivalent to

$$
\begin{aligned}
& (1-\delta)\Lambda^{b,\theta_{t-1}} \\
& +\delta(f_1-f_0)\Big[\alpha_b(U^{\text{private}}(g,b,f_1,f_0)-U^{\text{off}}(g))+(1-\alpha_b)(U^{\text{private}}(b,b,f_1,f_0)-U^{\text{off}}(b))\Big] \\
\geq\ & 0
\end{aligned}
$$

As f_1 increases and f_0 remains the same, $f_1 - f_0$ increases, and the deviation punishment $U^{\text{private}}(\theta_t, b, f_1, f_0) - U^{\text{off}}(\theta_t)$ increases. Therefore, the LHS increases and the constraint becomes more slacked. The opaque equilibrium is therefore easier to sustain. □

Proposition 7. *Let q represents the regulator's auditing intensity, that is, with probability q, at the end of a period t, a dealer is audited. If she provides a price weakly above $\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2$ but not $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$, then all her future payoffs are forfeited. Denote P_q as the set of trader's risk aversion coefficients when* ODTE *can be sustained. If $q_1 < q_2$, then $P_{q_2} \subseteq P_{q_1}$. Specifically, $P_0 = [\rho^{\text{ODTE}}, \infty]$ and $P_1 \subsetneqq [\rho^{\text{OSTE}}, \infty]$.*

Proof. Denote the on-path and off-path continuation payoffs are $U^{\text{ODTE}}(\theta_t, \theta_{t-1}; q)$ and $U^{\text{off}}(\theta_t, \theta_{t-1}; q)$ given the current and the last economic states are θ_t and θ_{t-1}, and the auditing intensity is q. Then along the equilibrium path, the probability that the dealer still stays in the game for the next period is $(1-q)$, and the discount factor is δ. It's as if along the path her discount factor is $\delta(1-q)$ rather than δ. Off the equilibrium path, she will pass the censorship even if being audited, and everything remains the same. That is, $\forall \theta_t, \theta_{t-1}$,

$$
\begin{aligned}
U_t^{\text{ODTE}}(\theta_t,\theta_{t-1};q) &= (1-\delta)(\varphi a_{t-1}+J_{\theta_t}-p_t^{*\text{UI}}(\theta_{t-1})) \\
&\quad +\delta(1-q)[\alpha_{\theta_t}U_{t+1}^{\text{ODTE}}(g,\theta_t)+(1-\alpha_{\theta_t})U_{t+1}^{\text{ODTE}}(b,\theta_t)], \\
U_t^{\text{off}}(\theta_t) &= (1-\delta)\mathbf{1}(\theta_t=g)\cdot\frac{\rho}{2}\sigma_g^2+\delta[\alpha_{\theta_t}U_{t+1}^{\text{off}}(g)+(1-\alpha_{\theta_t})U_{t+1}^{\text{off}}(b)].
\end{aligned}
$$

Similarly as the proof of Proposition 3, the sufficient and necessary condition for the existence of ODTE is dealer's IC constraint:

$$
U_t^{\text{ODTE}}(\theta_t,\theta_{t-1};q) \geq (1-\delta)\times 0+\delta[\alpha_{\theta_t}U_t^{\text{off}}(g)+(1-\alpha_{\theta_t})U_t^{\text{off}}(b)], \forall\theta_t,\theta_{t-1}. \tag{29}
$$

We first show the first part of the statement. We show that whenever an ODTE exists for a pair of parameters (ρ, q_2), then it also exists for a pair of parameters (ρ, q_1) if $q_1 < q_2$. To show this, we show that $U_t^{\text{ODTE}}(\theta_t, \theta_{t-1}; q)$ decreases in q. In fact, fix a period

t and economic states θ_t, in all future period $\tau \geq t+1$, the expected dealer's payoff from that period, conditional on economic state in period $\tau - 1$ is $\theta_{\tau-1}$ and the dealer is still in the game, is that

$$\begin{aligned}
&\alpha_{\theta_{\tau-1}}\Lambda^{g,\theta_{\tau-1}} + (1-\alpha_{\theta_{\tau-1}})\Lambda^{b,\theta_{\tau-1}} \\
&= \alpha_{\theta_{\tau-1}}(\varphi a_{\tau-1} + J_g - p^{*\mathsf{UI}}(\theta_{\tau-1})) + (1-\alpha_{\theta_{\tau-1}})(\varphi a_{\tau-1} + J_b - p^{*\mathsf{UI}}(\theta_{\tau-1})) \\
&= \frac{\rho}{2}[\alpha_{\theta_{\tau-1}}(1-\alpha_{\theta_{\tau-1}})(J_g - J_b)^2 + \alpha_{\theta_{\tau-1}}\sigma_g^2 + (1-\alpha_{\theta_{\tau-1}})\sigma_b^2] \\
&> 0, \quad \forall \tau \geq t+1, \forall \theta_{\tau-1}.
\end{aligned}$$

Therefore, all future periods expected payoff is constant over q and is positive. The weights of all future periods, or in other words, the probability to stay in the game for a future period and collect this payoff, decrease in q. Therefore, the continuation payoff $U_t(\theta_t, \theta_{t-1}; q)$, as a weighted sum of all future expected payoffs, also decrease in q.

So a decrease in q enlarges the left hand side of (29) and relaxes dealer's IC constraints, making ODTE easier to sustain.

We next show that $P_0 = [\rho^{\mathsf{ODTE}}, \infty]$ and $P_1 \subsetneqq [\rho^{\mathsf{OSTE}}, \infty]$. The former one is obvious as when $q = 0$, the definition of $U_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1}; 0)$ coincides with that of $U_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1})$ and the rest just follows the proof of Proposition 3. To see the latter one, notice that $U_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1}; 1) = (1-\delta)(\varphi a_{t-1} + J_{\theta_t} - p^{*\mathsf{UI}}(\theta_{t-1}))$.

From the proof of Proposition 3,

$$\begin{bmatrix} U^{\mathsf{off}}(g) \\ U^{\mathsf{off}}(b) \end{bmatrix} = (1-\delta)\begin{bmatrix} 1-\delta\alpha_g & -\delta(1-\alpha_g) \\ -\delta\alpha_b & 1-\delta(1-\alpha_b) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{\rho}{2}\sigma_g^2 \\ 0 \end{bmatrix}.$$

Hence,

$$\begin{aligned}
\alpha_{\theta_t}U_{t+1}^{\mathsf{off}}(g) + (1-\alpha_{\theta_t})U_{t+1}^{\mathsf{off}}(b) &= \begin{bmatrix} \alpha_{\theta_t} & 1-\alpha_{\theta_t} \end{bmatrix} \cdot \begin{bmatrix} U^{\mathsf{off}}(g) \\ U^{\mathsf{off}}(b) \end{bmatrix} \\
&= (1-\delta)\begin{bmatrix} \alpha_{\theta_t} & 1-\alpha_{\theta_t} \end{bmatrix} \cdot \begin{bmatrix} 1-\delta\alpha_g & -\delta(1-\alpha_g) \\ -\delta\alpha_b & 1-\delta(1-\alpha_b) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{\rho}{2}\sigma_g^2 \\ 0 \end{bmatrix} \\
&= \frac{((1-\delta)\alpha_{\theta_t} + \delta\alpha_{\theta_b})}{1-\delta(\alpha_g - \alpha_b)}\frac{\rho}{2}\sigma_g^2.
\end{aligned}$$

Dealer's IC constraint in bad times becomes

$$(1-\delta)(\varphi a_{t-1} + J_b - p_t^{*\mathsf{UI}}(\theta_{t-1})) \geq \delta \frac{\alpha_b}{1-\delta(\alpha_g - \alpha_b)} \frac{\rho}{2} \sigma_g^2 > 0.$$

Therefore, whenever the above IC holds, the IC constraints for OSTE also holds and OSTE exists. Moreover, when $\rho = \rho^{\mathsf{ODTE}}$, the left hand side becomes 0 and dealer's IC in bad times fail to sustain. In other words, we have $P_1 \subsetneqq [\rho^{\mathsf{OSTE}}, \infty]$. □

Proposition 8. *In period t, the expected social welfare for each equilibrium is explicitly given by:*

$$W^{\mathsf{OTE}} \equiv W^{\mathsf{OSTE}} = W^{\mathsf{ODTE}} = \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right],$$
$$W^{\mathsf{ISTE}} = \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2\right].$$

In the region where ISTE *and either* OSTE *or* ODTE *exist (i.e.,* $\rho \in [\rho^{\mathsf{ODTE}},\ \rho^{\mathsf{ISTE}}]$*), the welfare gap is*

$$W^{gap} \equiv W^{\mathsf{OTE}} - W^{\mathsf{ISTE}} = \frac{\rho}{2}\left[(1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right],$$

which is independent of σ_g *and is convex and increasing in* σ_b *as well as the spread* $J_g - J_b$.

Proof. In opaque equilibrium, along the equilibrium path in period t, traders learn no information from the price offers, therefore, his ex-post belief about the economic state is still his prior at the beginning of period t: $\xi_t = \alpha_{\theta_{t-1}}$. His ex-post utility after the trade becomes

$$p_t - \varphi a_{t-1} - \alpha_{\theta_{t-1}} J_g - (1-\alpha_{\theta_{t-1}}) J_b + \frac{\rho}{2}(\alpha_{\theta_{t-1}}(\alpha_{\theta_{t-1}}(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2)).$$

Dealer's ex-post utility becomes

$$\varphi + J_{\theta_t} - p_t.$$

At the beginning of period t, the expectation of this ex-post utility becomes

$$\varphi + \alpha_{\theta_{t-1}} J_g + (1-\alpha_{\theta_{t-1}}) J_b - p_t.$$

Therefore, in period t the expected ex-post surplus in opaque equilibrium, becomes

$$
\begin{aligned}
W^{\mathsf{OTE}} &= p_t - \varphi a_{t-1} - \alpha_{\theta_{t-1}} J_g - (1-\alpha_{\theta_{t-1}}) J_b + \frac{\rho}{2}(\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2)) \\
&\quad + \varphi a_{t-1} + \alpha_{\theta_{t-1}} J_g + (1-\alpha_{\theta_{t-1}}) J_b - p_t \\
&= \frac{\rho}{2}(\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2)).
\end{aligned}
$$

In informative equilibrium, along the equilibrium path in period t, if it is in bad times, then no trade happens and the ex-post surplus is 0. If it is in good times, then trader learns about θ_t after observing the price offer of good times. His ex-post utility becomes

$$p_t^g - \varphi a_{t-1} - J_g + \frac{\rho}{2}\sigma_g^2.$$

The dealer's ex-post utility becomes

$$\varphi a_{t-1} + J_g - p_t^g.$$

Thence, the social welfare in good times becomes $\frac{\rho}{2}\sigma_g^2$. At the beginning of time t, the expected ex-post utility becomes $\frac{\rho}{2}\alpha_{\theta_{t-1}}\sigma_g^2$.

□

Proposition 9.

(a) If OSTE *exists with trader's utility function U, then there still exists an* OSTE *when trader's utility function is V and V exhibits more risk aversion than U.*

(b) If ISTE *exists with trader's utility function V, then there still exists an* ISTE *when trader's utility function is U and V exhibits more risk aversion than U.*

(c) If ODTE *exists with trader's utility function U, then there still exists an* ODTE *when trader's utility function is V and V exhibits more risk aversion than U.*

(d) If a private history equilibrium exists with trader's utility function U, then there still exists a private history equilibrium when trader's utility function is V and V exhibits more risk aversion than U.

Proof. (a) In our construction of OSTE, dealer will offer a price that makes an uninformed trader indifferent in both good times and bad times. From the proof of

Proposition 1, the sufficient and necessary condition for the existence of OSTE is that

$$CE(a_t|\xi_t = \alpha_{\theta_{t-1}}) \leq \varphi a_{t-1} + J_b.$$

If OSTE exists for utility function U, then we have

$$CE_U(a_t|\xi_t = \alpha_{\theta_{t-1}}) \leq \varphi a_{t-1} + J_b.$$

Therefore,

$$CE_V(a_t|\xi_t = \alpha_{\theta_{t-1}}) \leq CE_U(a_t|\xi_t = \alpha_{\theta_{t-1}}) \leq \varphi a_{t-1} + J_b,$$

and OSTE also exists for utility function V.

(b) In our construction of ISTE, dealer will offer a price $CE(a_t|\xi_t = 1)$ in good times and offer a price low enough to decline trade in bad times. From the proof of Proposition 2, the sufficient and necessary condition for the existence of such kind of equilibrium becomes

$$\varphi a_{t-1} + J_b \leq CE(a_t|\xi_t = 1).$$

If a utility function V exhibits more risk aversion than U, then

$$CE_U(a_t|\xi_t = 1) \geq CE_V(a_t|\xi_t = 1).$$

If there exists an ISTE with trader's utility function V, then

$$CE_V(a_t|\xi_t = 1) \geq \varphi a_{t-1} + J_b.$$

Therefore,

$$CE_U(a_t|\xi_t = 1) \geq CE_V(a_t|\xi_t = 1) \geq \varphi a_{t-1} + J_b,$$

and ISTE exists if traders have utility function U.

(c) Similarly as the proof of Lemma 2.1, we only need to check that dealer's IC constraint in bad times is relaxed if the trader's utility function exhibits more risk aversion. That is, one only needs to check that the deviation gain from the current period decreases and the punishment losses from all future periods increase. That is,

$$-[\varphi a_{t-1} + J_b - CE_V(a_t|\xi_t = \alpha_{\theta_{t-1}})] \leq -[\varphi a_{t-1} + J_b - CE_U(a_t|\xi_t = \alpha_{\theta_{t-1}})]$$

and for $\forall \tau > t, h^\tau$,

$$\begin{aligned} & [\varphi a_{\tau-1} + J_{\theta_\tau} - CE_V(a_\tau|\xi_\tau = \alpha_{\theta_{\tau-1}})] - [\varphi a_{\tau-1} + J_{\theta_\tau} - CE_V(a_\tau|\xi_\tau = 1)] \\ \geq \; & [\varphi a_{\tau-1} + J_{\theta_\tau} - CE_U(a_\tau|\xi_\tau = \alpha_{\theta_{\tau-1}})] - [\varphi a_{\tau-1} + J_{\theta_\tau} - CE_U(a_\tau|\xi_\tau = 1)] \end{aligned}$$

Equivalently,

$$CE_V(a_t|\xi_t = \alpha_{\theta_{t-1}}) \leq CE_U(a_t|\xi_t = \alpha_{\theta_{t-1}})$$

and for $\forall \tau > t$,

$$CE_V(a_\tau|\xi_\tau = 1) - CE_V(a_\tau|\xi_\tau = \alpha_{\theta_{\tau-1}}) \geq CE_U(a_\tau|\xi_\tau = 1) - CE_U(a_\tau|\xi_\tau = \alpha_{\theta_{\tau-1}})$$

These directly come from Definition 7.

(d) The proof follows the similar steps as in (c).

□

Proposition 10. *Fix a trader's utility function U,*

(a) if an OSTE *and an* ISTE *exist, then there exists a private history equilibrium;*

(b) if a private history equilibrium and an ISTE *exist, then there exists an* ODTE.

Proof. In the proof of Proposition 5, replace $\Lambda^{\theta_t,\theta_{t-1}}$ with $\varphi a_{t-1} + J_{\theta_t} - CE_U(a_t|\xi_t = \alpha_{\theta_{t-1}})$, replace $\frac{\rho}{2}\sigma_g^2$ with $\varphi a_{t-1} + J_g - CE_U(a_t|\xi_t = 1)$, and the rest of the proof remains unchanged. □

Proposition 11. *When the trader's trading position is $\chi_t = -1$, that is, he is in demand of a unit of asset and needs to buy it from the dealer, there exists an opaque static trading equilibrium* (OSTE) *if and only if*

$$\rho \geq \rho^{\mathsf{OSTE}}_{\chi_t=-1} \equiv 2\frac{(1-\alpha_{\min})(J_g - J_b)}{\alpha_{\min}\sigma_g^2 + (1-\alpha_{\min})\sigma_b^2 + \alpha_{\min}(1-\alpha_{\min})(J_g-J_b)^2}, \quad \alpha_{\min} = \min\{\alpha_g, \alpha_b\}$$

Proof. In any opaque equilibrium, dealer's price offer reveals no extra information about the underlying economic state. Similar to the argument in OSTE when traders are potential sellers, an Bayesian trader will hold the same prior belief after observing the equilibrium price offer. Hence, along the equilibrium path, if trader does not accept the dealer's proposed price offer and $o_t = 0$, using (4), then his payoff at the end of period t becomes

$$\begin{aligned}\mathbb{E}[-a_t|h^{t-1}, p_t] - \frac{\rho}{2}\mathrm{Var}[-a_t|h^{t-1}, p_t] &= -\left(\varphi a_{t-1} + \mathbb{E}[J_{\theta_t}|h^{t-1}]\right) - \frac{\rho}{2}\left[\mathrm{Var}[\sigma_{\theta_t} z_t|h^{t-1}] + \mathrm{Var}[J_{\theta_t}|h^{t-1}]\right] \\ &\overset{(a)}{=} -\left(\varphi a_{t-1} + J_b + \alpha_{\theta_{t-1}}(J_g - J_b)\right) \\ &\quad - \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right],\end{aligned}$$

where (a) follows from the fact that trader's on-path posterior coincides with the prior. Hence, trader's indifference condition implies he will accept the offer if and only if $-p_t$, his end of period wealth for accepting the offer, is larger than $\mathbb{E}[-a_t|h^{t-1}, p_t] - \frac{\rho}{2}\mathrm{Var}[-a_t|h^{t-1}, p_t]$. That is, the highest price for the trader to accept is

$$\begin{aligned}p_t^{*\mathsf{UI}}(\theta_{t-1}, \chi_t = -1) \equiv\ & \varphi a_{t-1} + J_b + \alpha_{\theta_{t-1}}(J_g - J_b) + \\ & \frac{\rho}{2}\left[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2\right].\end{aligned}$$

Since trades occur in both good times and bad times, the dealer's IR constraints impose that $p_t^{*\mathsf{UI}}(\theta_{t-1}, \chi_t = -1) - (\varphi a_{t-1} + J_g) \geq 0$ and $p_t^{*\mathsf{UI}} - (\varphi a_{t-1} + J_b) \geq 0$. Hence, dealer's IR in good times is binding and gives us

$$\rho \geq 2\frac{(1-\alpha_{\theta_{t-1}})(J_g - J_b)}{\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2 + \alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2}, \forall \theta_{t-1} \in \{g, b\}$$

Since the right hand side is decreasing in $\alpha_{\theta_{t-1}}$, we have

$$\rho \geq \rho^{\mathsf{OSTE}}_{\chi_t=-1} \equiv 2\frac{(1-\alpha_{\min})(J_g - J_b)}{\alpha_{\min}\sigma_g^2 + (1-\alpha_{\min})\sigma_b^2 + \alpha_{\min}(1-\alpha_{\min})(J_g - J_b)^2}, \quad \alpha_{\min} = \min\{\alpha_g, \alpha_b\}$$

□

Proposition 12. *When the trader's trading position is $\chi_t = -1$, that is, he is in demand of a unit of asset and needs to buy it from the dealer, there exists an informative static trading equilibrium* (ISTE) *if and only if*

$$\rho \leq \rho^{\mathsf{ISTE}}_{\chi_t=-1} \equiv 2\frac{J_g - J_b}{\sigma_b^2}$$

Proof. At any ISTE, along the equilibrium path, dealer's price offer reveals all the information about the economic state. To avoid dealer to offer a good time ask offer in bad times, transaction can only occur in bad times and in good times traders always decline the offer.

We can still construct trader's off-path belief such that he believes the current period is always in bad times. Then in good times, we need to make sure if dealer deviates and offers a lower price, then either it is below $\varphi a_{t-1} + J_g$ so the dealer is experiencing a loss, or the trader will reject this price offer. Given his off path belief, then the highest price he can accept becomes

$$\begin{aligned} -p_t^b &= \mathbb{E}[-a_t|h^{t-1}, p_t^b] - \frac{\rho}{2}\mathrm{Var}[-a_t|h^{t-1}, p_t^b] \\ p_t^b &= \varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2. \end{aligned}$$

In other words, no price is above $\varphi a_{t-1} + J_g$ and below $\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$ because otherwise the dealer in good times would rather offer a price in between. Equivalently, this requires

$$\varphi a_{t-1} + J_g \geq \varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$$

$$\rho \leq \rho_{\chi_t=-1}^{\mathsf{ISTE}} \equiv 2\frac{J_g - J_b}{\sigma_b^2}$$

One can check that $p_t^b = \min\{\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2, \varphi a_{t-1} + J_g\}$ and $p_t^g > \varphi a_{t-1} + J_g + \frac{\rho}{2}\sigma_g^2$ satisfy dealer's IC constraint in bad times as well as dealer's and trader's IR constraints at all times. □

Corollary 4. *When the dealer is myopic and is a potential buyer, we have*

$$\rho_{\chi_t=-1}^{\mathsf{ISTE}} > \rho_{\chi_t=-1}^{\mathsf{OSTE}}$$

Proof. As shown, respectively, in Propositions 11 and 12

$$\rho_{\chi_t=-1}^{\mathsf{OSTE}} = 2\frac{(J_g - J_b)}{\alpha_{\min}(J_g - J_b)^2 + \frac{\alpha_{\min}}{1-\alpha_{\min}}\sigma_g^2 + \sigma_b^2},$$

and

$$\rho_{\chi_t=-1}^{\mathsf{ISTE}} = 2\frac{J_g - J_b}{\sigma_b^2}$$

By a little algebra it is immediate that

$$\frac{(J_g - J_b)}{(\alpha_{\min}(J_g - J_b)^2 + \frac{\alpha_{\min}}{1-\alpha_{\min}}\sigma_g^2 + \sigma_b^2} < \frac{J_g - J_b}{\sigma_b^2},$$

completing the proof. □

Proposition 13. *Given that $\sigma_g^2 - \sigma_b^2 + (1-\alpha_{\max})(J_g - J_b)^2 \geq 0$. Suppose the trader has a trading position χ_t in period t, then there exists a $\rho_{\chi_t}^{\mathsf{ODTE}} \leq \rho_{\chi_t}^{\mathsf{OSTE}}$ such that if and only if $\rho \geq \rho_{\chi_t}^{\mathsf{ODTE}}$, there exists an* ODTE.

Proof. The proof closely maps the three steps of the proof of Proposition 3. That is, we first show the "if" part by showing that whenever there exists an ODTE for traders' risk preference coefficient ρ_1, then for any $\rho_2 > \rho_1$, there exists an ODTE for for traders' risk preference coefficient ρ_2. We then show the "only if" part by showing that for any condition belief the trader may have, when trader is a potential buyer, than the dealer's ex-post ISTE payoffs are no larger than the min-max static payoffs given that the trader best responses to her strategy. Finally, we show that $\rho_{\chi_t}^{\mathsf{ODTE}} \leq \rho_{\chi_t}^{\mathsf{OSTE}}$ by showing that whenever an OSTE exists, then there must exist an ODTE.

Let's take a close look of each of the three steps. First, denote $U_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1}, \chi_t)$ as the on-path continuation payoff when the current and last period economic states are θ_t and θ_{t-1} respectively and trader of the current period has a trading position of χ_t. Then these payoffs can be recursively defined as follows. For $\forall \theta_t, \theta_{t-1}, \chi_t$,

$$\begin{aligned} U_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1}, \chi_t) &= (1-\delta)(\varphi a_{t-1} + J_{\theta_t} - p_t^{*\mathsf{UI}}(\theta_{t-1}, \chi_t))\chi_t + \\ &\quad \delta\beta\alpha_{\theta_t} U_{t+1}^{\mathsf{ODTE}}(g, \theta_t, 1) + \delta\beta(1-\alpha_{\theta_t}) U_{t+1}^{\mathsf{ODTE}}(b, \theta_t, 1) + \\ &\quad \delta(1-\beta)\alpha_{\theta_t} U_{t+1}^{\mathsf{ODTE}}(g, \theta_t, -1) + \delta(1-\beta)(1-\alpha_{\theta_t}) U_{t+1}^{\mathsf{ODTE}}(b, \theta_t, -1). \end{aligned}$$

Denote $U_t^{\mathsf{off}}(\theta_t, \theta_{t-1}, \chi_t)$ as the off-path continuation payoff when the current economic state is θ_t and trader of the current period has a trading position of χ_t. Off the equilibrium path, For $\forall \theta_t, \theta_{t-1}, \chi_t$,

$$\begin{aligned} U_t^{\mathsf{off}}(\theta_t, \chi_t) &= (1-\delta)\mathbf{1}(\theta_t = g)\mathbf{1}(\chi_t = 1)\frac{\rho}{2}\sigma_g^2 + \mathbf{1}(\theta_t = b)\mathbf{1}(\chi_t = -1)\frac{\rho}{2}\sigma_b^2 \\ &\quad \delta\beta\alpha_{\theta_t} U_{t+1}^{\mathsf{Off}}(g, 1) + \delta\beta(1-\alpha_{\theta_t}) U_{t+1}^{\mathsf{off}}(b, 1) + \\ &\quad \delta(1-\beta)\alpha_{\theta_t} U_{t+1}^{\mathsf{off}}(g, -1) + \delta(1-\beta)(1-\alpha_{\theta_t}) U_{t+1}^{\mathsf{off}}(b, -1). \end{aligned}$$

Then if $\chi_t = 1$, a myopic dealer may want to deviate and decline trade in bad times if OSTE fails to exist. Then the binding constraint in this scenario is the dealer's IC in bad times:

$$\begin{aligned} U_t^{\mathsf{ODTE}}(b,\theta_{t-1},1) \quad \geq \quad & (1-\delta)\cdot 0+\delta\beta\alpha_b U_{t+1}^{\mathsf{off}}(g,1)+\delta\beta(1-\alpha_b)U_{t+1}^{\mathsf{off}}(b,1)+ \\ & \delta(1-\beta)\alpha_b U_{t+1}^{\mathsf{off}}(g,-1)+\delta(1-\beta)(1-\alpha_b)U_{t+1}^{\mathsf{off}}(b,-1). \end{aligned}$$

Then if $\chi_t = -1$, a myopic dealer may want to deviate and decline trade in good times if OSTE fails to exist. Then the binding constraint in this scenario is

$$\begin{aligned} U_t^{\mathsf{ODTE}}(g,\theta_{t-1},-1) \quad \geq \quad & (1-\delta)\cdot 0+\delta\beta\alpha_g U_{t+1}^{\mathsf{off}}(g,1)+\delta\beta(1-\alpha_g)U_{t+1}^{\mathsf{off}}(b,1)+ \\ & \delta(1-\beta)\alpha_g U_{t+1}^{\mathsf{off}}(g,-1)+\delta(1-\beta)(1-\alpha_g)U_{t+1}^{\mathsf{off}}(b,-1). \end{aligned}$$

We now show that whenever the above two conditions hold for ρ_1, then for any $\rho_2 > \rho_1$, they still hold. A sufficient condition is that for any realization of $\{\theta_t\}$ and $\{\chi_t\}$, the current deviation gain decreases in ρ, whereas the future punishment loss in each period increases in ρ. Specifically, the current deviation gain is

$$-(\varphi a_{t-1}+J_{\theta_t}-p_t^{*\mathsf{UI}}(\theta_{t-1},\chi_t))\chi_t \propto -\frac{\rho}{2}[\alpha_{\theta_t}(1-\alpha_{\theta_t})(J_g-J_b)^2+\alpha_{\theta_t}\sigma_g^2+(1-\alpha_{\theta_t})\sigma_b^2],$$

and decreases in ρ. For any realization of future period's economic state θ_τ and trading position χ_τ, the deviation punishment becomes

$$\varphi a_{\tau-1}+J_{\theta_\tau}-p_t^{*\mathsf{UI}}(\theta_{\tau-1},1)-\frac{\rho}{2}\sigma_g^2 \propto \frac{\rho(1-\alpha_{\theta_{\tau-1}})}{2}[\alpha_{\theta_{\tau-1}}(J_g-J_b)^2+\sigma_b^2-\sigma_g^2]$$

if $\chi_t = 1$ or

$$-\varphi a_{\tau-1}-J_{\theta_\tau}+p_t^{*\mathsf{UI}}(\theta_{\tau-1},-1)-\frac{\rho}{2}\sigma_b^2 \propto \frac{\rho\alpha_{\theta_{\tau-1}}}{2}[(1-\alpha_{\theta_{\tau-1}})(J_g-J_b)^2-\sigma_b^2+\sigma_g^2]$$

if $\chi_t = -1$. The first one increases in ρ always; the second one increases in ρ given our parameter assumption.

Therefore, as ρ increases, the current economic gain from deviation decreases, whereas the future punishment caused by deviation increases, making deviation less profitable and the equilibrium easier to sustain.

Second, we prove the "only if" part by showing that when $\chi_t = -1$, the ISTE payoffs remain as minmax payoffs and consist of the harshest feasible punishment one can put on deviation. Formally speaking, similar to the case where $\chi_t = 1$, we will show that when $\chi_t = -1$,

$$\begin{aligned}
\min_{\zeta(p;\cdot)} \max_{p^g, p^b} \min_{o_t \in \mathsf{BR}_{\zeta(p;\cdot)}} (p^g - \varphi a_{t-1} - J_g) \cdot o_t(p^g) &= 0 \\
\min_{\zeta(\cdot|p)} \max_{p^g, p^b} \min_{o_t \in \mathsf{BR}_{\zeta(p;\cdot)}} (p^b - \varphi a_{t-1} - J_b) \cdot o_t(p^b) &= \frac{\rho}{2}\sigma_b^2
\end{aligned}$$

In fact, in good times, no matter what belief trader may have, dealer can always offer a high enough price to decline the trade and collect a zero payoff. If trader has a belief that the economic state is in bad times unless observing an offer of $\varphi a_{t-1} + J_g$, then he will only accept any offer below $\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$ or exactly at $\varphi a_{t-1} + J_g$. Offering a price below $\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$ leads to a negative payoff for the dealer; for price above $\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$, either trader will decline it or trader will accept it but the dealer's payoff is exactly 0. In conclusion, in good times, given the trader's belief and his optimization order decision over that belief, the best the dealer can collect is 0.

In bad times, the dealer can always offer $\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$, the trader will always accept it, and the dealer can collect a payoff of $\frac{\rho}{2}\sigma_b^2$. Now let the trader has the same belief that the current period is always in good times, then he will only accept offers weakly below $\varphi a_{t-1} + J_b + \frac{\rho}{2}\sigma_b^2$. Consequently, $\frac{\rho}{2}\sigma_b^2$ is dealer's best case ex-post payoff given this belief and trader best responding to it.

The rest of the "if only" proof then follows the logic of that in Proposition 3. Since the equilibrium we constructed has already employ the harshest punishment, it gives the lower bound for the traders' risk aversion coefficient, ρ, for the existence of an opaque equilibrium.

Third, we show that $\rho_{\chi_t}^{\mathsf{ODTE}} \leq \rho_{\chi_t}^{\mathsf{OSTE}}$. Since along the equilibrium path, dealer always offer uninformative prices and off the equilibrium path, dealer has to offer differently in bad times and in good times and hence release her information about the economic states, the on-path continuation payoff $U_t^{\mathsf{ODTE}}(\theta_t, \theta_{t-1}, \chi_t)$ is no less than the off-path continuation payoff $U_t^{\mathsf{off}}(\theta_t, \chi_t)$ due to this information rent. Consequently, if given the parameter values, an OSTE exists, then in the current period the deviation leads to a loss, in all future periods, it triggers the punishment and leads to lower continuation payoffs. As a result, the dealer will never have an incentive to deviate in the dynamic game and therefore, the constructed ODTE is valid. □

Proposition 14. *Given that $\sigma_g^2 - \sigma_b^2 + (1 - \alpha_{\max})(J_g - J_b)^2 \geq 0$. Suppose the trader has a trading position χ_t in period t, then there exists a $\rho_{\chi_t}^{\mathsf{private}} \in [\rho_{\chi_t}^{\mathsf{ODTE}}, \rho_{\chi_t}^{\mathsf{OSTE}}]$ such that if and only if $\rho \geq \rho_{\chi_t}^{\mathsf{private}}$, there exists a private history equilibrium.*

Proof. Since the signal of past trade is independent of trader's trading positions. The analysis of the private history equilibrium follows exactly the same steps as those in the scenario where trader is a potential seller. The exact mapping between the case where $\chi_t = 1$ and the case $\chi_t = -1$ is illustrated in the proof of Proposition 13. □

Proposition 15. *If and only if trader's risk aversion coefficient ρ lies in the interval $(\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$, there exists a semi-opaque static trading equilibrium. Moreover, there exists a semi-opaque static trading equilibrium that Pareto dominates the informative static trading equilibrium (*ISTE*), and is Pareto dominated by an opaque static trading equilibrium (*OSTE*). In conclusion,*

$$W^{\mathsf{ODTE}} = W^{\mathsf{OSTE}} > W^{\mathsf{semi}} > W^{\mathsf{ISTE}}.$$

Finally, the expected social surplus at time t in a semi-opaque static trading equilibrium increases in the trading probability in bad times q.

Proof. We first characterize the restriction of q for the constructed equilibrium to hold. We then prove the welfare result.

(I) if part

First, it is obvious that the constructed belief follows the Bayes' rule whenever it applies, and hence consistency is satisfied. Next, we check that dealer does not want to deviate at all times. Given $\rho \leq \rho^{\mathsf{ISTE}}$, we have $\varphi a_{t-1} + J_b \leq \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. In good times, by offering $\varphi a_{t-1} + J_b$, the dealer is making a positive profit, hence it does not make sense to lower the price and decline the offer. It also does not make sense to increase the price and earn less. In bad times, offering $\varphi a_{t-1} + J_b$ gives the dealer zero profit, while declining the trade also gives her zero. Hence she is indifferent. Plus offering a price weakly above $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ leads to non-positive profit and is always sub-optimal. Finally the only thing left to check is trader's IR constraint:

$$\varphi a_{t-1} + \hat{\alpha} J_g + (1-\hat{\alpha}) J_b - \frac{\rho}{2}[\hat{\alpha}(1-\hat{\alpha})(J_g - J_b)^2 + \hat{\alpha}\sigma_g^2 + (1-\hat{\alpha})\sigma_b^2] \leq \varphi a_{t-1} + J_b, \quad (30)$$

where $\hat{\alpha} = \frac{\alpha_{\theta_{t-1}}}{\alpha_{\theta_{t-1}} + (1-\alpha_{\theta_{t-1}})q}$ for some $q \in (0, 1)$.

Equivalently, trader's IR holds if and only if there exists $\hat{\alpha} \in (\alpha_{\theta_{t-1}}, 1)$ such that the inequality 30 holds:

$$h(\hat{\alpha}) \equiv \hat{\alpha}(J_g - J_b) - \frac{\rho}{2}[\hat{\alpha}(1-\hat{\alpha})(J_g - J_b)^2 + \hat{\alpha}\sigma_g^2 + (1-\hat{\alpha})\sigma_b^2] \leq 0. \quad (31)$$

$h(\cdot)$ is a quadratic function and $h(1) = J_g - J_b - \frac{\rho}{2}\sigma_g^2 \geq 0$ for $\rho \leq \rho^{\mathsf{ISTE}}$. Whence, there exists a $\hat{\alpha} \in [\alpha_{\theta_{t-1}}, 1]$ such that $h(\hat{\alpha}) \leq 0$, if and only if

$$
\begin{aligned}
& h(\alpha_{\theta_{t-1}}) < 0 \\
\Leftrightarrow \quad & \rho > \frac{2\alpha_{\theta_{t-1}}(J_g - J_b)}{\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2}, \quad \forall \theta_{t-1} = g, b \\
\Leftrightarrow \quad & \rho > \rho^{\mathsf{OSTE}}.
\end{aligned}
$$

(II) only if part

We just showed that $\rho > \rho^{\mathsf{OSTE}}$ is necessary for the existence of semi-opaque equilibria. To see that ρ can not exceed ρ^{ISTE}, we prove by contradiction. Suppose $\rho > \rho^{\mathsf{ISTE}}$, then $\varphi a_{t-1} + J_b > \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$, then in good times the dealer can deviate and offer a price at $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. Then the dealer will deviate to offer $\varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$. By doing so, the trader will accept the offer and the dealer earns more. As a result, the constructed equilibrium fails. In other words, we must have $\rho \in (\rho^{\mathsf{OSTE}}, \rho^{\mathsf{ISTE}}]$.

(III) welfare result

The social surplus in this semi-opaque static trading equilibrium in period t is

$$
\begin{aligned}
W^{\mathsf{semi}} = \; & \alpha_{\theta_{t-1}}[\varphi a_{t-1} + J_g - \varphi a_{t-1} - \hat{\alpha} J_g - (1-\hat{\alpha})J_b + \frac{\rho}{2}(\hat{\alpha}(1-\hat{\alpha})(J_g - J_b)^2 + \hat{\alpha}\sigma_g^2 + (1-\hat{\alpha})\sigma_b^2)] + \\
& (1-\alpha_{\theta_{t-1}})q[\varphi a_{t-1} + J_b - \varphi a_{t-1} - \hat{\alpha} J_g - (1-\hat{\alpha})J_b + \frac{\rho}{2}(\hat{\alpha}(1-\hat{\alpha})(J_g - J_b)^2 + \hat{\alpha}\sigma_g^2 + (1-\hat{\alpha})\sigma_b^2)] \\
& + (1-\alpha_{\theta_{t-1}})(1-q) \cdot 0 \\
= \; & \frac{\rho}{2}[\frac{\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})q}{(1-\alpha_{\theta_{t-1}})q + \alpha_{\theta_{t-1}}}(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})q\sigma_b^2]
\end{aligned}
$$

It then follows immediately that W^{semi} increases in the trading probability in bad times q. Therefore, we have:

$$W^{\mathsf{semi}} > W^{\mathsf{ISTE}} = \frac{\rho}{2}\alpha_{\theta_{t-1}}\sigma_g^2$$

and

$$W^{\mathsf{semi}} < W^{\mathsf{OSTE}} = \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2].$$

As $q = 0$ corresponds to the ISTE and $q = 1$ corresponds to an OSTE. □

Appendix-Proposition 1 (Informative Dynamic Trading Equilibrium). *If there exists a*

$\pi^b \in [0, \frac{\rho}{2}\sigma_b^2]$ *such that*

$$(1 - \frac{\delta(1-\alpha_g)}{(1-\delta)(1-\delta(\alpha_g - \alpha_b))})\pi^b \leq \frac{\rho\sigma_g^2}{2} - J_g + J_b \leq [1 + \delta\frac{1 - \delta\alpha_g - (1-\delta)\alpha_b}{(1-\delta)(1-\delta(\alpha_g - \alpha_b))})]\pi^b,$$

*then there exists an informative dynamic trading equilibrium (*IDTE*) where the dealer fully reveals her private information about current economic state by providing distinct price offers at different times, and trade occurs in both* good *times and* bad *times.*

Proof. We construct the following set of strategies and beliefs. At an on-path history h^t, dealer will offer a price $p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ in good times and a price p_t^b in bad times. Suppose $\varphi a_{t-1} + J_b - p_t^b$ is a constant. Therefore, at observing the price $p^{\tilde{\theta}_t}$, traders will update their posterior and believe the current economic state is $\tilde{\theta}_t$. If dealer deviates and offer something else, trader will believe that the current period is in good times. Since all previous offers and economic states are publicly observed, the dealer and all future traders can determine whether there is deviation or not. If there is at least one deviation in the previous periods, then the game moves into the punishment stage and both players choose the ISTE strategies and form the corresponding beliefs.

We now check all individual rationality constraints:

$$\begin{aligned}
p_t^g &\geq \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2 && \text{trader's IR in good times} \\
p_t^b &\geq \varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 && \text{trader's IR in bad times} \\
p_t^g &\leq \varphi a_{t-1} + J_g && \text{dealer's IR in good times} \\
p_t^b &\leq \varphi a_{t-1} + J_b && \text{dealer's IR in bad times}
\end{aligned}$$

$p_t^g = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$ implies that the IR constraints in good times are satisfied. The IR constraints in bad times are equivalent to $\varphi a_{t-1} + J_b - \frac{\rho}{2}\sigma_b^2 \leq p_t^b \leq \varphi a_{t-1} + J_b$. Denote π^{θ_t} as dealer's static payoff at period t if she follows the on-path strategy. Then in our construction $\pi^g = \frac{\rho}{2}\sigma_g^2$ and IR constraints are equivalent to $\pi^b \in [0, \frac{\rho}{2}\sigma_b^2]$.

We now look at dealer's IC constraints. In good times, if dealer deviates to offer a price below p_t^g other than p_t^b, then according to our belief construction, trader will interpret this price as a signal of good times and will reject this offer. By deviating dealer loses current period profit and triggers the punishment. If dealer increases her price offer, then she loses part of her current period profit and triggers the punishment. Clearly both kinds of strategies are dominated by p_t^g. The only deviation left to check is to pretend now is in bad times and offer p_t^b. If she offers p_t^b, then the trader will believe the current economic state is bad and accept this offer. Dealer then collects a static profit of $\varphi a_{t-1} + J_g - p_t^b = \varphi a_{t-1} + J_b - p_t^b + J_g - J_b = \pi^b + J_g - J_b$. In all future periods, the punishment is triggered.

$$U_t^{\mathsf{IDTE}}(g) \geq \underbrace{(1-\delta)(\pi^b + J_g - J_b)}_{\substack{\text{dealer's static payoff after} \\ \text{pretending it's in bad times}}} + \underbrace{\delta[\alpha_g U_t^{\mathsf{off}}(g) + (1-\alpha_g)U_t^{\mathsf{off}}(b)]}_{\substack{\text{dealer's continuous} \\ \text{off-path payoff}}}, \tag{32}$$

where $\forall \theta_t$,

$$\begin{aligned} U_t^{\mathsf{IDTE}}(\theta_t) &= (1-\delta)\pi^{\theta_t} + \delta[\alpha_{\theta_t} U_{t+1}^{\mathsf{IDTE}}(g) + (1-\alpha_{\theta_t})U_{t+1}^{\mathsf{IDTE}}(b)] \\ U_t^{\mathsf{off}}(\theta_t) &= (1-\delta)\mathbf{1}(\theta_t = g) \cdot \frac{\rho}{2}\sigma_g^2 + \delta[\alpha_{\theta_t} U_t^{\mathsf{off}}(g) + (1-\alpha_{\theta_t})U_t^{\mathsf{off}}(b)]. \end{aligned}$$

are dealer's on-path and off-path continuous payoffs respectively, given the current economic state is θ_t.

Similarly, in bad times, deviating to any price below p_t^g will decline the transaction and result in the loss of current profit; deviating to any price above p_t^g will let the trader agree to trade, but reduce the current profit. Hence, any deviation other than p_t^g is clearly not profitable. The only IC constraint left to check becomes

$$U_t^{\mathsf{IDTE}}(b) \geq \underbrace{(1-\delta)(\pi^g - J_g + J_b)}_{\substack{\text{dealer's static payoff after} \\ \text{pretending it's in good times}}} + \underbrace{\delta[\alpha_b U_t^{\mathsf{off}}(g) + (1-\alpha_b)U_t^{\mathsf{off}}(b)]}_{\substack{\text{dealer's continuous} \\ \text{off-path payoff}}}. \tag{33}$$

By the similar argument as in the proof of Proposition 3, one can show that $U_t^{\mathsf{IDTE}}(\theta_t)$ is stationary. Denote it as $U_t^{\mathsf{IDTE}}(\theta_t) = U^{\mathsf{IDTE}}(\theta_t)$.

$$\begin{bmatrix} 1-\delta\alpha_g & -\delta(1-\alpha_g) \\ -\delta\alpha_b & 1-\delta(1-\alpha_b) \end{bmatrix} \cdot \begin{bmatrix} U^{\mathsf{IDTE}}(g) \\ U^{\mathsf{IDTE}}(b) \end{bmatrix} = (1-\delta) \cdot \begin{bmatrix} \pi^g \\ \pi^b \end{bmatrix},$$

and $U_t^{\mathsf{off}}(\theta_t) = U^{\mathsf{off}}(\theta_t)$ is as calculated in the proof of Proposition 3.

Whence, there exists such an informative dynamic trading equilibrium where dealer offers distinct prices and trade occurs in both good times and bad times if equation (32) and equation (33) hold. Equivalently, solving $U^{\mathsf{IDTE}}(\theta_t)$ and $U^{\mathsf{off}}(\theta_t)$ shows that $sfIDTE$ exists if there exists $\pi^b \in [0, \frac{\rho\sigma_b^2}{2}]$ such that

$$(1 - \frac{\delta(1-\alpha_g)}{(1-\delta)(1-\delta(\alpha_g - \alpha_b))})\pi^b \leq \frac{\rho\sigma_g^2}{2} - J_g + J_b \leq [1 + \delta\frac{1-\delta\alpha_g - (1-\delta)\alpha_b}{(1-\delta)(1-\delta(\alpha_g - \alpha_b))})]\pi^b.$$

$\square$

Appendix-Corollary 1. *If $\alpha_g \geq \alpha_b$, then whenever an* IDTE *exists, there also exists an* ODTE.

Proof. Consider the equilibrium σ^{ODTE} constructed in the proof of Proposition 3, we show that given there exists an IDTE, then σ^{ODTE} is also an equilibrium. From the proof we know that we only need to check that condition (24) is satisfied:

$$U^{\mathsf{ODTE}}(b, \theta_{t-1}) \geq (1-\delta) \times 0 + \delta[\alpha_b U^{\mathsf{off}}(g) + (1-\alpha_b) U^{\mathsf{off}}(b)]$$

where $\forall \theta_t, \theta_{t-1}$,

$$\begin{aligned} U^{\mathsf{ODTE}}(\theta_t, \theta_{t-1}) &= (1-\delta)(\varphi a_{t-1} + J_{\theta_t} - p_t^{*\mathsf{UI}}(\theta_{t-1})) + \delta[\alpha_{\theta_t} U^{\mathsf{ODTE}}(g, \theta_t) + (1-\alpha_{\theta_t}) U^{\mathsf{ODTE}}(b, \theta_t)] \\ U^{\mathsf{off}}(\theta_t) &= (1-\delta)\mathbf{1}(\theta_t = g) \cdot \frac{\rho}{2}\sigma_g^2 + \delta[\alpha_{\theta_t} U^{\mathsf{off}}(g) + (1-\alpha_{\theta_t}) U^{\mathsf{off}}(b)]. \end{aligned}$$

are dealer's on-path and off-path continuous payoffs respectively, given the current and last economic states are θ_t and θ_{t-1}.

In fact, suppose σ is such an IDTE equilibrium. At any on-path history h^t, denote dealer's continuous payoff in this IDTE as $\hat{U}_t(\theta_t, h^{t-1})$. If the current period is in good times, she should have no incentive to deviate to pretend it is in bad times and offer a bad time price $p_t^b(h^{t-1})$. That is,

$$\hat{U}_t(g, h^{t-1}) \geq (1-\delta)(\varphi a_{t-1} + J_g - p_t^b(h^{t-1})) + \delta[\alpha_g U^{\mathsf{off}}(g) + (1-\alpha_g) U^{\mathsf{off}}(b)]$$

where $\forall \theta_t$

$$\hat{U}_t(\theta_t, h^{t-1}) = (1-\delta)(\varphi a_{t-1} + J_{\theta_t} - p_t^{\theta_t}(h^{t-1})) + \delta[\alpha_{\theta_t} \hat{U}_t(g, h^t) + (1-\alpha_{\theta_t}) \hat{U}_t(b, h^t)]$$

are dealer's on-path continuous payoffs for equilibrium σ and $U_t^{\mathsf{off}}(\theta_t)$ are the same off-path continuous payoff as those in the ODTE, given the current economic state is θ_t.

From the proof of Proposition 1 we learned that ρ^{IDTE} reaches its lower bound when the following condition is binding:

$$U^{\mathsf{IDTE}}(g) \geq (1-\delta)(\pi^b + J_g - J_b) + \delta[\alpha_g U^{\mathsf{off}}(g) + (1-\alpha_g) U^{\mathsf{off}}(b)],$$

where $\pi^b = \varphi a_{t-1} + J_b - p_t^b(h^{t-1})$ and $p_t^g(h^{t-1}) = \varphi a_{t-1} + J_g - \frac{\rho}{2}\sigma_g^2$, so $\hat{U}_t(g, h^{t-1}) = U^{\mathsf{IDTE}}(g)$.

Therefore, we only need to show that

$$\begin{aligned}
& U^{\mathsf{ODTE}}(b,\theta_{t-1}) - \delta[\alpha_b U^{\mathsf{off}}(g) + (1-\alpha_b)U^{\mathsf{off}}(b)] \\
\geq\ & U^{\mathsf{IDTE}}(g) - (1-\delta)(\pi^b + J_g - J_b) - \delta[\alpha_g U^{\mathsf{off}}(g) + (1-\alpha_g)U^{\mathsf{off}}(b)] \qquad (34)
\end{aligned}$$

In fact,

$$\begin{aligned}
& U^{\mathsf{ODTE}}(b,\theta_{t-1}) - \delta[\alpha_b U^{\mathsf{off}}(g) + (1-\alpha_b)U^{\mathsf{off}}(b)] \\
- & U^{\mathsf{IDTE}}(g) + (1-\delta)(\pi^b + J_g - J_b) + \delta[\alpha_g U^{\mathsf{off}}(g) + (1-\alpha_g)U^{\mathsf{off}}(b)] \\
= & U^{\mathsf{ODTE}}(b,\theta_{t-1}) - U^{\mathsf{IDTE}}(g) + (1-\delta)(\pi^b + J_g - J_b) + \delta(\alpha_g - \alpha_b)(U^{\mathsf{off}}(g) - U^{\mathsf{off}}(b))
\end{aligned}$$

$$\begin{aligned}
= & (1-\delta)\Lambda^{b,\theta_{t-1}} + \delta[\alpha_b U^{\mathsf{ODTE}}(g,\theta_{t-1}) + (1-\alpha_b)U^{\mathsf{ODTE}}(b,\theta_{t-1})] + (1-\delta)(\pi^b + J_g - J_b) \\
& -(1-\delta)\pi^b - \delta[\alpha_g U^{\mathsf{IDTE}}(g,\theta_{t-1}) + (1-\alpha_g)U^{\mathsf{IDTE}}(b,\theta_{t-1})] + \delta(\alpha_g - \alpha_b)(U^{\mathsf{off}}(g) - U^{\mathsf{off}}(b)) \\
= & (1-\delta)(\Lambda^{b,\theta_{t-1}} - \pi^b + \pi^b + J_g - J_b) + \delta(\alpha_g - \alpha_b)[(U^{\mathsf{off}}(g) - U^{\mathsf{off}}(b)) - (U^{\mathsf{IDTE}}(g) - U^{\mathsf{IDTE}}(b))] \\
& +\delta[\alpha_b U^{\mathsf{ODTE}}(g,\theta_{t-1}) + (1-\alpha_b)U^{\mathsf{ODTE}}(b,\theta_{t-1}) - \alpha_b U^{\mathsf{IDTE}}(g) - (1-\alpha_b)U^{\mathsf{IDTE}}(b)] \qquad (35) \\
\geq & 0
\end{aligned}$$

To see the last inequality, notice that

$$\begin{aligned}
& \Lambda^{b,\theta_{t-1}} - \pi^b + \pi^b + J_g - J_b \\
= & -\alpha_{\theta_{t-1}}(J_g - J_b) + \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2] + J_g - J_b \\
= & (1-\alpha_{\theta_{t-1}})(J_g - J_b) + \frac{\rho}{2}[\alpha_{\theta_{t-1}}(1-\alpha_{\theta_{t-1}})(J_g - J_b)^2 + \alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2] \\
\geq & 0;
\end{aligned}$$

$$
\begin{aligned}
& U^{\mathsf{off}}(g) - U^{\mathsf{off}}(b) - U^{\mathsf{IDTE}}(g) + U^{\mathsf{IDTE}}(b) \\
= \; & (1-\delta)\left\{[1,-1]\cdot\begin{bmatrix}1-\delta\alpha_g & -\delta(1-\alpha_g)\\ -\delta\alpha_b & 1-\delta(1-\alpha_b)\end{bmatrix}^{-1}\cdot\begin{bmatrix}\pi^g\\ 0\end{bmatrix} - [1,-1]\cdot\begin{bmatrix}1-\delta\alpha_g & -\delta(1-\alpha_g)\\ -\delta\alpha_b & 1-\delta(1-\alpha_b)\end{bmatrix}^{-1}\cdot\begin{bmatrix}\pi^g\\ \pi^b\end{bmatrix}\right\} \\
= \; & \frac{1-\delta}{(1-\delta)(1-\delta(\alpha_g-\alpha_b))}\left\{[1,-1]\cdot\begin{bmatrix}1-\delta(1-\alpha_b) & \delta(1-\alpha_g)\\ \delta\alpha_b & 1-\delta\alpha_g\end{bmatrix}\cdot\begin{bmatrix}0\\ -\pi^b\end{bmatrix}\right\} \\
= \; & \frac{-\pi^b}{1-\delta(\alpha_g-\alpha_b)}\left\{[1,-1]\cdot\begin{bmatrix}\delta(1-\alpha_g)\\ 1-\delta\alpha_g\end{bmatrix}\right\} \\
= \; & \frac{(1-\delta)\pi^b}{1-\delta(\alpha_g-\alpha_b)} \\
\geq \; & 0;
\end{aligned}
$$

and the non-negativity of the last part in (35) is due to the fact that in opaque equilibrium the dealer can extract more information rent and gain a higher ex-ante expected payoff along the equilibrium path. Therefore, we just show that condition (34) holds and complete the proof. □

Appendix-Proposition 2. *If an* IDTE *exists, then the expected social welfare in period t is*

$$W^{\mathsf{IDTE}} = \frac{\rho}{2}[\alpha_{\theta_{t-1}}\sigma_g^2 + (1-\alpha_{\theta_{t-1}})\sigma_b^2].$$

Moreover, if the dealer is pricing discriminatingly, then this is the most social welfare one can get.

Proof. As in the proof of Proposition 8, in any informative equilibrium, as long as there is trade in bad times, trader's ex-post utility becomes $p_t^b - \varphi a_{t-1} - J_b + \frac{\rho}{2}\sigma_b^2$, while dealer's ex-post utility is $\varphi a_{t-1} + J_b - p_t^b$. Therefore, in any informative equilibrium, the social welfare in bad times is $\frac{\rho}{2}\sigma_b^2$, providing there is trade. Therefore, IDTE provides the maximal social welfare in all informative equilibria (the ones where the dealer prices discriminatingly), which is $\frac{\rho}{2}\sigma_g^2$ in good times and $\frac{\rho}{2}\sigma_b^2$ in bad times. □

Appendix-Proposition 3. *If for certain parameters and the auditing intensity q, there exists an* IDTE, *then for any $q' \in [q,1]$, there also exists an* IDTE.

Proof. Similarly as in the proof of Proposition 7, we can rewrite the off-path continuation payoffs as $U_t^{\mathsf{off}}(\theta_t;q)$, given the current economic state is θ_t and the auditing intensity is q. The recursive formula for this off-path continuation payoff is

$$U_t^{\mathsf{off}}(\theta_t;q) \;=\; (1-\delta)\mathbf{1}(\theta_t=g)\cdot\frac{\rho}{2}\sigma_g^2 + \delta(1-q)[\alpha_{\theta_t}U_t^{\mathsf{off}}(g;q) + (1-\alpha_{\theta_t})U_t^{\mathsf{off}}(b;q)].$$

Since if the dealer follows the IDTE strategy, she will never be punished, the on-path continuation payoff remains the same:

$$U_t^{\mathsf{IDTE}}(\theta_t) \quad = \quad (1-\delta)\pi^{\theta_t} + \delta[\alpha_{\theta_t} U_{t+1}^{\mathsf{IDTE}}(g) + (1-\alpha_{\theta_t}) U_{t+1}^{\mathsf{IDTE}}(b)].$$

The IC constraint becomes

$$\begin{aligned}
U_t^{\mathsf{IDTE}}(g) \quad &\geq \quad (1-\delta)(\pi^b + J_g - J_b) + \delta(1-q)[\alpha_g U_t^{\mathsf{off}}(g;q) + (1-\alpha_g) U_t^{\mathsf{off}}(b;q)]; \\
U_t^{\mathsf{IDTE}}(b) \quad &\geq \quad (1-\delta)(\pi^b + J_b - J_g) + \delta(1-q)[\alpha_b U_t^{\mathsf{off}}(g;q) + (1-\alpha_b) U_t^{\mathsf{off}}(b;q)].
\end{aligned}$$

A similar argument can show that the off-path continuation payoff $U_t^{\mathsf{off}}(\theta_t; q)$ is decreasing in q. Therefore, with a higher auditing intensity, the IC constraint is relaxed, which completes the proof. $\square$